Elastodynamic plane wave Marchenko redatuming: Theory and examples

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Summary

The Marchenko method is capable to create virtual sources inside a medium that is only accessible from an open-boundary (Broggini et al., 2012; Van der Neut et al., 2015). The resulting virtual data can be used to retrieve images free of artefacts caused by internal multiples. Conventionally, the Marchenko method retrieves a so-called focusing wavefield that focuses the data from the recording surface to a point inside the medium. Recently, Meles et al. (2017) suggested to modify the focusing condition such that the new focusing wavefield creates a virtual plane wave source inside the medium, instead of a virtual point source. The virtual plane wave data can be used to image an entire surface inside the medium in a single step rather than imaging individual points on the surface. Consequently, the imaging process is accelerated significantly. We provide an extension of plane wave Marchenko redatuming for elastodynamic waves and demonstrate its performance numerically.

Introduction

In many imaging applications the medium of interest is observed using reflection measurements acquired on an open-boundary. Recently, a novel imaging technique, the Marchenko method, has been developed with the aim to retrieve images free of artefacts caused by internal multiples. The conventional Marchenko method creates a virtual point source inside the medium, accounting for primary as well as multiply scattered waves (Broggini et al., 2012; Van der Neut et al., 2015). From the virtual response an image at the virtual source location is computed (e.g. Behura et al., 2014). Hence, the imaging process is performed point-wise. Meles et al. (2017) combine the areal-source methods for primaries by Rietveld et al. (1992) with the spatially-extended virtual source Marchenko method presented by Broggini et al. (2012) to create a virtual plane wave source at an arbitrary surface inside the medium. Using the virtual plane wave response the entire surface can be imaged in one step rather than imaging each point on the surface individually. We extend plane wave Marchenko redatuming for elastodynamic waves, analogous to the elastodynamic extension of conventional Marchenko redatuming by Wapenaar (2014).

Elastodynamic plane wave Marchenko redatuming: Theory

First, we will introduce the conventional elastodynamic single-sided Green's function representations that allow to create virtual point sources inside a medium. For a detailed derivation we refer to Wapenaar (2014). Second, we will modify these equations such that they create virtual plane wave sources instead of virtual point sources. Consider an elastic medium without losses. Suppose the medium has infinite lateral extent and is bounded by a reflection-free surface $\partial\mathbb{D}_0$ at the top. Moreover, we consider elastodynamic power-flux normalised wavefield potentials. The single-sided Green's function representations can be written as,

$$\mathbf{G}^{-,+}(\mathbf{x}',\mathbf{x}_f,\omega) = \int_{\partial\mathbb{D}_0} \mathbf{R}(\mathbf{x}',\mathbf{x},\omega)\mathbf{F}^{+}(\mathbf{x},\mathbf{x}_f,\omega)\mathrm{d}^2\mathbf{x} - \mathbf{F}^{-}(\mathbf{x}',\mathbf{x}_f,\omega), \tag{1}$$

$$\{\mathbf{G}^{-,-}(\mathbf{x}',\mathbf{x}_f,\omega)\}^{*} = \int_{\partial\mathbb{D}_0} \mathbf{R}^{*}(\mathbf{x}',\mathbf{x},\omega)\mathbf{F}^{-}(\mathbf{x},\mathbf{x}_f,\omega)\mathrm{d}^2\mathbf{x} - \mathbf{F}^{+}(\mathbf{x}',\mathbf{x}_f,\omega). \tag{2}$$

Here, the one-way Green's functions $\mathbf{G}^{-,\pm}(\mathbf{x}',\mathbf{x}_f,\omega)$ are 3×3 matrices,

$$\mathbf{G}^{-,\pm}(\mathbf{x}',\mathbf{x}_f,\omega) = \begin{pmatrix} G^{-,\pm}_{\Phi,\Phi} & G^{-,\pm}_{\Phi,\Psi} & G^{-,\pm}_{\Phi,\Upsilon} \\ G^{-,\pm}_{\Psi,\Phi} & G^{-,\pm}_{\Psi,\Psi} & G^{-,\pm}_{\Psi,\Upsilon} \\ G^{-,\pm}_{\Upsilon,\Phi} & G^{-,\pm}_{\Upsilon,\Psi} & G^{-,\pm}_{\Upsilon,\Upsilon} \end{pmatrix}(\mathbf{x}',\mathbf{x}_f,\omega), \tag{3}$$

where the superscript "$+$" describes downgoing waves, the superscript "$-$" describes upgoing waves. The subscripts Φ, Ψ and Υ represent P-, S1- and S2-wavefield potentials, respectively. The first super- and subscripts refer to the wavefield at the receiver position $\mathbf{x}'$ on $\partial\mathbb{D}_0$, the second super- and subscripts refer to the wavefield at the source position $\mathbf{x}_f$ inside the medium. The spatial coordinates and the frequency are denoted as $\mathbf{x} = (x_1,x_2,x_3)^T$ and ω, respectively. The superscript "T" denotes a transpose and the superscript "$*$" denotes a complex-conjugate. The quantity $\mathbf{R}(\mathbf{x}',\mathbf{x},\omega)$ is the reflection response of the medium recorded on $\partial\mathbb{D}_0$. The focusing function $\mathbf{F}^{\pm}(\mathbf{x},\mathbf{x}_f,\omega)$ is defined in a truncated medium which is identical to the physical medium between $\partial\mathbb{D}_0$ and $\partial\mathbb{D}_f$ ($x_3 = x_{3,f}$) but reflection-free above $\partial\mathbb{D}_0$ and below $\partial\mathbb{D}_f$. We formulate the focusing functions in the space-time domain to emphasise its spatial and temporal behaviour. The downgoing part of the focusing function $\mathbf{F}^{+}(\mathbf{x}',\mathbf{x}_f,t)$ is the inverse of the transmission response $\mathbf{T}^{+}(\mathbf{x},\mathbf{x}',t)$ of the truncated medium (Wapenaar et al., 2016b),

$$\int_{-\infty}^{t}\int_{\partial\mathbb{D}_0} \mathbf{T}^{+}(\mathbf{x},\mathbf{x}',t-t')\mathbf{F}^{+}(\mathbf{x}',\mathbf{x}_f,t')\,\mathrm{d}^2\mathbf{x}'_H\mathrm{d}t'\bigg|_{x_3=x_{3,f}} = \delta(t)\,\delta(\mathbf{x}_H-\mathbf{x}_{H,f})\,\mathbf{I}. \tag{4}$$

The subscript "H" refers to the horizontal coordinates $\mathbf{x}_H = (x_1,x_2)^T$ and $\mathbf{I}$ is an identity matrix of appropriate size. From Eq. 4 follows that the downgoing focusing function satisfies the focusing condition,

$$\mathbf{F}^{+}(\mathbf{x},\mathbf{x}_f,t)\big|_{x_3=x_{3,f}} = \delta(t)\,\delta(\mathbf{x}_H-\mathbf{x}_{H,f})\,\mathbf{I}. \tag{5}$$

Thus, the focusing function $\mathbf{F}(\mathbf{x},\mathbf{x}_f,t)$ focuses in time and in space. The upgoing part of the focusing function $\mathbf{F}^-(\mathbf{x},\mathbf{x}_f,t)$ is the reflection response of the downgoing focusing function in the truncated medium. In physical interpretation the single-sided Green's function representations (Eqs. 1 and 2) can be understood as follows. The focusing function focuses, or inverse propagates, the source-side of the reflection response $\mathbf{R}(\mathbf{x}',\mathbf{x},\omega)$ from the recording surface $\partial\mathbb{D}_0$ to the focusing point $\mathbf{x}_f$ inside the medium. Hence, a virtual source is created inside the medium at $\mathbf{x}_f$.
Following the acoustic plane wave Marchenko redatuming by Meles et al. (2017), we suggest to define a modified focusing function $\bar{\mathbf{F}}^{\pm}(\mathbf{x},\mathbf{p}_H,x_{3,f},t)$ that focuses as a plane wave in time but not in space. Therefore, the modified downgoing focusing function obeys the modified focusing condition,

$$\bar{\mathbf{F}}^{+}(\mathbf{x},\mathbf{p}_H,x_{3,f},t)\big|_{x_3=x_{3,f}} = \delta(t-\mathbf{p}_H\cdot\mathbf{x}_H)\,\mathbf{I}, \tag{6}$$

where $\mathbf{p}_H=(p_1,p_2)^T$ denotes the horizontal ray-parameter. The conventional and the modified focusing conditions in Eqs. 5 and 6 are very similar but the temporal focus $\delta(t)$ is replaced by an offset-dependent focus in time $\delta(t-\mathbf{p}_H\cdot\mathbf{x}_H)$ and the spatial focus $\delta(\mathbf{x}_H-\mathbf{x}_{H,f})$ is removed. In the space-frequency domain, we obtain the modified focusing function $\bar{\mathbf{F}}^{\pm}(\mathbf{x},\mathbf{p}_H,x_{3,f},t)$ by multiplying the focusing function $\mathbf{F}^{\pm}(\mathbf{x},\mathbf{x}_f,\omega)$ by the plane wave $\mathrm{e}^{\mathrm{j}\omega\mathbf{p}_H\cdot\mathbf{x}_{H,f}}$ and by integrating the result over the focusing surface $\partial\mathbb{D}_f$,

$$\bar{\mathbf{F}}^{\pm}(\mathbf{x},\mathbf{p}_H,x_{3,f},\omega) = \int_{\partial\mathbb{D}_f} \mathbf{F}^{\pm}(\mathbf{x},\mathbf{x}_{H,f},x_{3,f},\omega)\,\mathrm{e}^{\mathrm{j}\omega\mathbf{p}_H\cdot\mathbf{x}_{H,f}}\mathrm{d}^2\mathbf{x}_{H,f}. \tag{7}$$

Further, we define the plane wave responses $\bar{\mathbf{G}}^{-,\pm}(\mathbf{x},\mathbf{p}_H,x_{3,f},\omega)$ that are associated to a plane wave source $\delta(t-\mathbf{p}_H\cdot\mathbf{x}_{H,f})$ at $\partial\mathbb{D}_f$ and recorded at $\mathbf{x}$ on $\partial\mathbb{D}_0$,

$$\bar{\mathbf{G}}^{-,\pm}(\mathbf{x},\mathbf{p}_H,x_{3,f},\omega) = \int_{\partial\mathbb{D}_f} \mathbf{G}^{-,\pm}(\mathbf{x},\mathbf{x}_{H,f},x_{3,f},\omega)\,\mathrm{e}^{\mathrm{j}\omega\mathbf{p}_H\cdot\mathbf{x}_{H,f}}\mathrm{d}^2\mathbf{x}_{H,f}. \tag{8}$$

Next, we multiply Eqs. 1 and 2 by $\mathrm{e}^{\mathrm{j}\omega\mathbf{p}_H\cdot\mathbf{x}_{H,f}}$, integrate the result over the focusing surface $\partial\mathbb{D}_f$, use Eqs. 7 and 8 and find the modified single-sided Green's function representations,

$$\bar{\mathbf{G}}^{-,+}(\mathbf{x}',\mathbf{p}_H,x_{3,f},\omega) = \int_{\partial\mathbb{D}_0} \mathbf{R}(\mathbf{x}',\mathbf{x},\omega)\bar{\mathbf{F}}^{+}(\mathbf{x},\mathbf{p}_H,x_{3,f},\omega)\mathrm{d}^2\mathbf{x} - \bar{\mathbf{F}}^{-}(\mathbf{x}',\mathbf{p}_H,x_{3,f},\omega), \tag{9}$$

$$\{\bar{\mathbf{G}}^{-,-}(\mathbf{x}',-\mathbf{p}_H,x_{3,f},\omega)\}^* = \int_{\partial\mathbb{D}_0} \mathbf{R}^*(\mathbf{x}',\mathbf{x},\omega)\bar{\mathbf{F}}^{-}(\mathbf{x},\mathbf{p}_H,x_{3,f},\omega)\mathrm{d}^2\mathbf{x} - \bar{\mathbf{F}}^{+}(\mathbf{x}',\mathbf{p}_H,x_{3,f},\omega). \tag{10}$$

Eqs. 9 and 10 are nearly equivalent to the conventional single-sided Green's function representations (Wapenaar, 2014). However, the focusing function $\mathbf{F}$ is replaced by a modified version $\bar{\mathbf{F}}$ that focuses in time but not in space. Besides, the Green's functions $\mathbf{G}^{-,\pm}$ are replaced by the virtual plane wave responses $\bar{\mathbf{G}}^{-,\pm}$ which are associated to a virtual plane wave source $\delta(t-\mathbf{p}_H\cdot\mathbf{x}_H)$ at $x_{3,f}$ instead of a virtual point source at $\mathbf{x}_f$.
The modified single-sided Green's function representations (Eqs. 9 and 10) are an underdetermined system that can only be solved if a separation operator exists. For the acoustic case, Meles et al. (2017) postulate that such an operator exists, and demonstrate its performance numerically. The (acoustic) separation operator is based on the kinematics of a direct transmission associated to plane wave source at the focusing depth and recorded at the surface $\partial\mathbb{D}_0$. Correspondingly, we hypothesise that in the elastodynamic case a separation operator $\bar{\mathbf{W}}$ exists. The separation operator $\bar{\mathbf{W}}$ is based on the kinematics of a forward-scattered transmission $\bar{\mathbf{T}}^{-}_{fs}(\mathbf{x}',\mathbf{p}_H,x_{3,f},\omega)$ associated to a plane wave source defined by the horizontal ray-parameter $\mathbf{p}_H$ at $\partial\mathbb{D}_f$ and recorded at $\partial\mathbb{D}_0$. Now we define the modified operator $\bar{\mathbf{W}}$,

$$\bar{W}_{\Phi,\Phi}(\mathbf{x}',\mathbf{p}_H,x_{3,f},t) = H(t^d_{\Phi,\Phi}-\varepsilon-t+\mathbf{p}_H\cdot\mathbf{x}'_H) - H(-t^d_{\Phi,\Phi}+\varepsilon-t+\mathbf{p}_H\cdot\mathbf{x}'_H). \tag{11}$$

For better readability we only defined one component of the modified operator $\bar{\mathbf{W}}$. The remaining elements are defined analogously. The function $H()$ is the Heaviside function. The variable $t^d_{\Phi,\Phi}$ is the travel time of the first event of the plane wave response $\bar{G}^{-,-}_{\Phi,\Phi}(\mathbf{x}',\mathbf{p}_H=\mathbf{0},x_{3,f},t)$. In addition, we introduced a small positive constant ε to account for the finite width of the wavelet. Note that the modified operator $\bar{\mathbf{W}}$ is applied to the modified single-sided Green's function representations in form of a Hadamard product. Under the assumption that the operator $\bar{\mathbf{W}}(\mathbf{x}',\mathbf{p}_H,x_{3,f},t)$ exists the modified single-sided Green's function representations can be solved analogous to the conventional Marchenko scheme (Wapenaar, 2014) by replacing $\mathbf{G}^{-,\pm}(\mathbf{x}',\mathbf{x}_f,\omega)$ by $\bar{\mathbf{G}}^{-,\pm}(\mathbf{x}',\mathbf{p}_H,x_{3,f},\omega)$, $\mathbf{F}(\mathbf{x}',\mathbf{x}_f,\omega)$ by $\bar{\mathbf{F}}(\mathbf{x}',\mathbf{p}_H,x_{3,f},\omega)$, and $\mathbf{W}(\mathbf{x}',\mathbf{x}_f,t)$ by $\bar{\mathbf{W}}(\mathbf{x}',\mathbf{p}_H,x_{3,f},t)$.

Elastodynamic plane wave Marchenko redatuming: Numerical example

We evaluate the performance of the presented plane wave Marchenko redatuming for an elastic 1.5D model shown in Fig. 1a, where we use the coordinates $\mathbf{x} = (x_1, x_3)^T$. We model reflection data $\mathbf{R}(\mathbf{x}', \mathbf{x}, t)$

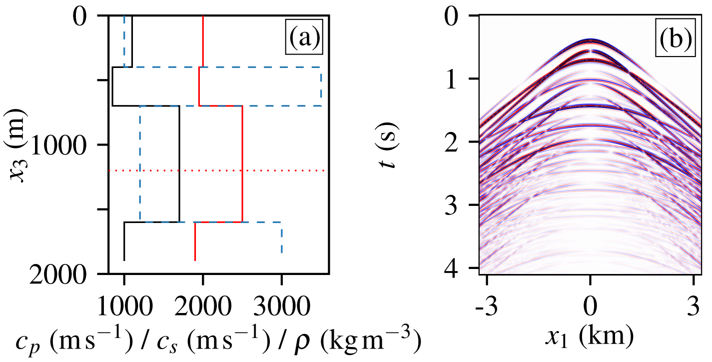

Figure 1: (a) Layered model (1.5D) with a lateral distance range from -12812.5 m to 12800 m. The model parameters are P-wave velocity c_p (red solid line), S-wave velocity c_s (black solid line) and density ρ (blue dashed line). The red dashed line indicates the focusing depth $x_{3,f} = 1200$ m. (b) Reflection response $\mathbf{R}(\mathbf{x}', \mathbf{x}, t)$. To visualise the late arrivals we applied a temporal gain $e^{0.5t}$.

for a point source (see Fig. 1b). We choose a focusing depth $x_{3,f} = 1200$ m. For better visualisation, in all figures we only show wavefields related to a P-wave source and a P-wave focus. The initial downgoing focusing function $\bar{\mathbf{F}}_0^+$ (see Fig. 2a) is computed by inverting the forward-scattered transmission response $\bar{\mathbf{T}}_{fs}^-(\mathbf{x}', p_1, x_{3,f}, t)$ related to an incident plane wave defined by the horizontal ray-parameter $p_1 = 5 \times 10^{-5}\,\mathrm{s\,m^{-1}}$. When we inject the initial downgoing focusing function $\bar{\mathbf{F}}_0^+$ on the surface $\partial\mathbb{D}_0$ in the truncated medium and record it on $\partial\mathbb{D}_f$ we observe a plane wave $\delta(t - p_1 x_1)$ plus a coda (see Fig. 2b), i.e. the initial downgoing focusing function does not focus in time on $\partial\mathbb{D}_f$. Next, we evaluate five iterations of the modified Marchenko series and use the resulting downgoing focusing function $\bar{\mathbf{F}}^+$ (see Fig. 2c) to repeat the experiment. Now we observe a temporal focus $\delta(t - p_1 x_1)$ on $\partial\mathbb{D}_f$ without a coda (see Fig. 2d), indicating that we retrieved the correct downgoing focusing function. We evaluate Eqs. 9 and 10 using the retrieved modified focusing functions to obtain the virtual plane wave responses $\bar{\mathbf{G}}^{-,\pm}(\mathbf{x}', p_1, x_{3,f}, t)$. Fig. 2e shows a superposition of $\bar{\mathbf{G}}^{-,+}(\mathbf{x}', p_1, x_{3,f}, t)$ and $\bar{\mathbf{G}}^{-,-}(\mathbf{x}', p_1, x_{3,f}, t)$. The retrieved virtual plane wave responses include primary, multiply-scattered as well as converted waves. To illustrate that the choice of the ray-parameter p_1 is arbitrary we repeat the above experiments using a different ray-parameter $p_1 = 0\,\mathrm{s\,m^{-1}}$ (see Figs. 2f-j). In this case we observe less events because at zero-incidence there are no conversions between P- and S-waves.

Conclusions

We extended the modified Marchenko redatuming by Meles et al. (2017) for elastodynamic waves. By modifying the focusing condition we obtained modified single-sided Green's function representations that allowed to create virtual plane wave sources (and receivers) inside the medium, only using the medium's reflection response recorded at an open-boundary and the forward-scattered transmission response between the recording and the focusing surfaces. The virtual plane wave responses are retrieved for single ray-parameters, i.e. they might be used for AVA inversion. Further, the virtual plane wave responses can be used to image the focusing surface $\partial\mathbb{D}_f$ in a single step. By imaging an entire surface, instead of a single point, the imaging process is accelerated significantly.

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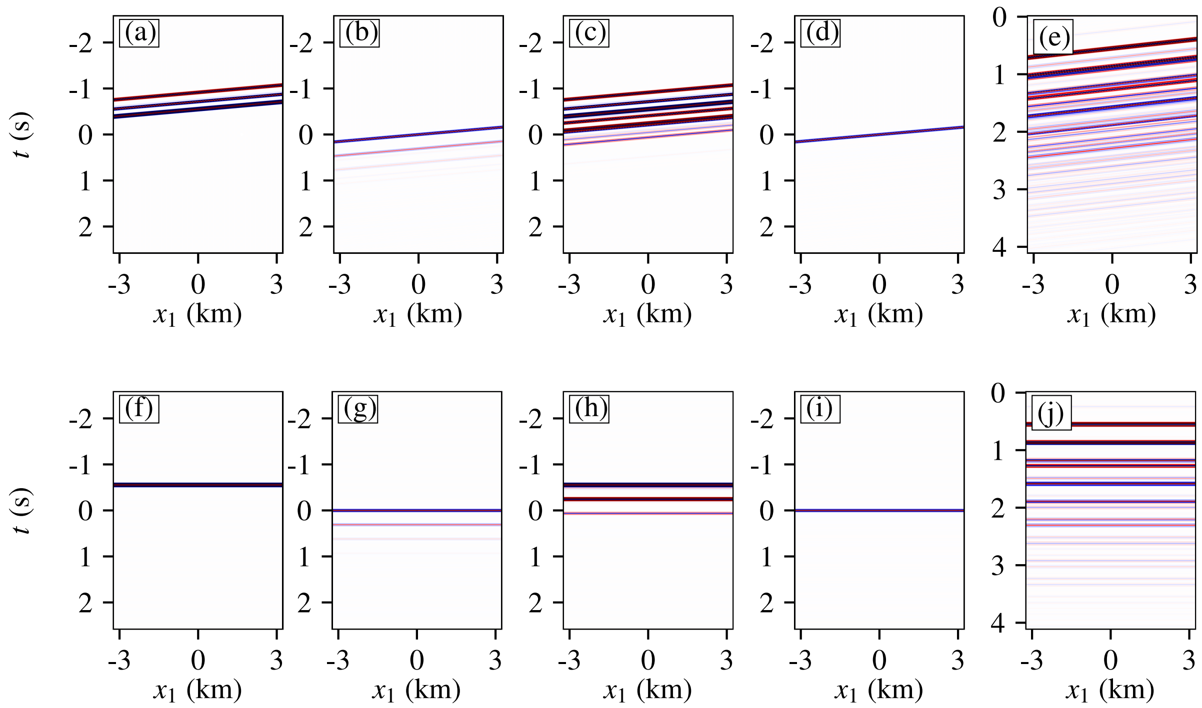

Figure 2: Initial modified focusing function $\bar{\mathbf{F}}_0^+(\mathbf{x}, p_1, x_{3,f}, t)$ for a ray-parameter $p_1 = 5 \times 10^{-5}\,\mathrm{s\,m^{-1}}$ at the surfaces (a) $\partial\mathbb{D}_0$ and (b) $\partial\mathbb{D}_f$. Modified focusing function $\bar{\mathbf{F}}^+(\mathbf{x}, p_1, x_{3,f}, t)$ at the surfaces (c) $\partial\mathbb{D}_0$ and (d) $\partial\mathbb{D}_f$. (e) Superposition of the up- and downgoing virtual plane wave responses $\bar{\mathbf{G}}^{-,\pm}(\mathbf{x}', p_1, x_{3,f}, t)$. (f-j) Repetition of (a-e) for a different ray-parameter $p_1 = 0\,\mathrm{s\,m^{-1}}$.

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