

Green's second theorem and the extraction of Green's functions

Evert Slob^{1,2}, Roel Snieder² and Kees Wapenaar¹

¹Dept of Geotechnology, Delft University of Technology.

²Dept of Geophysics, Colorado School Mines.

We derive four distinct identities for Green's functions, based on the generalized second theorem of Green. One of them is the generalized first theorem of Green, which can be used to derive the source-receiver reciprocity relation for Green's functions. Two identities give the sum or difference of the Green's function between two locations A and B as an integral sum of cross-convolutions of Green's functions from sources throughout the volume and/or over the surface, and received in A and B. Two other identities give the sum or difference of the Green's function and its time-reversed version between two locations A and B as an integral sum of cross-correlations of Green's functions from sources throughout the volume and/or over the surface, and received in A and B.

For any linear field response, we demonstrate that the difference of the Green's function between A and B and its time-reversed version can be obtained from the integral sum of cross-correlations of Green's functions for point sources on a boundary and received at A and B. The only condition is that the medium dissipates no energy. This holds for all linear fields including moving and rotating media as well as bianisotropic media. For dissipative media sources with autocorrelations equal to the dissipative part of the medium property function must be distributed throughout the volume.

For any linear field response, we demonstrate that the sum of the Green's function between A and B and its time-reversed version can be obtained from the integral sum of cross-correlations of Green's functions for point sources throughout the volume and received at A and B. The source autocorrelation is equal to the non-dissipative part of the medium property functions.

When the medium dissipates energy we show that the fluctuation-dissipation theorem can be applied to the identity for the difference of the Green's function and its time-reversed version. This directly leads to the desired noise sources, which have autocorrelations proportional to the dissipative part of the medium property functions, and with a proportionality factor equal to the thermal energy of the system. This implies that for systems in thermal equilibrium, thermal noise provides the desired sources to extract the Green's function.

Thermal noise sources cannot exist inside the non-dissipative part. If the non-dissipative medium is embedded in a dissipative medium in thermal equilibrium, equivalent thermal noise sources on the boundary exist and can be used to extract the Green's function.

We show possible applications of these identities for potential, diffusive and wave fields.