

# LAYERED MEDIUM FULL WAVEFORM INVERSION AS A TWO-STEP LINEAR PROCESS

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## Abstract

*Recently, Broggin et al. discussed several connections between wave field focusing, interferometry, and inverse scattering [1]. In different ways it is demonstrated how to construct the 1D seismic wave field measured in the homogeneous upper half space and generated by a source in the interior of an unknown medium. The input data is a reflection measurement from a source and a receiver in the homogeneous upper half space. In seismic interferometry [2] a physical receiver is necessary inside the unknown heterogeneous medium and physical sources must be present in both homogeneous half-spaces in which the heterogeneous medium is embedded. As an alternative approach focusing theory [3] is applied to reflection data to construct the wave field recorded at the receiver from a virtual source inside the heterogeneous medium [1]. Both methods do not require knowledge about medium properties other than assuming the waves propagate without dissipation. The important advantage of the second method over the first is that the location of the virtual source is obtained without having a physical receiver at that location. This creates data that is the starting point of the virtual source method of [4], but now without the need for borehole data. The data that is obtained from reflection data is a virtual Vertical Seismic Profile (VSP) data set. In a companion paper Wapenaar et al. showed how this could be extended to three-dimensional media [5]. Once the 3D wave fields are reconstructed with a virtual source in the subsurface, a dual VSP can be created that allows directional decomposition at the subsurface virtual source depth level followed by multi-dimensional deconvolution to create a 3D internal multiple-free image [6].*

*It is interesting that a number of successive data driven filter steps lead to creating wave fields as if they were generated, or measured, at locations where no physical source, or receiver, was placed and in addition lead to the construction of images free of internal multiples. The focusing theory of Rose, being a specific implementation of solving the Marchenko equation [7], is a linear data driven filter method of iteratively focusing a wave field at a specified one-way travel-time, or virtual depth level. This implies that the filter itself is a Green's function. In this talk it is shown that this filter contains the local reflection response for a source and receiver at the*

virtual depth level. For 3D wave fields in a horizontally layered medium the obtained image contains the local reflection coefficients as a function of one-way intercept time and horizontal slowness. This is a non-recursive method, because the depth levels can be chosen independently. The solution to the Marchenko equation is analyzed for plane waves incident at arbitrary angles of incidence on a horizontally layered model. The image containing only local primary reflection coefficients at correct one-way travel times is extracted from the filter solution. Creating an image is the first step in an inversion approach. Once the image is obtained with local reflection coefficient amplitudes, the electric permittivity and magnetic permeability can be obtained from inverting the reflection coefficients for a number of slowness values. This is a direct matrix inversion solution that can be obtained in the least squares sense. The inversion step is recursive, because knowledge of the upper half space medium parameters is needed to start the scheme.

**Index Terms** – Focusing, Imaging, Inversion, Migration.

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