Title (max. 16 words)

Quantitative fracture imaging using least-squares migration and linear-slip model: theory and application to single-well reflection imaging

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5th EAGE Workshop on Borehole Geophysics
7. Acoustic Sonic and Non-Seismic Methods

Summary (max. 200 words)

Characterizing subsurface fractures is a key to developing hydrocarbon and geothermal fields, as well as providing fundamental information on fracture system relevant to regional seismotectonics. Seismic characterization of fractures has generally been based on the effective medium theory, which considers seismically invisible small fractures. Therefore, there is a considerable scale gap between the fracture properties obtained by seismic methods and those from borehole logging. Recent studies of single-well reflection imaging using acoustic borehole logging data show the potential of filling the scale gap by providing fracture properties around the borehole up to a few tens of meter away from the borehole location. In the context of reflection imaging of individual fractures, in this study, we develop least-squares migration (LSM) coupled with linear-slip model. LSM solves the linearized waveform inversion to provide high-resolution quantitative images. Linear-slip model can describe wave reflection at a fracture accurately. We show numerical modelling examples of the proposed approach considering a vertical fracture with coupling compliances, and acoustic dipole measurements of a dipping fracture embedded in a background random velocity distribution. The results show that the proposed LSM provides higher resolution images than reverse-time migration, and more accurate images than the conventional LSM without linear-slip model.

Introduction

Recent developments of single-well reflection imaging using acoustic logging data show the potential of detailed imaging of structures around a borehole. For example, dipole acoustic data successfully detect the azimuth angle of dipping reflectors, and also can image structures around a borehole up to distance of a few tens of meters from the borehole location (Lee et al., 2019). This earlier studies suggest the potential of filling the scale gap between the surface seismic, VSP, and borehole sonic logging.

Seismic characterization of fractures is crucial for hydrocarbon exploration and geothermal field development. Characterization of fractures using seismic anisotropy is based on the effective medium theory assuming seismically invisible small fractures. We argue that the use of a single-well reflection imaging technique with frequency range between 2kHz – 10kHz will enable locating individual fractures, joints and faults around a borehole as seismically visible, thin structures. When such seismically-thin structures are considered as individual scatterers and reflectors, the linear-slip interface model (Schoenberg, 1980) is useful to relate the seismic scattering to the mechanical compliances of the thin structures. The mechanical compliances reflect the small-scale properties at the structure, e.g., surface shape, contact asperities, and fracture infill materials (e.g., Worthington and Hudson, 2000). Our recent laboratory experiments (Minato et al., 2018a) successfully characterize the fluid-filled, spatially varying fracture using wave reflection and linear-slip model.

Toward quantitative, high-resolution imaging of thin structures/fractures around a borehole, in this study, we explore the potential of the least-squares migration (e.g., Chen and Sacchi, 2017, and references therein) coupled with the linear-slip interface model. The least-squares migration (LSM) solves linearized waveform inversion, and offers high-resolution, quantitative images. We present numerical modelling results considering a vertical fracture having coupling compliances, and a dipping fracture in the dipole acoustic logging geometry, in order to show the advantages of the new LSM over the conventional LSM and the reverse-time migration (RTM).

Elastic least-squares migration coupling linear-slip model

The linear slip boundary condition at a horizontal fracture can be written as,

$$\Delta u_j = Z_{ij}\tau_{zi} \qquad (i, j = x, y, z), \tag{1}$$

where Δu_j is a displacement discontinuity across the fracture, τ_{zi} is a traction vector acting on the fracture plane, and Z_{ij} is the fracture compliance tensor. Z_{ij} contains small-scale properties at the fracture, e.g., fracture aperture, contact asperity distribution, and elasticity of fracture infill materials (Worthington and Hudson, 2000). The simplest form of Z_{ij} is a diagonal matrix with rotational symmetry (Schoenberg, 1980), i.e., $\mathbf{Z} = diag(\eta_T, \eta_T, \eta_N)$, where η_T is the tangential compliance and η_N is the normal compliance. The off-diagonal components in **Z** or coupling compliances reflect the shape of the fracture surface, and they are sensitive to the shear-induced coupling changes (Nakagawa et al., 2000).

For high-resolution, quantitative reflection imaging we propose a new LSM coupling - the linear-slip interface model - by considering the linearized equation:

$$\mathbf{d} = \mathbf{Lm}, \tag{2}$$

where **d** contains reflection waveform data, **m** the fracture compliances, and **L** a Born approximation operator. We derive equation (2) using the general formulation for the boundary integral representation of seismic wavefield including linear-slip interfaces (Wapenaar, 2007). We consider a 2D problem, i.e., P-SV and SH wavefields, but an extension to 3D should be straightforward. More details of equation (2) for P-SV wavefield can be found in Minato et al. (2018b). In the case of P-SV wavefield, the data vector **d** contains a particle velocity vector (v_x and/or v_z), and the compliance matrix becomes $Z_{xx} = \eta_T$, $Z_{zz} = \eta_N$, and $Z_{xz} = Z_{zx} = \eta_C$ where η_C is the coupling compliance. For the 2D SH wavefield, the data vector **d** contains the horizontal particle velocity (v_y), and the fracture compliance matrix becomes a scalar function of η_T. Elastic least-squares migration (LSM) involves solving equation (2) for **m**. In this study, we solve equation (2) in the frequency domain using the conjugate gradient method. Note that we construct the full matrix **L** which requires a large memory storage. It is also possible to use matrix-free approach to solve equation (2), e.g., using least-squares reverse-time migration (e.g., Chen and Sacchi, 2017).

Numerical modelling results: a vertical fracture

We first show how the proposed elastic LSM works when a vertical fracture is represented by linear-slip interface with coupling compliance. We consider the experimental configuration (Figure 1a) similar to our previous ultrasonic laboratory experiments (Minato et al., 2018a). We install two sources in order to consider uneven illumination of the fracture from sources (Figure 1a). The compliance values along the fracture (Figure 1b) are taken from the laboratory experiments (Lubbe et al., 2008) assuming the relative magnitude of the coupling compliance (Nakagawa et al., 2000) to be 0.5. The example of the calculated shot gather is shown in Figure1c where we assume horizontal force sources, i.e., f_x in P-SV wavefield and f_y in SH wavefield.

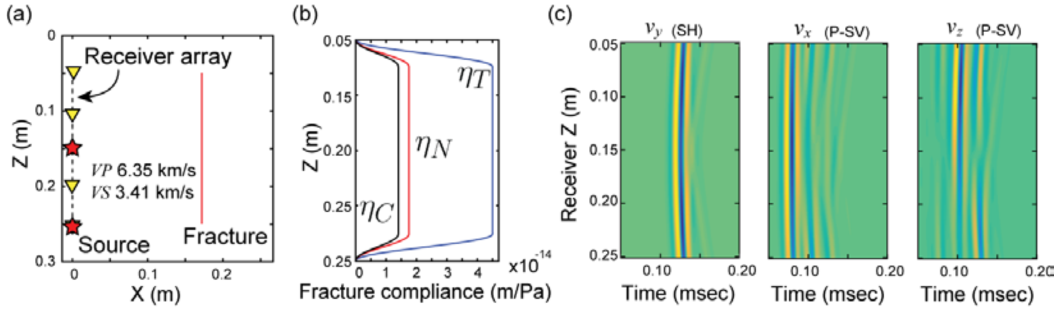

Figure 1 *(a) Configuration of sources, receivers and a fracture. (b) Fracture compliances. (c) Example of modelled common source gathers.*

The imaging results using SH wavefield are shown in Figure 2. Compared to the result of RTM (Figure 2a) or that using the conjugate operator $\mathbf{L}^{\dagger}\mathbf{d}$, the proposed LSM (Figure 2b) using the conjugate gradient method (CGLS) improves the imaging result by suppressing the migration artefacts. Furthermore, LSM estimates the quantitative values of the fracture compliance. The derived compliance (Figure 2b) is smaller than the true value; there are three main reasons: (1) Born approximation overpredicts amplitudes and underestimates compliance, (2) the conjugate gradient method estimates the fracture compliances including negative values, and (3) due to the limited bandwidth, a single interface with true compliance is represented almost equally well by multiple interfaces with smaller compliances. In order to mitigate the effect of the second reason, we constrain the solution of **m** to be only positive compliance values in solving equation (2). To this end, we use the Krylov subspace based approach (Gazzola and Wiaux, 2017). The result (Figure 2c) shows that the additional constraint greatly improves the resolution and, consequently, almost the true compliance values are obtained; the fracture is represented nearly as a single interface. Figure 2d is the imaging result using the conventional LSM for the volumetric distribution of elastic constant (μ), e.g., see Beydoun and Mendes (1989). We see that the result of the new LSM (Figure 2c) shows similar resolution to the conventional LSM (Figure 2d), which corresponds to the fact that the linear-slip interface model includes a thin layer of isotropic material in the long wavelength assumption ($\eta_T = h/\mu$, where h is the layer thickness). We conclude that in the case of SH wavefield illuminating a vertical fracture, the new LSM gives high-resolution information of the fracture compliance.

The imaging results using P-SV wavefield (Figure 3) show distinct differences between the proposed LSM (Figure 3a) and the conventional LSM (Figure 3b). This is because the coupling fracture compliance produces converted shear waves due to normally incident P wave (Nakagawa et al., 2000), which may not be properly described by a thin layer of isotropic material. In this case, the proposed LSM gives accurate imaging results than conventional LSM, and provides detailed insights on the mechanical compliance of the fracture.

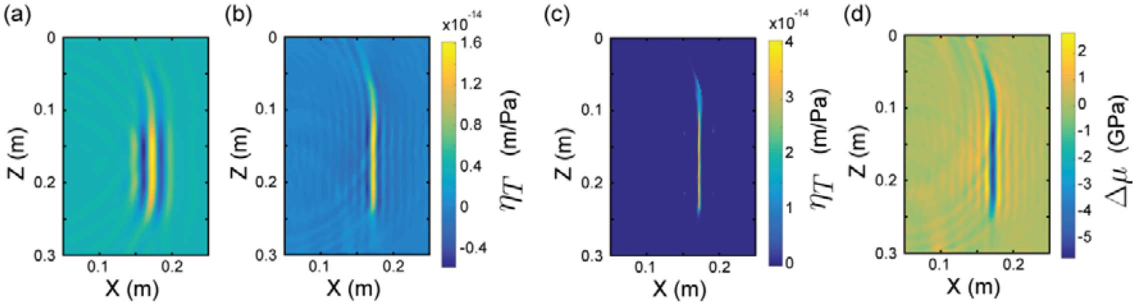

Figure 2 *Imaging results for SH waves using (a) RTM, (b) newly proposed LSM, (c) newly proposed LSM with positive-compliance constraint, and (d) conventional LSM.*

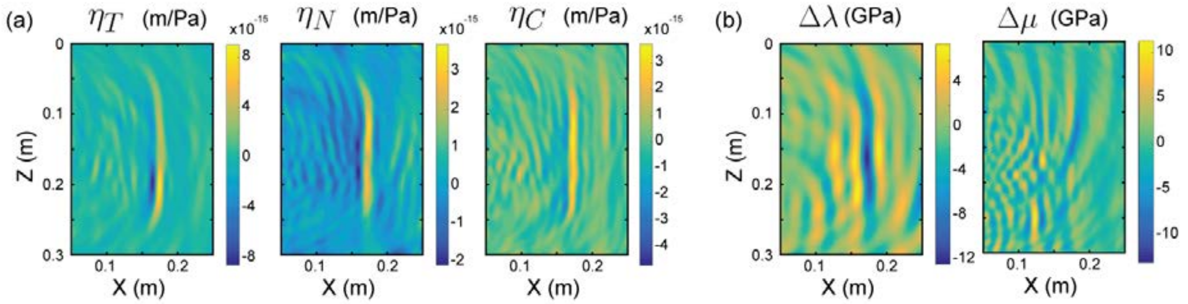

Figure 3 *(a) Imaging results for P-SV waves using newly proposed LSM, and (b) conventional LSM.*

Numerical modelling results: Acoustic logging geometry

We consider the application of the proposed LSM to the dipole acoustic logging geometry. We consider the complex background model represented by a random velocity distribution (Figure 4a). We image a single dipping fracture intersecting the borehole. For simplicity, we ignore the effect of the presence of the fluid-filled borehole, and we assume that the data has been filtered to suppress direct body waves and borehole waves. The fracture compliance values are η_T=1×10^{-11} m/Pa and η_N=1×10^{-12} m/Pa, respectively, assuming a fluid-filled rotationally invariant fracture. The magnitude of the compliances are calculated from the scaling behaviour of fracture compliances (Hobday and Worthington, 2012). Realistic source and receiver spacings are considered: the receiver array consists of 5 receivers with a minimum offset of 0.3 m and receiver spacing 0.15 m, and the tool moves along the borehole in steps of 0.5 m (e.g., Li et al., 2017). The dominant frequency for dipole measurements is considered in the source wavelet (3 kHz). Figures 4b and 4c show the modelled pseudo-dipole acoustic logging data (receiver gather); each dataset represent the responses of horizontal particle velocity due to horizontal force source. The response of the homogeneous medium has been subtracted in order to obtain the reflected waves. Due to the random background velocity, this subtraction is not perfect, and the input data is contaminated by scattered waves.

We apply the proposed LSM to each dataset where the Green's function in the homogeneous background medium is used for the Born operator. The results of the proposed LSM using the positive-compliance constraint (Figure 4d and 4e) show higher resolution images than RTM results. The result of P-SV waves (Figure 4e) shows an X shaped artefact mainly due to the lack of vertical component receiver. Nevertheless, the proposed LSM successfully images quantitatively the fracture compliances in case of a random background medium.

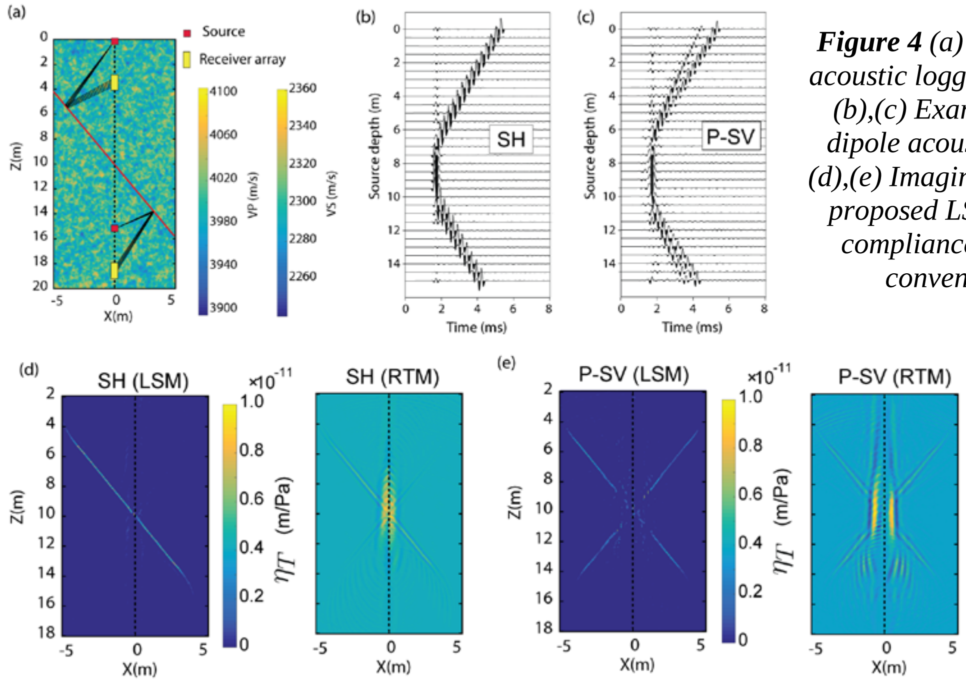

Figure 4 *(a) Configuration for acoustic logging measurements. (b),(c) Example of modelled dipole acoustic logging data. (d),(e) Imaging results using the proposed LSM with positive-compliance constraint and conventional RTM.*

Conclusion

Towards high-resolution, quantitative imaging of fractures around a borehole, we propose a new least-squares migration (LSM) approach that couples the linear-slip interface model. Numerical experiments show that the proposed LSM provides higher resolution images than RTM, and more accurate images than conventional LSM where a fracture is represented by coupling compliance of the fracture. Numerical experiments considering dipole acoustic logging geometry show the potential of the proposed LSM to imaging fractures around a borehole.

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