

Topic 1: constitutive modeling and continuum mechanics

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## A REVIEW OF BIOT'S ELASTIC COEFFICIENTS

The past 45 years the theory of Biot has been very successful in predicting the mechanical behavior of linear poroelastic materials. Nevertheless, Biot's theory has been criticized by some researchers, since some basic parts of this theory were not developed from the fundamental axioms and principles of mechanics and thermodynamics. For instance, the coupled stress-strain relations of Biot were intuitively obtained by Biot; hence, it is not easy to identify the precise definition and clear physical meaning of the stress tensors and material constants. Although some fundamental questions still remain to be solved, we note that Biot's stress-strain relations (and other crucial parts of Biot's theory) can be obtained from first principles by a direct volume averaging approach.

The elastic coefficients appearing in Biot's stress-strain relations are related to measurable quantities in a rather complex way. We do not dispute the correctness of these expressions, but in all the approaches we have seen so far one finds some crucial assumptions and/or definitions that are not easy to interpret. More importantly, in almost all approaches the derived expressions are not based explicitly on the constitutive and continuity equations describing the solid and fluid phase in the poroelastic material. The aim of our present work is to show that the expressions for the elastic coefficients can be obtained in a straightforward and elegant way, which results in a better insight in what kind of ingredients are needed to derive Biot's stress-strain relations.

Our derivation is set up as follows. We first write down the two linearized continuity equations for the solid and fluid phase, respectively, in the porous medium. Each continuity equation contains a part related to

the density change of the fluid or solid phase, a part related to the porosity change and a part containing Biot's fluid or solid strains. Here, the fluid and solid density changes can be expressed in Biot's stresses by using two constitutive equations defining the solid and fluid bulk moduli, respectively. Furthermore, the porosity change can also be expressed in Biot's stresses, i.e., we have derived such an expression by combining the specific characterizations of theunjacketed and jacketed test: at this point theunjacketed and jacketed bulk moduli come into play. Thus, the fluid and solid density changes and the porosity change can all be expressed in Biot's stresses and after substitution in the linearized continuity equations one obtains two coupled equations relating Biot's stresses to Biot's fluid and solid strains. After some simple mathematics and by making use of the fact that for a linear material the strain energy is quadratic (apparently, this implies that the unjacketed bulk modulus is identical to the solid bulk modulus), we obtain expressions in which the Biot's elastic coefficients are related to measurable quantities (porosity and solid, fluid, and jacketed bulk moduli).

It is clear that we cannot present all the details of our derivation in this abstract. We also omitted a short review of all the relevant results obtained by other researchers so far. All this will be presented in the full-length paper and we are sure that it will help to understand the fundamentals of poroelastic mechanics.