Scattering representations of dynamic fields: implications for Green's function retrieval

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Abstract

Imaging, inversion and property monitoring techniques many times rely on the retrieval of scattered or perturbed fields. To target the reconstruction of only the desired scattered fields, here we present a generalized perturbation-based formulation that retrieves the causal scattered fields between two receivers as if one acts as a pseudo-source in general dynamic systems. With this formulation we show that causal scattered fields can be obtained via cross-correlations without a requirement for energy equipartitioning. This has practical implications for experiments whose objective is precisely the retrieval of scattered fields.

Introduction

In most prior formulations of Green's function retrieval e.g. [1], [2], [3], [4], one retrieves a superposition of all fields in both positive and negative times, as long as the condition for energy equipartitioning is satisfied [1], [2]. From retrieved full-fields, the extraction of desired scattered waves is not always straightforward. Representation theorems for general physical systems which give rise to the retrieval of Green's functions are provided in [3] and [2]. Based on these representations, we analyze the special case of perturbed systems to derive representations that are tailored specifically for scattering problems.

1 Scattering & Green's function retrieval

Let the general frequency-domain matrix-vector differential equation $\hat{\mathbf{A}}\,(i\omega + \mathbf{v}\cdot\nabla)\,\hat{\mathbf{u}} + \hat{\mathbf{B}}\hat{\mathbf{u}} + \hat{\mathbf{D}}_{\mathbf{r}}\hat{\mathbf{u}} = \hat{\mathbf{s}}$ describe physical phenomena such as wave propagation and diffusion/transport. $\hat{\mathbf{u}} = \hat{\mathbf{u}}(\mathbf{r},\omega)$ and $\hat{\mathbf{s}} = \hat{\mathbf{s}}(\mathbf{r},\omega)$ are field and source vectors, respectively. The matrices $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ describe spatially-varying medium parameters. The operator $\hat{\mathbf{D}}_{\mathbf{r}}$ contains the spatial differential operators $\partial_{1,2,3}$; and $\mathbf{v}$ is the spatially-varying velocity of the moving medium. Examples of the matrices $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$ and $\hat{\mathbf{D}}_{\mathbf{r}}$ for attenuative acoustic and elastodynamic waves, mass diffusion, electromagnetic and seismo-electric phenomena are given in e.g. [3]; examples for the Schrödinger equation and advection are found in [2].

A Green's matrix correlation-type theorem (e.g., [3]) that can be derived from perturbed and unperturbed field states reads

$$\begin{aligned}\hat{\mathbf{G}}_S(\mathbf{r}_B,\mathbf{r}_A) &= \oint_{\partial\mathbb{V}} \hat{\mathbf{G}}_0^{\dagger}(\mathbf{r}_A,\mathbf{r})\,\hat{\mathbf{M}}_3^P\,\hat{\mathbf{G}}_S(\mathbf{r}_B,\mathbf{r})\;d^2\mathbf{r}\\ &+ \int_{\mathbb{V}} \hat{\mathbf{G}}_0^{\dagger}(\mathbf{r}_A,\mathbf{r})\,\hat{\mathbf{M}}_4^P\,\hat{\mathbf{G}}_S(\mathbf{r}_B,\mathbf{r})\;d^3\mathbf{r}\\ &+ \int_{\mathbb{V}} \hat{\mathbf{G}}_0^{\dagger}(\mathbf{r}_A,\mathbf{r})\,\hat{\mathbf{V}}\,\hat{\mathbf{G}}_0(\mathbf{r}_B,\mathbf{r})\;d^3\mathbf{r}\;;\end{aligned}\quad(1)$$

where $\hat{\mathbf{G}}_0$ and $\hat{\mathbf{G}}$ are the frequency-domain Green's tensors for the unperturbed and perturbed states, respectively. $\hat{\mathbf{G}}_S = \hat{\mathbf{G}} - \hat{\mathbf{G}}_0$ is the scattered-field matrix. $\hat{\mathbf{M}}_3^P = \mathbf{N}_{\mathbf{r}} - \hat{\mathbf{A}}_0^{\dagger}(\mathbf{v}_0\cdot\mathbf{n})$, $\hat{\mathbf{M}}_4^P = \hat{\mathbf{A}}\,(i\omega+\mathbf{v}\cdot\nabla) - \hat{\mathbf{A}}_0^{\dagger}\,(i\omega+\mathbf{v}_0\cdot\nabla) + \hat{\mathbf{B}} + \hat{\mathbf{B}}_0^{\dagger}$ and $\hat{\mathbf{V}} = \hat{\mathbf{A}}\,(i\omega+\mathbf{v}\cdot\nabla) - \hat{\mathbf{A}}_0\,(i\omega+\mathbf{v}_0\cdot\nabla) + \hat{\mathbf{B}} - \hat{\mathbf{B}}_0$. where $\mathbf{N}_{\mathbf{r}}$ contains the vector $\mathbf{n}$ normal to the integration surface. The superscript † denotes the adjoint matrix.

1.1 Retrieving scattered fields

In general, equation 1 is not immediately practical for Green's function retrieval, because it requires sources everywhere in the interior of $\mathbb{V}$ and complete knowledge of the medium parameters. Now, let us consider a special case: that of a non-moving media (i.e., $\mathbf{v} = \mathbf{v}_0 = \mathbf{0}$), with $\mathbf{r}_B \notin \mathbb{P}$ and $\hat{\mathbf{M}}_4^P = \hat{\mathbf{V}} = \mathbf{0}\ \forall\ \mathbf{r} \notin \mathbb{P}$ (Figure 1a-c). Thus, scattering occurs only inside $\mathbb{P}$. Also, outside $\mathbb{P}$, the fields behave as loss-less propagating waves, but the scattered fields inside $\mathbb{P}$ can be lossy and diffusive. In addition, let $\oint_{\partial\mathbb{V}} = \int_{\partial\mathbb{V}_b\cup\partial\mathbb{V}_t}$ (Figure 1). We fix the sets $\partial\mathbb{V}_t$ and $\mathbb{P}$, and have two choices for $\partial\mathbb{V}_b$ such that in (b) $\mathbb{P}\subset\mathbb{V}$, and in (c) $\mathbb{P}\not\subset\mathbb{V}$. In Figures 1a through 1d, all physical contributions to scattered waves propagating between $\mathbf{r}_B$ and $\mathbf{r}_A$ come from the sources in the top surface $\partial\mathbb{V}_t$. In Figure 1c, the volume integrals in equation 1 vanish because $\mathbb{P}\not\subset\mathbb{V}$. In that case, the bottom surface integral also vanishes because the contributions of monopole sources are cancelled by those of dipole sources inside the integrand

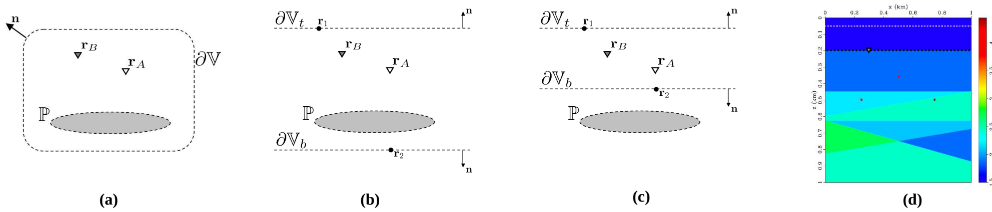

Figure 1: Panels (a) - (c) are schematic illustrations of configurations for retrieval of remotely scattered fields. Panel (d) shows the perturbed acoustic wavespeed (in km/s) model used in a numerical experiment. The medium perturbations in (d) consist of the scatterers and interfaces located below the depth of 0.3 km.

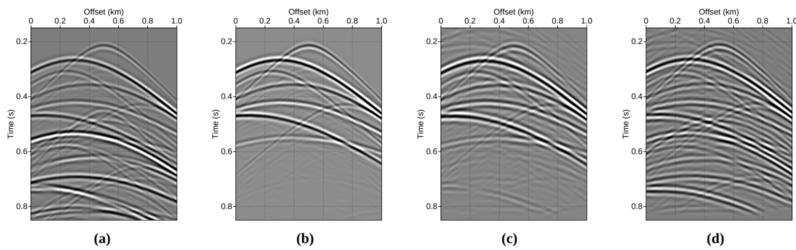

Figure 2: Comparisons of true scattered-wave responses with pseudo-source responses obtained by cross-correlating reference and scattered waves from the model in Figure 1d.

(equation 1). Thus, for Figure 1c, we obtain

$$\hat{\mathbf{G}}_S(\mathbf{r}_B, \mathbf{r}_A) = \int_{\partial\mathbb{V}_\mathbf{t}} \hat{\mathbf{G}}_0^\dagger(\mathbf{r}_A, \mathbf{r})\, \hat{\mathbf{M}}_3^P\, \hat{\mathbf{G}}_S(\mathbf{r}_B, \mathbf{r})\; d^2\mathbf{r}\;. \tag{2}$$

This equation is exact, while an approximation is derived in [4]. Equation 2 is also valid for Figure 1b, since it is not affected by changes to $\partial\mathbb{V}_b$. In Figure 1b, sources on $\partial\mathbb{V}_b$ result in unphysical scattered-wave arrivals, but their contributions are canceled by the volume integrals (equation 1). Equation 2 is suitable for retrieving the scattered-wave Green's functions between $\mathbf{r}_B$ and $\mathbf{r}_A$ by cross-correlations of waves excited by sources on the open surface $\partial\mathbb{V}_t$. Note that the open surface integral in equation 2 implies uneven energy radiation at $\mathbf{r}_B$. This demonstrates that the retrieval of causal scattered waves does not require energy equipartitioning e.g. [1], [2].

In Figure 2 we show results from reconstructing scattered Green's functions from an acoustic example in the model in Figure 1d. Simulating an ocean-bottom seismic experiment, sources are placed in horizontal line at $z = 0.1$km with 2 m spacing and receivers are at the ocean bottom ($z =$0.2 km) at every 2 m. The true scattered-wave responses for a physical source at $(0.3\,\text{km}, 0.2\,\text{km})$ (triangle in Figure 1d) are displayed in Figure 2a modeled *with* a free-surface (at $z = 0\,$km), and in Figure 2b, where it is modeled *without* a free-surface. The responses in Figure 2c-d correspond to scattered-wave responses retrieved via cross-correlations. The result in Figure 2c is obtained with equation 2 using combined pressure and particle velocity measurements; while Figure 2d results from pressure measurements alone. It is important to note that the input data *to both* (c) and (d) were modeled *with* a free-surface. While the original input data contains the effects of the free-surface, proper manipulation of the scattering representation in equation 2 can retrieve scattered with (Figure 2d) or without (Figure 2c) the influence of the ocean surface.

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