Deconvolution and Multiple Elimination to Enhance Temporal Resolution in Ultrasonic Volume-scan

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Abstract

Traditionally, data obtained with a C-scan only gives one value for each particular point of the target. If the whole time-signal is preserved, however, it is possible to obtain more information. Since the time-signal resulting from a pulse-echo measurement might be very complex, advanced algorithms are needed. Applying the inverse formulation of the theory describing the interaction of a plane wave with a layered medium can be used in order to obtain the acoustic parameters of the target material.

Keywords: c-scan, ultrasonic, wave theory, deconvolution, multiple elimination, resolution, processing

1. Introduction

The goal of ultrasonic inspection of materials is to obtain information about the materials without destroying them. Ultrasonic waves are sent to the target, where they interact with the material. The resulting wavefield can be measured, processed and analysed. The process of ultrasonic inspection can be divided into three subprocesses: data-acquisition, data-processing and the analysis of the processed data. The quality of the results is highly dependend on the quality of the acquisition and the processing.
We focus on the inspection of laminated materials, constituting of thin layers – with a thickness ranging from 0.2 mm till 2 mm. We inspect these materials in order to search for delaminations between 2 layers. The results of the inspection are an indication about the material properties of the different layers, and the quality of the bonds between them. In order to obtain finer details, waves with a higher frequency have to be used. Practical problems put a limit on the frequency we can use. Due to advanced processing techniques, however, we are able to obtain a lot of information.

2. Data Acquisition

The target is a laminated material. Ultrasonic waves are sent to the material. The target is embedded in a fluid – e.g. water – in order to avoid the damping of waves propagating through the air. The wavefield propagates from the transducer through the water, and arrives at the target. There, it interacts with the material. Part of the energy will be reflected back to the transducer where it came from – the reflected wavefield –, another part will be transmitted through the material and travel through the water to a transducer at the other side of the material – the transmitted wavefield. Both the reflected and the transmitted wavefield can be recorded.
The inspection method is developped for thin laminated materials, especially for glare. Glare consists of alternating layers of aluminium and prepreg. The acoustic parameters of the glare-components and water are given in *Table 1*.

acoustic paramters		
Medium	density $[kg/m^3]$	velocity $[m/s]$
Water	1000	1480
Aluminium	2790	6350
Prepreg	1600	3100

Table 1: acoustic parameters of glare-components and water

An A/D-converter converts the recorded data into a digital format, and the data is sent to a computer where it can be processed. When chosing an A/D-converter, two parameters are very important: the sampling rate and the number of bits one output sample consists of. We want the sampling rate as high as possible, since we want to obtain fine details about the wavefield. Also, we want the number of bits as high as possible since we want to distinguish both strong as well as weak reflections. Because of technical limitations, the resulting choice is a compromise between the sampling rate and the number of bits. We have chosen for an A/D-converter with a 100 MHz sampling rate and 12 bits per output sample.
In order to avoid interference of waves, we want the emitted wavefield to be short. In general, high-frequency transducers emit less energy than low-frequency transducers, so a compromise is necessary here too. We work with Panametrics transducers with a center frequency of 15 MHz.

3. Processing

Consider a target material with no variations of the acoustic parameters in the x- en y-directions. The density and the velocity vary as a dicrete function of the z-coordinate. The material consists of n layers. Layer 1 is considered to be the top of the material, while layer n is the bottom of layer.

In each layer, the acoustic parameters are assumed constant. Each layer i is characterized by the values of three paramters:

1. ρ_i, the density
2. v_i, the acoustic velocity
3. z_i, the layer thickness

Given these parameters, we are able to calculate the resulting reflected wavefield for a given emitted wavefield – using the wave theory. This is done in the forward processing. On the other hand, given the resulting reflected wavefield and the used source wavefield, we are able to calculate the acoustic parameters. This is done in the inverse processing.

3.1. The forward model

The acoustic impedance of a layer is defined as

$$Z_i = \rho_i v_i \tag{1}$$

When a wave inpinges at the boundary of two layers, part of the energy will be reflected, and part of it will be transmitted. The reflection coefficient at the boundary of layers i and $i+1$ is given by

$$R_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i} \tag{2}$$

The acoustic pressure at the boundary of layers $i-1$ and i and assuming only upgoing waves is denoted by p_i^0 in the time domain, and by $\hat{p}_i^0$ in the frequency domain. The total wavefield – taking into account the downgoing waves due to reflection at the boundary between layers $i-1$ and i is denoted by p_i in the time domain and by $\hat{p}_i$ in the frequency domain.

In the following, we assume that the source and the receiver are located at the same place – which means, in practice, that we use one transducer which is both the transmitter and the receiver. When the transducer is located at the boundary between the layers $n-1$ and n, the pressure of the upgoing wavefield is given by

$$\hat{p}_n^0 = \hat{X}_n^0 \hat{S} \tag{3}$$

in which $\hat{X}_n^0$ is the response function for layer n with a non-reflecting surface, and is given by

$$\hat{X}_n^0 = \hat{W}_n R_n \hat{W}_n \tag{4}$$

$\hat{S}$ is the source-function and $\hat{W}_n$ is the operator associated with the propagation of the wavefield in layer n, defined by

$$\hat{W}_i = \exp(j\omega t_i) \tag{5}$$

where

$$t_i = z_i / v_i \tag{6}$$

When there is an acoustic contrast between layers $i-1$ and i, part of the upgoing wavefield will be reflected back in the layer. As a consequence, not only the source-function will be sent into the material. The total pressure at the boundary is given by

$$\hat{p}_n = \hat{X}_n^0 \left(\hat{S} - R_{n-1} \hat{p}_n \right) \tag{7}$$

From this, it follows that the total pressure can be written as

$$\hat{P}_n = \frac{\hat{X}_n^0 \hat{S}}{1 + R_{n-1} \hat{X}_n^0} \tag{8}$$

or, by defining $\hat{X}_n$ as the total response of the structure under layer $n-1$:

$$\hat{p}_n = \hat{X}_n \hat{S} \tag{9}$$

with

$$\hat{X}_n = \frac{\hat{X}_n^0}{1 + R_{n-1} \hat{X}_n^0} \tag{10}$$

A relationship between the pressure of the upgoing wavefield and the total pressure is obtained by combining Equations (25) and (8):

$$\hat{p}_n = \frac{\hat{p}_n^0 \hat{S}}{S + R_{n-1} \hat{p}_n^0} \tag{11}$$

Given the total wavefield at z_n, we can calculate the upgoing wavefield at position z_{n-1} when the source is also moved to that position. The downgoing wavefield will interact with the structure beneath layer $n-1$. Part of the energy will be reflected at the boundary, and part of it will be transmitted. The upgoing wavefield due to a reflection at the boundary between layers $n-1$ and n is written as

$$\hat{W}_{n-1} R_{n-1} \hat{W}_{n-1} \hat{S} \tag{12}$$

A portion $(1 + R_{n-1})$ of the downgoing wavefield will be transmitted from layer $n-1$ to layer n. This will give raise to an upgoing wavefield at the boundary at z_n. From this wavefield, a portion $(1 - R_{n-1})$ is transmitted from layer n to layer $n-1$. The second contribution to the upgoing wavefield in z_{n-1} can thus be written as

$$\hat{W}_{n-1} (1 + R_{n-1}) \hat{X}_n (1 - R_{n-1}) \hat{W}_{n-1} \hat{S} \tag{13}$$

The total upgoing wavefield is then given by

$$\hat{p}_{n-1}^0 = \hat{W}_{n-1} \left[\hat{X}_n \left(1 - R_{n-1}^2 \right) + R_{n-1} \right] \hat{W}_{n-1} \hat{S} \tag{14}$$

Defining X_{n-1}^0 as the response function for the structure consisting of layer $n-1$ till layer n, and with a non-reflecting surface, we can write

$$\hat{p}_{n-1}^0 = \hat{X}_{n-1}^0 \hat{S} \tag{15}$$

with

$$\hat{X}_{n-1}^0 = \hat{W}_{n-1} \left[\hat{X}_n \left(1 - R_{n-1}^2 \right) + R_{n-1} \right] \hat{W}_{n-1} \tag{16}$$

Iteratively repeating these steps gives the general equations for the acoustic pressure with or without a reflecting surface:

$$\hat{p}_i^0 = \hat{X}_i^0 \hat{S} \tag{17}$$

$$\hat{p}_i = \hat{X}_i \hat{S} \tag{18}$$

where the response functions are defined by

$$\hat{X}_i^0 = \hat{W}_i \left[\hat{X}_{i+1} \left(1 - R_i^2 \right) + R_i \right] \hat{W}_i \tag{19}$$

$$\hat{X}_i = \frac{\hat{X}_i^0}{1 + R_{i-1} \hat{X}_i^0} \tag{20}$$

Combining Equations 17, 18, 19 and 20 gives

$$\hat{p}_i^0 = \hat{W}_i \left[\hat{p}_{i+1} \left(1 - R_i^2 \right) + R_i \hat{S} \right] \hat{W}_i \tag{21}$$

$$\hat{p}_i = \frac{\hat{p}_i^0 \hat{S}}{\hat{S} + R_{i-1} \hat{p}_i^0} \tag{22}$$

The iteration is started with

$$\hat{X}_n = \hat{W}_n R_n \hat{W}_n \tag{23}$$

$$\hat{p}_n = \hat{W}_n R_n \hat{W}_n \hat{S} \tag{24}$$

The receiver is located in the medium surrounding the target. Assuming no surface-multiples will occur in the measured time-frame, we can write

$$\hat{P}_0 = \hat{p}_0^0 = \hat{W}_0 \left[\hat{p}_1 \left(1 - R_0^2 \right) + R_0 \hat{S} \right] \hat{W}_0 \tag{25}$$

$$\hat{X}_0 = \hat{X}_0^0 = \hat{W}_0 \left[\hat{X}_1 \left(1 - R_0^2 \right) + R_0 \right] \hat{W}_0 \tag{26}$$

$\hat{X}_0$ is the response function of the structure, while $\hat{P}_0$ is the representation of the measured wavefield in the frequency domain.

3.2. The inverse model

Starting from the total response function X_0, we can iteratively strip the effect of one layer, and calculate the response functions $\hat{X}_i$ and $\hat{X}_i^0$. From Equations 19 and 20, it follows that

$$\hat{X}_i = \frac{\hat{X}_{i-1}^0 - \hat{W}_{i-1} R_{i-1} \hat{W}_{i-1}}{\hat{W}_0 \left(1 - R_0^2 \right) \hat{W}_0} \tag{27}$$

$$\hat{X}_i^0 = \frac{\hat{X}_i}{1 - \hat{X}_i R_{i-1}} \tag{28}$$

From Equations 21 and 22, we can write

$$\hat{p}_i^0 = \frac{\hat{p}_i \hat{S}}{\hat{p}_i R_{i-1} - \hat{S}} \tag{29}$$

$$\hat{p}_{i+1} = \frac{\hat{p}_i^0 - \hat{W}_i R_i \hat{S} \hat{W}_i}{\hat{W}_i \left(1 - R_i^2 \right) \hat{W}_i} \tag{30}$$

In the inverse proces, we start from $\hat{p}_0$. We calculate respectively $\hat{p}_0 \rightarrow \hat{p}_0^0 \rightarrow \hat{p}_1 \rightarrow \hat{p}_1^0, \rightarrow \ldots \rightarrow \hat{p}_n \rightarrow \hat{p}_n^0$. Assuming no surface-related multiples results in

$$\hat{p}_0^0 = \hat{p}_0 \tag{31}$$

It follows from Equation 30 that

$$\hat{p}_1 = \frac{p_0^0 - \hat{W}_0 R_0 \hat{S} \hat{W}_0}{\hat{W}_0 \left(1 - R_0^2\right) \hat{W}_0} \tag{32}$$

In order to calculate $\hat{p}_1$, we need to know the source-signal $\hat{S}$, the propagation-operator $\hat{W}_0$ and the reflection coefficient R_0. This will be discussed in the next section.
An expression for $\hat{p}_1^0$ follows from Equation 29:

$$\hat{p}_1^0 = \frac{\hat{p}_1 \hat{S}}{\hat{p}_1 R_0 - \hat{S}} \tag{33}$$

The surface-multiple free pressure p_i^0 can be calculated from the total pressure p_i without the need of extra information.

3.3. Obtaining the acoustic parameters

As mentionned before, we need to know the reflection coefficients R_i and the propagation-operators $\hat{W}_i$ to be able to strip one layer. The propagation-operator $\hat{W}_i$ – as defined in Equation 5 – can be calculated once we know the traveltime in layer i. The two-way traveltime is the elapsed time before the source-funtion occurs in the time-signal $p_i(t)$. When the length of the source-function in the time-domain is shorter than the two-way traveltime in layer i, the first arrival will be completely finished before other reflections or multiples occur. We will prove now that an auto-correlation of the source-function $s(t)$ with the signal $p_i(t)$ will have a local maximum at $t = t_i$. This allows us to retrieve the propagation-operator from the signal $p_i(t)$.
We denote the impulse response of the total structure by $\hat{h}$. In the frequency domain, the measured signal is then given by

$$\hat{p}_0 = \hat{h}\hat{s} \tag{34}$$

Applying an inverse Fourier transform gives the pressure in the time domain:

$$p(t) = \int_{t"} h\left(t - t"\right) s(t")dt" \tag{35}$$

A time-correlation of this function with the source signal gives:

$$c(t) = \int_{t'} p\left(t + t'\right) s(t')dt' \tag{36}$$

Using Equation 35, this results in

$$c(t) = \int_{t'} s(t')dt' \int_{t"} h\left(t + t' - t"\right) s(t")dt" \tag{37}$$

Since the response-function in the time domain consists of a discrete number of Dirac-functions, we can write

$$h(t) = \sum_{i=1}^{n} a_i \delta\left(t - t_i\right) \tag{38}$$

Using this in Equation 37, we obtain

$$c(t) = \int_{t'} s(t')dti \int_{t"} \left[i \sum_{i=1}^{n} a_i \delta\left(t + t' - t" - t_i\right)\right] s(t")dt" \tag{39}$$

from which it follows that

$$c(t) = \sum_{i=1}^{n} a_i \int_{t'} s(t')s(t - t_i + t')dt' \tag{40}$$

The first factor in the integrand, $s(t')$ is only non-zero in the source-time domain B. If we restrict t to be in the vicinity of t_i, the second factor in the integrand will vanish for all contributions a_j except for $i = j$. As a consequence, we obtain

$$c(t) = \int_{t'} s(t')s\left(t' + t - t_i\right) dt' \tag{41}$$

It can easily be shown that this function has a maximum for $t = t_i$.

Since this correlation has to be done with a discrete number of samples, the error on t_i is of order $O(\delta t)$, with δt the sampling rate. Applying a cubic interpolation in the vicinity of t_i allows us to obtain the maximum with a better precision.

The reflection coefficient R_i can be calculated from the values $p_i(t_i)$, $s(0)$ and the previous reflection coefficients R_0 ... R_{i-1}. It is easy to show that

$$p_i(t_i) = R_i s(0) \sum_{j=0}^{i-1} \left(1 - R_j^2\right) \tag{42}$$

from which it follows that

$$R_i = \frac{p_i(t_i)}{s(0) \sum_{j=0}^{i-1} \left(1 - R_j^2\right)} \tag{43}$$

Starting from R_0, we can iteratively calculate the different reflection coefficients.

3.4. Deconvolution

The signal emitted by the transducer has a certain shape and time-length. A transformation can alter these properties. A simple shape and a short length enhance the visual aspect of the signal. When the actual source wavefield is defined as $\hat{S}$ in the frequency domain, and the desired source signal is written as $\hat{D}$ – also in the frequency domain –, there exists a transfer function $\hat{T}$ for which it holds that

$$\hat{D} = \hat{T}\hat{S} \tag{44}$$

or

$$\hat{T} = \frac{\hat{D}}{\hat{S}} \tag{45}$$

In order to apply the multiple elimination scheme as described in the previous paragraph, the same transformation has to applied on the measured signal p_0. This is done in the frequency domain:

$$\hat{Y} = \hat{T}\hat{P}_0 \tag{46}$$

$\hat{Y}$ is called the deconvolved signal.

From definition (45), it follows that the transfer fucnction $\hat{T}$ becomes unbounded for frequencies ω that obey $\hat{S}(\omega) = 0$ where $\hat{D}(\omega) \neq 0$.

Applying such transfer functions to the measured wavefield will cause the layer stripping algorithm to fail. Therefore, it doesn't make sence to transform the source function $\hat{S}$ to a function $\hat{D}$ with a larger bandwidth.

A well-shaped wavelet is the Ricker Wavelet. In Figure (2), a Ricker Wavelet with a center frequency of $15MHz$ is shown. The source-function, obtained with a $15MHz$-transducer is shown in Figure (**??**). Comparing the frequency spectra of the two functions shows that the Ricker Wavelet has a larger bandwidth than the source function. Deconvolving the signal to a Ricker Wavelet with a center frequency of $15MHz$ would lead to numerical problems, since the denominator of the transfer function is very small while the nominator still has normal values. A Ricker Wavelet with a lower center frequency, or a modified Ricker Wavelet has to be chosen. This is still a subject of research.

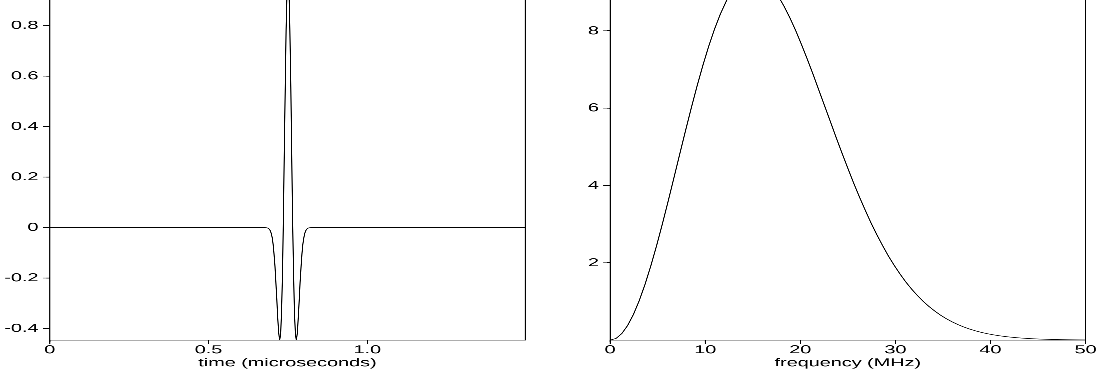

Figure 1: time- and frequency behaviour of the ricker wavelet

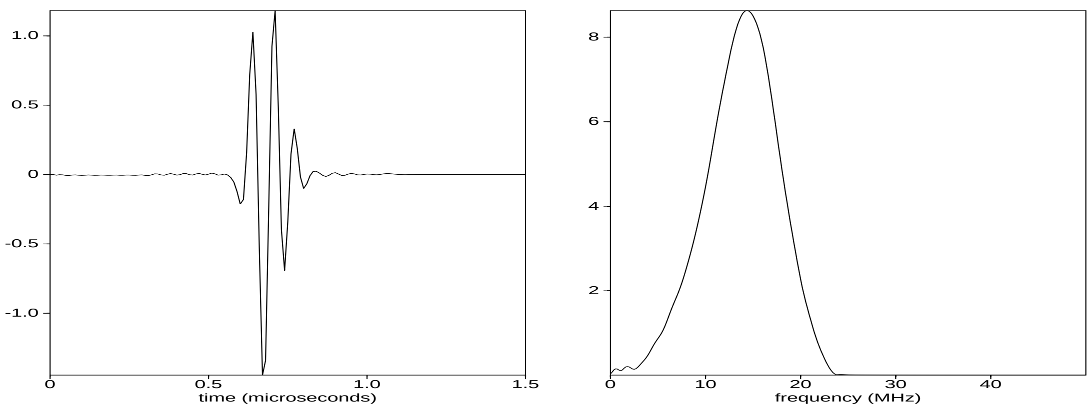

Figure 2: time- and frequency behaviour of the actualsource wavelet

4. Analysis of the results

The layer stripping algorithm calculates the reflection coefficients for each boundary. These values can be compared with the expected values. When the calculated reflection coefficient is much stronger than the expected value, this is an indication that the wave travels from a medium with a high acoustic impedance to a medium with a low acoustic impedance – e.g. air. This can be due to a delamination. Taking advantage of the iterative character of the layer stripping algorithm, one is able to determine at which boundary the delamination occurs.