

Removal of surface effects, using elastic reciprocity

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Summary

When performing land seismic experiments, the data is often polluted with surface related phenomena, such as Rayleigh and Love waves. They often make up for most of the energy in a seismogram, and are hardly separable from reflection data by conventional techniques.

This research is focused on developing a technique, useful for elimination of surface related phenomena from seismic data. This technique will be similar to that of van Borselen et al. (1996), who used acoustic reciprocity for the removal of multiples from marine seismic data. In our case, elastic reciprocity will be used. The focus will be on the removal of Love waves in SH wave data.

Introduction

The data obtained in a seismic experiment can be divided into two parts: the desired data, and noise. The noise itself can be divided also into two parts: random (stochastic) noise and shot generated noise. An example of shot generated noise is the occurrence of surface waves like Rayleigh and Love waves. Surface waves are considered noise because they bear no information about the subsurface. Also, since they propagate along the surface, they attenuate slowly, thus obscuring the weaker subsurface reflection data. An extra problem with Rayleigh and Love waves is that their velocity is almost the same as the shear wave velocity. This problem makes it difficult to separate dispersive surface waves with for example f-k analysis. For a discussion on the behavior of Rayleigh and Love waves, we refer to Aki and Richards (1980).

Not many techniques have been developed to filter surface waves from seismic data, and they often do not produce satisfactory results. We present a technique that aims to remove all surface effects from seismic data.

Theory

In this section we will derive the reciprocity integrals, and from there we will derive a set of integral equations of the second kind, with which we can calculate the wave field as if there is no surface present. But first we will give some basic theory about the Laplace transform.

The Laplace transform

Our equations are time invariant and causal. The causality condition can most easily be met by using a Laplace transformation. The Laplace transform of a causal

function is defined as:

$$\hat{u}(\mathbf{x}, s) = \int_0^\infty e^{-st} u(\mathbf{x}, t) dt. \quad (1)$$

Here, $\text{Re}(s) > 0$. We arrive at the Fourier transform to the angular frequency domain if we take $s = j\omega$. But for our theoretical analysis we use the Laplace transform. The Laplace transform has the following property with regard to differentiation to time: $\partial_t u(\mathbf{x}, t) \rightarrow s\hat{u}(\mathbf{x}, s)$.

The Betti-Rayleigh integral

What we do with reciprocity is comparing two different states with each other in a convenient way. The first state is denoted as state *A*. We write for the elastodynamic equations in the Laplace domain:

$$\partial_j \hat{\tau}_{i,j}^A - s\rho^A \hat{v}_i^A = -\hat{f}_i^A, \quad (2)$$

$$\frac{1}{2}(\partial_p \hat{v}_q^A + \partial_q \hat{v}_p^A) - sS_{p,q,i,j}^A \hat{\tau}_{i,j}^A = \hat{h}_{p,q}^A. \quad (3)$$

And for state *B* we write:

$$\partial_j \hat{\tau}_{i,j}^B - s\rho^B \hat{v}_i^B = -\hat{f}_i^B, \quad (4)$$

$$\frac{1}{2}(\partial_p \hat{v}_q^B + \partial_q \hat{v}_p^B) - sS_{p,q,i,j}^B \hat{\tau}_{i,j}^B = \hat{h}_{p,q}^B. \quad (5)$$

In these equations, $\hat{\tau}_{i,j}$ is the elastic stress tensor, \hat{v}_i is the particle velocity vector, ρ is the volume density of mass of the material, $S_{p,q,i,j}$ is the compliance tensor (the inverse is known as the stiffness tensor $C_{i,j,p,q}$), \hat{f}_i is the volume source density of external forces, and finally, $\hat{h}_{p,q}$ is the volume source density of deformation. For the derivation of these equations we refer to de Hoop (1995). These states are summarized in Table 1. Next, we con-

	State <i>A</i>	State <i>B</i>
Field State	$\{\hat{\tau}_{k,j}^A, \hat{v}_k^A\}(\mathbf{x}, s)$	$\{\hat{\tau}_{k,j}^B, \hat{v}_k^B\}(\mathbf{x}, s)$
Material State	$\{\rho^A, S_{i,j,p,q}^A\}(\mathbf{x})$	$\{\rho^B, S_{i,j,p,q}^B\}(\mathbf{x})$
Source State	$\{\hat{h}_{j,k}^A, \hat{f}_k^A\}(\mathbf{x}, s)$	$\{\hat{h}_{j,k}^B, \hat{f}_k^B\}(\mathbf{x}, s)$
Domain \mathbb{D}		

Table 1: States in the elastodynamic reciprocity theorem

sider the following scalar interaction quantity between the two states: $\partial_j(\hat{\tau}_{k,j}^A \hat{v}_k^B - \hat{\tau}_{k,j}^B \hat{v}_k^A)$. We substitute the constitutive relations, integrate over a volume, called domain

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\mathbb{D} , and apply Gauss' theorem. Analogous to the acoustic case (Fokkema and van den Berg, 1993), we obtain:

$$\begin{aligned} & \int_{\mathbf{x} \in \partial \mathbb{D}} (\hat{\tau}_{k,j}^A \hat{v}_k^B - \hat{\tau}_{k,j}^B \hat{v}_k^A) \nu_j dA = \\ & \int_{\mathbf{x} \in \mathbb{D}} [s (S_{j,k,p,q}^B - S_{p,q,j,k}^A) \hat{\tau}_{j,k}^A \hat{\tau}_{p,q}^B - \\ & \quad s (\rho^B - \rho^A) \hat{v}_k^A \hat{v}_k^B] dV + \\ & \int_{\mathbf{x} \in \mathbb{D}} [\hat{f}_k^B \hat{v}_k^A + \hat{h}_{j,k}^B \hat{\tau}_{j,k}^A - \hat{f}_k^A \hat{v}_k^B - \hat{h}_{j,k}^A \hat{\tau}_{j,k}^B] dV. \quad (6) \end{aligned}$$

This is the global form of the Betti-Rayleigh reciprocity theorem. The media are assumed to be reciprocal. The reciprocity condition implies the symmetry relation: $S_{j,k,p,q} = S_{p,q,j,k}$.

The removal procedure

The two states we want to compare, are the actual state, where there is a stress free surface causing multiples, Rayleigh waves and Love waves and the like, and a desired state, where there is no surface, and therefore no surface effects. Figure 1 shows a graphical representation of the two states.

In the actual state, we take a point source of force, $\hat{f}_k^a(s)\delta(\mathbf{x} - \mathbf{x}^S)$, at the source position, and in the desired state, we take a point source of force, $\hat{f}_k^d(s)\delta(\mathbf{x} - \mathbf{x}^R)$, at the receiver position. The resulting wave fields are summarized in Table 2. We substitute this into eq. (6), and

	State A (actual wave field)	State B (desired wave field)
Field	$\{\hat{\tau}_{k,j}^a, \hat{v}_k^a\}(\mathbf{x} \mathbf{x}^S, s)$	$\{\hat{\tau}_{k,j}^d, \hat{v}_k^d\}(\mathbf{x} \mathbf{x}^R, s)$
Material	$\{\rho, S_{i,j,p,q}\}$ in \mathbb{D} $\{\rho', S'_{i,j,p,q}\}$ in \mathbb{D}'	$\{\rho, S_{i,j,p,q}\}$ in \mathbb{D} $\{\rho', S'_{i,j,p,q}\}$ in \mathbb{D}'
Source	$\hat{f}_k^a(s)\delta(\mathbf{x} - \mathbf{x}^S)$	$\hat{f}_k^d(s)\delta(\mathbf{x} - \mathbf{x}^R)$
Domain $\mathbb{D} \cup \mathbb{D}'$ (see Figure 1)		

Table 2: States for the procedure for removing surface effects

integrate over the lower half space. The integral at infinity becomes zero ($\mathcal{O}(\Delta^{-1})$ as $\Delta \rightarrow \infty$), due to causality (Fokkema and van den Berg, 1993), and we are left only with an integral over the surface. Realizing that in the actual state the surface is stress free, we obtain:

$$\begin{aligned} & - \int_{(x_1, x_2) \in \mathbb{R}^2} \hat{\tau}_{k,3}^d(x_1, x_2, 0|\mathbf{x}^R, s) \hat{v}_k^a(x_1, x_2, 0|\mathbf{x}^S, s) dA = \\ & \quad \hat{f}_k^d(s) v_k^a(\mathbf{x}^R|\mathbf{x}^S, s) - \hat{f}_k^a(s) v_k^d(\mathbf{x}^S|\mathbf{x}^R, s). \quad (7) \end{aligned}$$

In the next sections we will modify this integral, and obtain a set of coupled integral equations of the second

kind, that together will eliminate surface effects from land seismic data. Whilst the above equation assume buried sources and receivers, we will derive a similar equation where the sources and receivers are located on the surface.

The actual state

We review the situation as is shown in Figure 1a. The domain of integration is the lower halfspace $\mathbb{D} \cup \mathbb{D}' = \{\mathbf{x} \in \mathbb{R}^3 | -\infty < x_1, x_2 < \infty, 0 < x_3 < \infty\}$ (The x_3 axis is pointing downwards) this halfspace consists of a homogeneous layer \mathbb{D} , no matter how thin, and further earth layers \mathbb{D}' , which can be homogeneous or inhomogeneous. The boundary between the two domains is denoted as $\partial \mathbb{D}'$.

In the actual situation, we have a stress free surface, $\hat{\tau}_{i,3} = 0$ at $x_3 = 0$. We place our sources and receivers at this surface. Then, instead of taking a point source of force at the source position, we describe the source as a boundary condition in the stress field. This implies that the surface is stress free, except at a certain point, where we the stress is given by: $t_i = \hat{t}_i(s)\delta_{i,m}\delta(x_1 - x_1^S)\delta(x_2 - x_2^S)$, i.e. we take a surface source specifically pointing in the x_m direction. Here, we wrote $t_i = \hat{\tau}_{i,3}\nu_3$, where t_i is defined as the traction on the surface. We write the resulting wave field with an extra subscript: $\hat{\tau}_{i,j;m}^a(\mathbf{x}|\mathbf{x}_1^S, \mathbf{x}_2^S, 0, s)$ and $\hat{v}_{i;m}^a(\mathbf{x}|\mathbf{x}_1^S, \mathbf{x}_2^S, 0, s)$, where $\mathbf{x} \in (\mathbb{D} \cup \mathbb{D}')$. This means that we have a wave field $\hat{\tau}_{i,j}$ and \hat{v}_i , caused by a surface source in the x_m direction.

The desired state

In the desired state, the domain of integration is again the lower halfspace. However the surface is extended to $x_3 \rightarrow -\infty$. This means that the plane $x_3 = 0$ is just an artificial boundary. It is clear that no surface effects can take place in this configuration. Now we take a point source of force in the point \mathbf{x}^R , at $x_3 = 0$, i.e. $\hat{f}_i^d = \hat{f}^d(s)\delta_{i,n}\delta(x_1 - x_1^R)\delta(x_2 - x_2^R)\delta(x_3)$. This source is pointing specifically in the x_n direction. Again, we write the resulting wave field with an extra subscript: $\hat{\tau}_{i,j;n}^d(\mathbf{x}|\mathbf{x}_1^R, \mathbf{x}_2^R, 0, s)$ and $\hat{v}_{i;n}^d(\mathbf{x}|\mathbf{x}_1^R, \mathbf{x}_2^R, 0, s)$, where $\mathbf{x} \in (\mathbb{D} \cup \mathbb{D}')$. Similar to the former section, this means that we have a wave field $\hat{\tau}_{i,j}$ and \hat{v}_i , caused by a source in the x_n direction.

The reciprocity integrals

We start off by summarizing the results achieved so far in Table 3. We apply the reciprocity theorem of eq. (6) to the domain $\mathbb{D} \cup \mathbb{D}'$, while using the states given in Table 3, following the same procedure as Rademakers (1996). Note that the surface of the actual state is not completely stress free in this case. We obtain:

$$\begin{aligned} & \int_{(x_1, x_2) \in \mathbb{R}^2} [\hat{\tau}_{i,3;m}^a(\mathbf{x}|\mathbf{x}^S, s) \hat{v}_{i;n}^d(\mathbf{x}|\mathbf{x}^R, s) \\ & \quad - \hat{\tau}_{i,3;n}^d(\mathbf{x}|\mathbf{x}^R, s) \hat{v}_{i;m}^a(\mathbf{x}|\mathbf{x}^S, s)] dA = \\ & \int_{\mathbf{x} \in (\mathbb{D} \cup \mathbb{D}')} \hat{f}^d(s) \delta_{i,n} \delta(\mathbf{x} - \mathbf{x}^R) \hat{v}_{i;m}^a(\mathbf{x}|\mathbf{x}^S, s) dV, \quad (8) \end{aligned}$$

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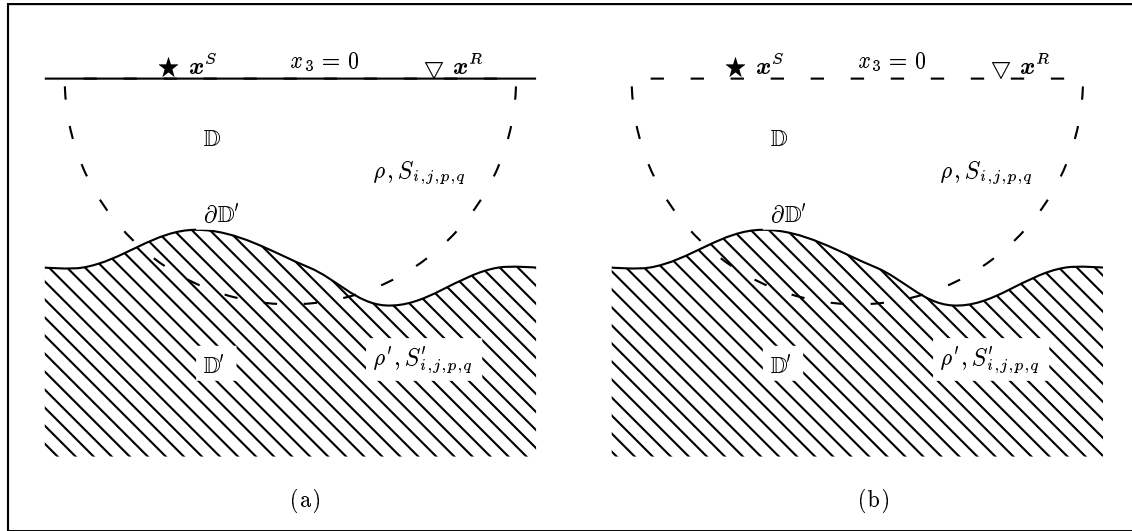


Fig. 1: The two states for the reciprocity theorem: a) with stress-free surface, b) without surface. The dashed line is the path of integration, which goes to infinity.

	State A (actual wave field)	State B (desired wave field)
Field	$\{\hat{\tau}_{i,j;m}^a, \hat{v}_{i;m}^a\}(\mathbf{x} \mathbf{x}^S, s)$	$\{\hat{\tau}_{i,j;n}^d, \hat{v}_{i;n}^d\}(\mathbf{x} \mathbf{x}^R, s)$
Material	$\{\rho, S_{i,j,p,q}\}$ in \mathbb{D} $\{\rho', S'_{i,j,p,q}\}$ in \mathbb{D}'	$\{\rho, S_{i,j,p,q}\}$ in \mathbb{D} $\{\rho', S'_{i,j,p,q}\}$ in \mathbb{D}'
Source	0	$\hat{f}^d(s)\delta_{i,n}\delta(\mathbf{x} - \mathbf{x}^R)$
Boundary	Surface is stress free, except for a traction source: $\hat{\tau}_{i,3;m}^a = \hat{t}^a(s)\delta_{i,m}$ $\delta(x_1 - x_1^S)\delta(x_2 - x_2^S)$	N.A.
Domain $\mathbb{D} \cup \mathbb{D}'$ (see Figure 1)		

Table 3: States for the removal of surface effects

where \mathbf{x}^R and \mathbf{x}^S are located on the surface ($x_3 = 0$). The special integral sign on the left hand side of this equation means that we perform a Cauchy principal value integral, as described by Tan (1975). This is an integral where one integrates over a surface, but excludes a singular point t , when necessary. The singular point t in this integral is caused by the sources on the surface. Since the point source of force $\hat{f}^d(s)\delta_{i,n}\delta(\mathbf{x} - \mathbf{x}^R)$ is located on the surface, the integration over it is only over “half” a delta-function. We then obtain:

$$-\int_{(x_1, x_2) \in \mathbb{R}^2} \hat{\tau}_{i,3;n}^d(\mathbf{x}|\mathbf{x}^R, s)\hat{v}_{i;m}^a(\mathbf{x}|\mathbf{x}^S, s)dA = \frac{1}{2}\hat{f}^d(s)\hat{v}_{n,m}^a(\mathbf{x}^R|\mathbf{x}^S, s) - \hat{t}^a(s)\hat{v}_{m,n}^d(\mathbf{x}^S|\mathbf{x}^R, s). \quad (9)$$

As a final step, we separate the desired wavefield into an incoming and a reflected wavefield: $\{\hat{\tau}_{i,3;n}^d, \hat{v}_{i;n}^d\} = \{\hat{\tau}_{i,3;n}^{\text{inc}}, \hat{v}_{i;n}^{\text{inc}}\} + \{\hat{\tau}_{i,3;n}^{\text{ref}}, \hat{v}_{i;n}^{\text{ref}}\}$. With this final step, we can give a relation between what we want (the reflected wave field), and what we measured (the actual wave field):

$$-\int_{(x_1, x_2) \in \mathbb{R}^2} \hat{\tau}_{i,3;n}^{\text{ref}}(\mathbf{x}|\mathbf{x}^R, s)\hat{v}_{i;m}^a(\mathbf{x}|\mathbf{x}^S, s)dA = -\hat{t}^a(s)\hat{v}_{m,n}^{\text{ref}}(\mathbf{x}^S|\mathbf{x}^R, s) + \mathcal{A}_{m,n}(\mathbf{x}^R, \mathbf{x}^S, s), \quad (10)$$

with

$$\mathcal{A}_{m,n}(\mathbf{x}^R, \mathbf{x}^S, s) = \frac{1}{2}\hat{f}^d(s)\hat{v}_{n,m}^a(\mathbf{x}^R|\mathbf{x}^S, s) - \hat{t}^a(s)\hat{v}_{m,n}^{\text{inc}}(\mathbf{x}^S|\mathbf{x}^R, s) + \int_{(x_1, x_2) \in \mathbb{R}^2} \hat{\tau}_{i,3;n}^{\text{inc}}(\mathbf{x}|\mathbf{x}^R, s)\hat{v}_{i;m}^a(\mathbf{x}|\mathbf{x}^S, s)dA. \quad (11)$$

These equations constitute a set of 9 coupled integral equations of the second kind. All the known terms are grouped together in the term $\mathcal{A}_{m,n}$. $\hat{\tau}_{i,3;n}^{\text{inc}}$ and $\hat{v}_{m,n}^{\text{inc}}$ are analytical expressions, and $\hat{v}_{i;m}^a$ is measured. The term $\hat{\tau}_{i,3;n}^{\text{ref}}$ on the left hand side of eq. (10) has to be written in terms of the particle velocity with the help of the elastodynamic equation: $\hat{\tau}_{i,3;n}^{\text{ref}} = (C_{i,3,p,q}/s)(\partial_p \hat{v}_{q;n}^{\text{ref}})$. The kernel of the integral equations is $\hat{v}_{i;m}^a(\mathbf{x}|\mathbf{x}^S, s)$.

There is one case for which the nine equations simplify significantly: sources and receivers in the x_2 direction, with x_2 invariant media. This configuration measures SH-waves. Since SH-waves are decoupled from the other waves in x_2 invariant media, there is only one integral equation left, with $m = 2$ and $n = 2$.

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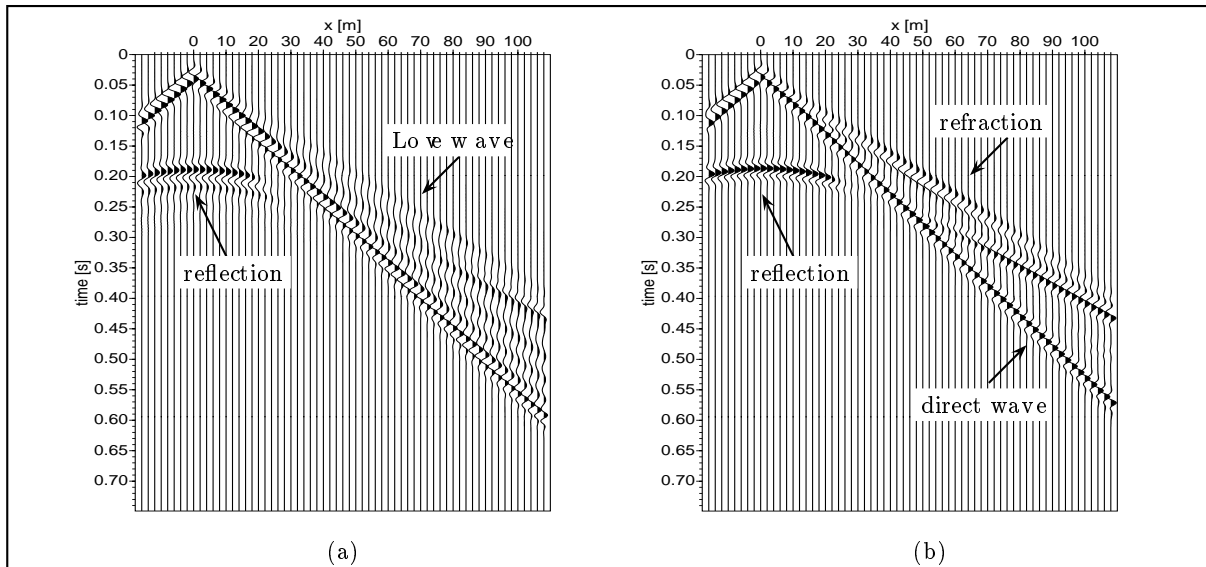


Fig. 2: SH-wave data, a) with stress free surface, b) without surface

Example with finite difference modeling

In order to show some results we hope to accomplish, we made an example with finite difference modeling, developed by Falk (1998). We computed the wave field resulting from both sources and receivers put in the x_2 direction, so we computed the SH-wave field. The example was calculated with and without a stress free surface. The subsurface model is as follows: First, there is a small layer with a depth of 1.2 m, having an S-wave velocity of 200 m/s. Then there is a second layer with a depth of 22.0m, which has an S-wave velocity of 300 m/s. Finally, the lower halfspace underneath has an S-wave velocity of 350 m/s. The results of the finite difference modeling were somewhat "improved" by using AGC, so the reflections would be more clearly visible.

The Figures 2a) and 2b) show the result for the SH-wave data. In the case when there is a stress free surface, there is a Love wave present, which obscures the lower layer reflection. This is exactly the problem with shear wave seismics. There is only a small part of the reflection visible, making it difficult to find a rms-velocity, and leaving only a low stack fold. When there is no surface, the Love wave has obviously disappeared, and the lower layer reflection is no longer obscured. This can for example lead to a better velocity estimation. Unfortunately, the reflection of the first layer is obscured by the direct wave. The refraction however does come forward. When applying the filter as described in eq. (10), this direct wave will also be filtered, and the reflections will be even more clear.

Conclusions

The procedure presented in this paper for removing surface effects from land seismic data is a promising

technique. Further work will concentrate on making this technique operational. The theory should be expanded to buried sources and receivers. As in the acoustic case, the source wavelet is needed to eliminate the surface effects. Hence the next step to be taken is a procedure for wavelet estimation. After that we can test the technique. First on artificial data, next on real seismic data.

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