Two-way and one-way representations of seismic data for highly heterogeneous media

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Summary

The '3-D non-linear two-way representation' as well as the '3-D generalized primary one-way representation' of seismic data both account for the anisotropic dispersive effects of the small scale variations ('fine detail') of the medium parameters. In the former representation the fine detail is included deterministically in the contrast function whereas in the latter it is included statistically in the propagation operators. Hence, given the bandlimitation of the seismic source, it appears that the generalized primary representation allows an upscaling of the fine detail in the *macro model* and a spatial filtering of the *scattering operator.* Therefore this representation is a well suited starting point for the development of 3-D modeling-as well as imaging techniques that take the effects of the small scale variations into account.

Introduction

In seismic modeling as well as in seismic imaging it is common use to ignore the rapid spatial variations of the medium parameters at a scale smaller to much smaller than the seismic wavelength ('fine detail'). However, it has been recognized by many researchers that these small scale variations may seriously affect the propagation properties of the seismic wave field. In particular, in the last three decades much research has been done on wave propagation through finely layered media (the 1-D problem). These studies have shown that the small scale variations manifest themselves as an apparent anisotropic dispersion of the seismic wave field. Hence, ignoring these variations in modeling or imaging implies that AVO effects are not properly handled. In this paper we review various two-way and one-way representations of 3-D seismic data and we briefly discuss their potential for incorporating the effects of fine detail in true amplitude modeling and imaging.

Two-way representations

General considerations

Consider an inhomogeneous acoustic medium, characterized by the compression modulus $K(\boldsymbol{x})$ and the mess density $\rho(\boldsymbol{x})$, where $\boldsymbol{x}$ is the Cartesian coordinate vector (x, y, z). The acoustic pressure in this medium due to a source distribution $S(\boldsymbol{x})$ is denoted (in the frequency domain) by $P(\boldsymbol{x})$. The frequency variable ω is suppressed for notational convenience. We define a *macro model,* characterized by $\bar{K}(\boldsymbol{x})$ and $\bar{\rho}(\boldsymbol{x})$. The Green's function in this macro model is denoted by $G(\boldsymbol{x}, \boldsymbol{x}')$. Moreover we define a contrast model, characterized by $\Delta K(\boldsymbol{x}) = K(\boldsymbol{x}) - \bar{K}(\boldsymbol{x})$ and $\Delta\rho(\boldsymbol{x}) = \rho(\boldsymbol{x}) - \bar{\rho}(\boldsymbol{x})$. The wave equations for $P(\boldsymbol{x})$ and $G(\boldsymbol{x}, \boldsymbol{x}')$ read, respectively,

$$\nabla \cdot \left(\frac{1}{\bar{\rho}}\nabla P\right) + \frac{\omega^2}{\bar{K}}P = -\underbrace{\left[S - \frac{\omega^2 \Delta K}{\bar{K}K}P - \nabla \cdot \left(\frac{\Delta\rho}{\bar{\rho}\rho}\nabla P\right)\right]}_{\text{'source distribution'}} \qquad \text{(1a)}$$

and

$$\nabla \cdot \left(\frac{1}{\bar{\rho}}\nabla G\right) + \frac{\omega^2}{\bar{K}}G = -\underbrace{\delta(\boldsymbol{x} - \boldsymbol{x}')}_{\text{'point source'}} . \qquad \text{(1b)}$$

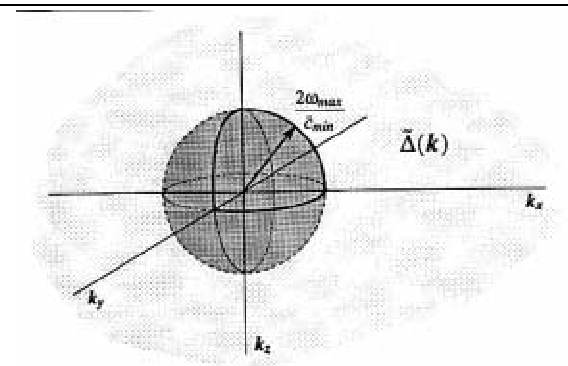

Figure 1: *Fourier transformed contrast function. The dark shaded area contributes to the scattered wave field, as described by the linearized two-way representation (3).*

$P(\boldsymbol{x})$ and $G(\boldsymbol{x}, \boldsymbol{x}')$ satisfy the same wave equation, with different 'source terms'. Hence, the two-way representation for the acoustic pressure is found by applying the superposition principle, according to

$$P(\boldsymbol{x}) = P^i(\boldsymbol{x}) + P^s(\boldsymbol{x}), \qquad \text{(2a)}$$

where the incident field is given by

$$P^i(\boldsymbol{x}) = \int_{\mathcal{R}^3} G(\boldsymbol{x}, \boldsymbol{x}')S(\boldsymbol{x}')d^3\boldsymbol{x}' \qquad \text{(2b)}$$

and the scattered field by

$$P^s(\boldsymbol{x}) = \int_{\mathcal{R}^3} G(\boldsymbol{x}, \boldsymbol{x}')\Big[\{\frac{-\omega^2 \Delta K}{\bar{K}K}\}(\boldsymbol{x}')P(\boldsymbol{x}') - \nabla' \cdot \left(\{\frac{\Delta\rho}{\bar{\rho}\rho}\}(\boldsymbol{x}')\nabla' P(\boldsymbol{x}')\right)\Big]d^3\boldsymbol{x}', \qquad \text{(2c)}$$

(Morse and Ingard, 1968). Equation (2) is an integral equation of the second kind for $P(\boldsymbol{x})$. In the following we assume for convenience $S(\boldsymbol{x}) = S_0(\boldsymbol{x})\delta(\boldsymbol{x} - \boldsymbol{x}_S)$, hence $P^i(\boldsymbol{x}) = G(\boldsymbol{x}, \boldsymbol{x}_S)S_0(\boldsymbol{x}_S)$ Moreover, we assume $\Delta\rho(\boldsymbol{x}) = 0$ and we introduce the contrast function $\Delta(\boldsymbol{x}) = (-\omega^2 \Delta K)/(\bar{K}K)$.

Linearized two-way representation

Assuming the contrasts are sufficiently small, then $P(\boldsymbol{x}')$ in the right-hand side of (2c) may be replaced by the incident field $P^i(\boldsymbol{x}')$ (Born approximation). Hence, we obtain for the scattered wave field

$$P^s(\boldsymbol{x}) \approx \int_{\mathcal{R}^3} G(\boldsymbol{x}, \boldsymbol{x}')\Delta(\boldsymbol{x}')G(\boldsymbol{x}', \boldsymbol{x}_S)S_0(\boldsymbol{x}_S)d^3\boldsymbol{x}'. \qquad (3)$$

Note that the Born approximation accounts only for *primary* scattering. We now want to investigate to what extent the small scale variations of $\Delta(\boldsymbol{x})$ are included in $P^s(\boldsymbol{x})$. We are only interested in a qualitative indication, so for convenience we replace the inhomogeneous macro model by a homogeneous background mediun with velocity $\bar{c}_{min} = \text{Min}\{\bar{K}(\boldsymbol{x})/\bar{\rho}(\boldsymbol{x})\}^{1/2}$. We introduce $\tilde{\Delta}(\boldsymbol{k})$ a

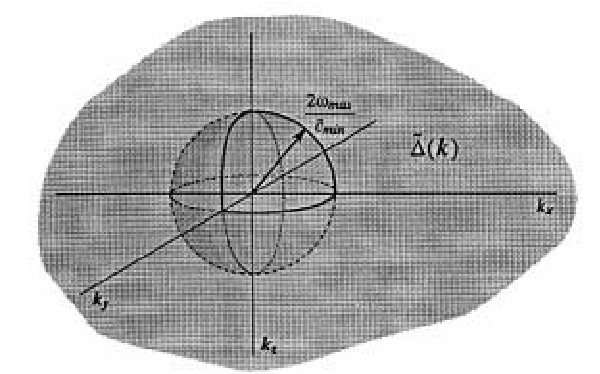

Figure 2: Fourier transformed contrast function. The dark shaded area (i.e., the entire domain) contributes to the scattered wave field, as described by the non-linear two-way representation (4).

the 3-D spatial Fourier transform of $\Delta(\boldsymbol{x})$. This is rcprcscnted by the shaded area in Figure 1. The spherical surface indicates the wavenumber limitation of the scattered wave field P”(x), projected on $\tilde{\Delta}(\boldsymbol{k})$ (see for example Wu and Toksöz (1987), who analyze the wavenumber coverage for different acquisition configurations). The radius of this spherical surface is given by $2\omega_{max}/\bar{c}_{min}$, where ω_{max} is the maximum frequency of the source *S*. Since $\Delta(\boldsymbol{x})$ appears *linearly* in equation (3), only the wavenumber components of $\tilde{\Delta}(\boldsymbol{k})$ within the spherical surface (the dark shaded area in Figure 1) contribute to P”(z). Hence, the Born approximation does ***not*** account for the small scale variations of $\Delta(\boldsymbol{x})$, represented by the light shaded area in Figure 1. This also implies, on the other hand, that for those situations where the Born approximation is justified, it suffices to insert a spatially filtered version of $\Delta(\boldsymbol{x})$ in equation (3).

Non-linear two-way representation

An iterative solution of equation (2) is given by the following Neumann series expansion

$$P^{(n)}(\boldsymbol{x}) = P^{i}(\boldsymbol{x}) + \int_{\mathcal{R}^3} G(\boldsymbol{x},\boldsymbol{x}')\Delta(\boldsymbol{x}')P^{(n-1)}(\boldsymbol{x}')d^3\boldsymbol{x}', \tag{4}$$

for $n > 0$, with $P^{(0)}(\boldsymbol{x}) = P^{i}(\boldsymbol{x})$. For $n = 1$ this reduces to the Born approximation. For $n > 1$ equation (4) accounts for primary as well as multiple scattering. In order to investigate qualitatively to what extent the small scale variations of the contrast parameters are included in $P^{(n)}(\boldsymbol{x})$, we consider again a homogeneous background medium with velocity $\bar{c}_{min} = \mathrm{Min}\{\bar{K}(\boldsymbol{x})/\bar{\rho}(\boldsymbol{x})\}^{1/2}$. Obviously for $n > 1$ equation (4) is *non-linear* in the contrast parameters. Hence, we cannot longer state that only the wavenumber components of $\tilde{\Delta}(\boldsymbol{k})$ within the spherical surface with radius $2\omega_{max}/\bar{c}_{min}$ contribute to $P^{(n)}(\boldsymbol{x})$. On the contrary, for $n \to \infty$ the full function $\tilde{\Delta}(\boldsymbol{k})$, denoted by the dark shaded area in Figure 2, contributes to $P^{(n)}(\boldsymbol{x})$. Hence, this non-linear representation *does* account for the small scale variations of $\Delta(\boldsymbol{x})$. This also implies that the spatial filtering of $\Delta(\boldsymbol{x})$, which is to some extent unavoidable in practice, should be carried out with utmost care.

One-way representations

General considerations

At the basis of the one-way representations lies the one-way wave equation for downgoing and upgoing waves, which will be briefly reviewed here. First consider the two-way wave equation in matrix form (Woodhouse, 1974)

$$\frac{\partial \boldsymbol{Q}}{\partial z} - \hat{\boldsymbol{A}}\boldsymbol{Q} = \boldsymbol{D}, \quad \text{where} \quad \boldsymbol{Q} = \begin{pmatrix} -\boldsymbol{T}_z \\ \boldsymbol{V} \end{pmatrix}. \tag{5a,b}$$

$\boldsymbol{Q}(\boldsymbol{x})$ is the two-way wave vector. In the acoustic situation -T,(z) is the acoustic pressure and $\boldsymbol{V}(\boldsymbol{x})$ the vertical component of the velocity; in the elastodynamic situation $\boldsymbol{T}_z(\boldsymbol{x})$ is the traction vector and $\boldsymbol{V}(\boldsymbol{x})$ the velocity vector. $\boldsymbol{D}(\boldsymbol{x})$ is the two-way source vector and $\hat{\boldsymbol{A}}(\boldsymbol{x})$ is the two-way operator matrix. The circumflex denotes an operator containing the horizontal differentiation operators $\partial/\partial x$ and $\partial/\partial y$. Next, we decompose the acoustic two-way wave equation into a system of one-way wave equations for downgoing and upgoing waves. We introduce a *one-way wave vector* $\boldsymbol{P}(\boldsymbol{x})$ and a *one-way source vector* $\boldsymbol{S}(\boldsymbol{x})$ according to

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{P}^{+} \\ \boldsymbol{P}^{-} \end{pmatrix} \quad \text{and} \quad \boldsymbol{S} = \begin{pmatrix} \boldsymbol{S}^{+} \\ \boldsymbol{S}^{-} \end{pmatrix} \tag{6a,b}$$

In the acoustic situation $\boldsymbol{P}^{+}(\boldsymbol{x})$ and $\boldsymbol{P}^{-}(\boldsymbol{x})$ represent the downgoing and upgoing normalized acoustic pressure; in the elastodynamic situation $\boldsymbol{P}^{+}(\boldsymbol{x})$ and $\boldsymbol{P}^{-}(\boldsymbol{x})$ are vectors, containing the normalized downgoing and upgoing quasi-P, quasi-S1 and quasi-S2 waves. Similar remarks apply to $\boldsymbol{S}^{+}(\boldsymbol{x})$ and $\boldsymbol{S}^{-}(\boldsymbol{x})$. Our aim is to find an equation for $\boldsymbol{P}(\boldsymbol{x})$ of the same form as equation (5), in such a way that the leading term of the operator is (block-) diagonal. To this end we introduce *pseudo-differential* operator matrices $\hat{\boldsymbol{L}}(\boldsymbol{x})$, $\hat{\boldsymbol{\Lambda}}(\boldsymbol{x})$ and $\hat{\boldsymbol{L}}^{-1}(\boldsymbol{x})$ that satisfy the relation

$$\hat{\boldsymbol{A}} = -j\omega \hat{\boldsymbol{L}}\hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{L}}^{-1}, \tag{7}$$

in such a way that $\hat{\boldsymbol{\Lambda}}(\boldsymbol{x})$ is (block-) diagonal. For an extensive list of references on the theoretical and numerical aspects of this decomposition for the acoustic situation, see Fishman et al (1987). For the elastodynamic situation see Wapenaar and Berkhout (1989) and de Hoop and de Hoop (1993). For an overview of the analogous eigenvalue decomposition problem in stratified media, see Ursin (1983). Finally we introduce the following *composition* relations

$$\boldsymbol{Q} = \hat{\boldsymbol{L}}\boldsymbol{P} \quad \text{and} \quad \boldsymbol{D} = \hat{\boldsymbol{L}}\boldsymbol{S}. \tag{8a,b}$$

Upon substitution of equations (7) and (8) into equation (5), we obtain after some straightforward manipulations the following system of coupled one-way wave equations

$$\frac{\partial \boldsymbol{P}}{\partial z} - \hat{\boldsymbol{B}}\boldsymbol{P} = \boldsymbol{S}, \quad \text{where} \tag{9}$$

$$\hat{\boldsymbol{B}} = -j\omega\hat{\boldsymbol{\Lambda}} + \hat{\boldsymbol{\Delta}}_0, \quad \text{with} \quad \hat{\boldsymbol{\Delta}}_0 = -\hat{\boldsymbol{L}}^{-1}\frac{\partial \hat{\boldsymbol{L}}}{\partial z}. \tag{10a,b}$$

The operator matrices $\hat{\boldsymbol{\Lambda}}(\boldsymbol{x})$ and $\hat{\boldsymbol{\Delta}}_0(\boldsymbol{x})$ have the following structure

$$\hat{\boldsymbol{\Lambda}} = \begin{pmatrix} \hat{\boldsymbol{\Lambda}}^{+} & \boldsymbol{O} \\ \boldsymbol{O} & -\hat{\boldsymbol{\Lambda}}^{-} \end{pmatrix} \quad \text{and} \quad \hat{\boldsymbol{\Delta}}_0 = \begin{pmatrix} \hat{\boldsymbol{T}}^{+} & \hat{\boldsymbol{R}}^{-} \\ -\hat{\boldsymbol{R}}^{+} & -\hat{\boldsymbol{T}}^{-} \end{pmatrix}. \tag{11a,b}$$

It appears that the one-way wave equation distinguishes explicitly between propagation ((block-) diagonal operator $\underline{\hat{\Lambda}}(\boldsymbol{x})$) and *scattering* (non-diagonal operator $\underline{\hat{\Delta}}_0(\boldsymbol{x})$). This property is exploited in the various one-way representations below.

We introduce a reference operator $\underline{\hat{\bar{B}}}$ which governs a one-way Green's matrix $\underline{G}(\boldsymbol{x},\boldsymbol{x}')$. The equations for $\boldsymbol{P}(\boldsymbol{x})$ and $\underline{G}(\boldsymbol{x},\boldsymbol{x}')$ thus read

$$\frac{\partial \boldsymbol{P}}{\partial z} - \underline{\hat{\bar{B}}}\boldsymbol{P} = \underbrace{\boldsymbol{S} + \{\underline{\hat{B}} - \underline{\hat{\bar{B}}}\}\boldsymbol{P}}_{\text{'source distribution'}}, \tag{12a}$$

and

$$\frac{\partial \underline{G}}{\partial z} - \underline{\hat{\bar{B}}}\underline{G} = \underbrace{\underline{I}\delta(\boldsymbol{x}-\boldsymbol{x}')}_{\text{'point source'}}. \tag{12b}$$

$\boldsymbol{P}(\boldsymbol{x})$ and $\underline{G}(\boldsymbol{x},\boldsymbol{x}')$ satisfy the same wave equation, with different 'source terms'. Hence, the one-way representation for the wave vector $\boldsymbol{P}(\boldsymbol{x})$ is found by applying the superposition principle, according to

$$\boldsymbol{P}(\boldsymbol{x}) = \boldsymbol{P}^i(\boldsymbol{x}) + \boldsymbol{P}^s(\boldsymbol{x}), \tag{13a}$$

where the incident field is given by

$$\boldsymbol{P}^i(\boldsymbol{x}) = \int_{\mathcal{R}^3} \underline{G}(\boldsymbol{x},\boldsymbol{x}')\boldsymbol{S}(\boldsymbol{x}')d^3\boldsymbol{x}' \tag{13b}$$

and the scattered field by

$$\boldsymbol{P}^s(\boldsymbol{x}) = \int_{\mathcal{R}^3} \underline{G}(\boldsymbol{x},\boldsymbol{x}')\{\underline{\hat{B}}(\boldsymbol{x}') - \underline{\hat{\bar{B}}}(\boldsymbol{x}')\}\boldsymbol{P}(\boldsymbol{x}')d^3\boldsymbol{x}', \tag{13c}$$

(Wapenaar, 1993). Equation (13) is an integral equation of the second kind for $\boldsymbol{P}(\boldsymbol{x})$. In the following we assume for convenience $\boldsymbol{S}(\boldsymbol{x}) = \boldsymbol{S}_0(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{x}_S)$, hence $\boldsymbol{P}^i(\boldsymbol{x}) = \underline{G}(\boldsymbol{x},\boldsymbol{x}_S)\boldsymbol{S}_0(\boldsymbol{x}_S)$.

Primary one-way representation

For the reference operator we choose $\underline{\hat{\bar{B}}} = -j\omega\underline{\hat{\Lambda}}$, hence, the contrast operator is given by $\underline{\hat{B}} - \underline{\hat{\bar{B}}} = \underline{\hat{\Delta}}_0$. Since $\underline{\hat{\Lambda}}$ is a (block-) diagonal operator matrix (in the true medium!), $\underline{G}$ will also get a (block-) diagonal structure, according to

$$\underline{G} = \begin{pmatrix} \underline{W}_p^+ & \underline{O} \\ \underline{O} & -\underline{W}_p^- \end{pmatrix}. \tag{14}$$

We refer to $\underline{W}_p^+(\boldsymbol{x},\boldsymbol{x}')$ and $\underline{W}_p^-(\boldsymbol{x},\boldsymbol{x}')$ as the propagation operators for the *primary* downgoing and upgoing waves in the true medium. Note that $\underline{W}_p^+(\boldsymbol{x},\boldsymbol{x}') = [\underline{W}_p^-(\boldsymbol{x}',\boldsymbol{x})]^T = Q$ for $z < z'$. Assuming that the contrast operator $\underline{\hat{\Delta}}_0$ is sufficiently small, we may replace $\boldsymbol{P}(\boldsymbol{x}')$ in the right-hand side of equation (13c) by the incident one-way wave vector $\boldsymbol{P}^i(\boldsymbol{x}')$. Hence

$$\boldsymbol{P}^s(\boldsymbol{x}) \approx \int_{\mathcal{R}^3} \underline{G}(\boldsymbol{x},\boldsymbol{x}')\underline{\hat{\Delta}}_0(\boldsymbol{x}')\underline{G}(\boldsymbol{x}',\boldsymbol{x}_S)\boldsymbol{S}_0(\boldsymbol{x}_S)d^3\boldsymbol{x}'. \tag{15}$$

Next we consider the special situation for which $\boldsymbol{S}_0^- = 0$. We now easily find from equations (6), (11b), (13a,b), (14) and (15)

$$\boldsymbol{P}^-(\boldsymbol{x}) \approx \int_{\mathcal{R}^3} \underline{W}_p^-(\boldsymbol{x},\boldsymbol{x}')\underline{\hat{R}}^+(\boldsymbol{x}')\underline{W}_p^+(\boldsymbol{x}',\boldsymbol{x}_S)\boldsymbol{S}_0^+(\boldsymbol{x}_S)d^3\boldsymbol{x}'. \tag{16}$$

Equation (16) is a straightforward one-way representation of primary reflection data. It was introduced (in a discrete formulation) by Berkhout (1982) for acoustic one-way wave fields in fluids and modified by Wapenaar and Berkhout (1989) for elastodynamic one-way wave fields in solids. The main difference with equation (3) is that the reflection operator $\underline{\hat{R}}^+(\boldsymbol{x})$ is proportional to the *vertical variations* of the medium parameters (see equations 10b and 11b). Hence, for the special situation of a horizontal interface between two homogeneous half-spaces equation (16) reduces to a surface integral, unlike equation (3). Finally, consider a highly heterogeneous acoustic medium and approximate the operator matrix $\underline{\hat{R}}^+(\boldsymbol{x})$ for convenience by a scalar reflection coefficient $R^+(\boldsymbol{x})$. From the similarity between equations (3) and (16) we may thus conclude that the primary one-way representation (16) does *not* account for the small scale variations of $R^+(\boldsymbol{x})$, represented by the light shaded area in Figure 3.

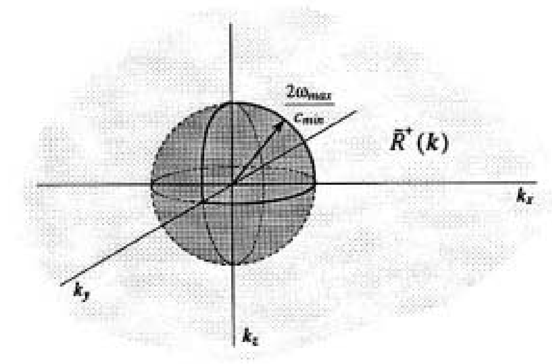

Figure 3: Fourier transfomed reflection coefficient. The dark shaded area contributes to the upgoing wave field, as described by the primary one-way representation (16) as well as by the generalized primary one-way representation (23). (In the latter representation the detail is included in the propagation operator!).

Generalized primary one-way representation

In order to include multiple scattering we could formulate an iterative solution of equation (13), analogous to equation (4). However, here we follow a different approach that exploits the natural distinction between propagation and scattering in the one-way operator. For the reference operator we write

$$\underline{\hat{\bar{B}}}(\boldsymbol{x}|\zeta) = -j\omega\underline{\hat{\Lambda}}(\boldsymbol{x}) + H(\zeta - z)\underline{\hat{\Delta}}_0(\boldsymbol{x}), \tag{17}$$

where $H(z)$ is the Heaviside step function. Hence, for a given value of ζ, $\underline{\hat{\bar{B}}}(\boldsymbol{x}|\zeta)$ applies to a configuration that is identical to the true medium for the upper half-space $z < \zeta$ and that is 'scatter-free' (along the z-axis) for the lower half-space $z > \zeta$. We let $\underline{\hat{\bar{B}}}(\boldsymbol{x}|\zeta)$ govern a Green's matrix $\underline{G}(\boldsymbol{x},\boldsymbol{x}'|\zeta)$ and we let a similar operator $\underline{\hat{\bar{B}}}(\boldsymbol{x}|\xi)$ govern a reference wave vector $\bar{\boldsymbol{P}}(\boldsymbol{x}|\xi)$. Note that $\bar{\boldsymbol{P}}(\boldsymbol{x}|\infty) = \boldsymbol{P}(\boldsymbol{x})$. Now instead of equation (13) we may write

$$\bar{\boldsymbol{P}}(\boldsymbol{x}|\xi) - \bar{\boldsymbol{P}}(\boldsymbol{x}|\zeta) =$$

$$\int_{\mathcal{R}^3} \underline{G}(\boldsymbol{x},\boldsymbol{x}'|\zeta)\{\underline{\hat{\bar{B}}}(\boldsymbol{x}'|\xi) - \underline{\hat{\bar{B}}}(\boldsymbol{x}'|\zeta)\}\bar{\boldsymbol{P}}(\boldsymbol{x}'|\xi)d^3\boldsymbol{x}', \tag{18a}$$

where

$$\bar{\boldsymbol{P}}(\boldsymbol{x}|\zeta) = \underline{G}(\boldsymbol{x},\boldsymbol{x}_S|\zeta)\boldsymbol{S}_0(\boldsymbol{x}_S). \tag{18b}$$

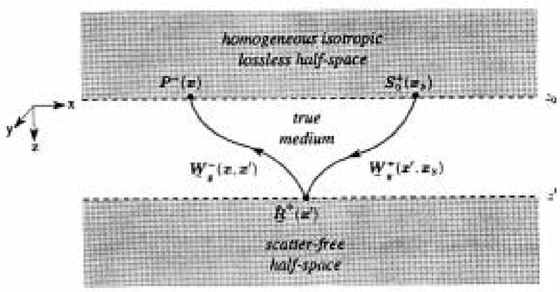

Figure 4: The generalized primary representation. $\underset{\sim}{W}_g^+(x', x_S)$ *and* $\underset{\sim}{W}_g^-(x, x')$ *are defined in the true medium between* z_0 *and* z' *and a scatter-free half-space below* z'. $P^-(x)$ *is defined in the true medium (*z' *ranges from* $-\infty$ *to* ∞ *in equation (23)).*

Next we choose $\xi = \zeta + d\zeta$ and we take the limit for $d\zeta \to 0$. We thus obtain

$$\frac{\partial \bar{P}(x|\zeta)}{\partial \zeta} = \int_{\mathcal{R}^3} \underset{\sim}{G}(x, x'|\zeta) \frac{\partial \hat{\underset{\sim}{B}}(x'|\zeta)}{\partial \zeta} \bar{P}(x'|\zeta) d^3x'. \tag{19}$$

Substituting equations (17) and (18b) into (19) and replacing ζ by z' in the result yields

$$\frac{\partial \bar{P}(x|z')}{\partial z'} = \int_{\mathcal{R}^2} \underset{\sim}{G}(x, x'|z') \hat{\underset{\sim}{\Delta}}_0(x') \underset{\sim}{G}(x', x_S|z') S_0(x_S) d^2x'_H, \tag{20}$$

where x'_H = (x', y'). Integrating both sides with respect to z' from $-\infty$ to ∞ yields

$$P(x) = P^i(x) + P^s(x), \tag{21a}$$

where

$$P^i(x) = \underset{\sim}{G}(x, x_S|-\infty) S_0(x_S) \tag{21b}$$

and

$$P^s(x) = \int_{\mathcal{R}^3} \underset{\sim}{G}(x, x'|z') \hat{\underset{\sim}{\Delta}}_0(x') \underset{\sim}{G}(x', x_S|z') S_0(x_S) d^3x'. \tag{21c}$$

Next we choose x_S and x in a homogeneous upper half-space $z \leq z_0$, so that for $z' > z_0$ we may write

$$\underset{\sim}{G}(x', x_S|z') = \begin{pmatrix} \underset{\sim}{W}_g^+ & \underset{\sim}{O} \\ \underset{\sim}{O} & \underset{\sim}{O} \end{pmatrix} (x', x_S) \tag{22a}$$

and

$$\underset{\sim}{G}(x, x'|z') = \begin{pmatrix} \underset{\sim}{O} & \underset{\sim}{O} \\ \underset{\sim}{O} & -\underset{\sim}{W}_g^- \end{pmatrix} (x, x'), \tag{22b}$$

(remember that for $\underset{\sim}{G}$ the half-space below z' is scatter-free, see Figure 4). We refer to $\underset{\sim}{W}_g^+$ and $\underset{\sim}{W}_g^-$ as the propagation operators for the *generalized primary* downgomg and upgoing waves, respectively. Finally, we consider again the special situation for which S_0^- = *O.* Hence, we obtain in a similar way as before

$$P^-(x) = \int_{\mathcal{R}^3} \underset{\sim}{W}_g^-(x, x') \hat{\underset{\sim}{R}}^+(x') \underset{\sim}{W}_g^+(x', x_S) S_0^+(x_S) d^3x', \tag{23}$$

see Figure 4. Note that no approximations have been made in this section, hence, the effects of the small scale variations arc included. As a matter of fact, they are included in the generalized primary propagation operators $\underset{\sim}{W}_g^+(x', x_S)$ and $\underset{\sim}{W}_g^-(x, x')$ For practical applications these operators may bc parameterized (Herrmann and Wapenaar, 1993) or they may be defined in an extended macro model (with anisotropic anelastic losses) that accounts for the effects of the small scale variations (Wapenaar and Slot, 1994). In both cases the deterministic fine detail is *upscaled* to statistic *macro* parameters. Last, but not least, since equation (23) is truly *linear* in $\hat{\underset{\sim}{R}}^+(x')$[1], it suffices to replace this operator by a spatially filtered version, as indicated qualitatively by the dark shaded area in Figure 3.

Conclusions

From the 3-D representations discussed in this paper, only the 'non-linear two-way representation' and the 'generalized primary one-way representation' account for the anisotropic dispersive effects of the small scale variations of the medium parameters (fine detail). In the former representation the fine detail is included deterministically in the contrast function whereas in the latter it is included statistically in the propagation operators. Given the bandlimitation of the seismic source, the generalized primary representation allows an upscaling of the fine detail in the *macro model* operators and a spatial filtering of the *scattering operator*. Since it is truly linear in the scattering operator, it is also an excellent starting point for true amplitude imaging techniques that take the anisotropic dispersion effects into account (Wapcnaar and Herrmann, 1993).

References

Berkhout, A.J.,1982, Seismic migration: Elsevier, Amsterdam

de Hoop, M.V., and de Hoop, A.T., 1993, Elastic wave up/down decomposition in inhomogeneous and anisotropic media: an operator approach and its approximations: submitted to Geophysical Prospecting

Fishman, L., McCoy, J.J., and Wales, S.C., 1987, Factorization and path integration of the Helmholtz equation: Numerical algorithms: J. Acoust. Soc. Am., 81, 1355-1376

Herrmann, F.J., and Wapenaar, C.P.A., 1993, Wave propagation in finely layered media, a parametric approach: 63rd annual SEG meeting, Washington, 909-912

Morse, P.M., and Ingard, K.U., 1968, Theoretical acoustics: McGraw-Hill, New York

Ursin, B., 1983, Review of elastic and electromagnetic wave propagation in horizontally layered media: Geophysics, 48, 1063-1081

Wapenaar, C.P.A., and Berkhout, A.J., 1989, Elastic wave field extrapolation: Elsevier, Amsterdam

Wapenaar, C.P.A., 1993, Representations of seismic reflection data. Part II: new developments: J. Seism. Expl., 2, 247-255

Wapenaar, C.P.A., and Herrmann, F.J., 1993, True amplitude migration taking fine-layering into account: 63rd annual SEG meeting, Washington, 653-656

Wapenaar, C.P.A., and Slot, R.E., 1994, Towards an extended macro model, that takes fine-layering into account: J. Seism. Expl., (accepted)

Woodhouse, J.H., 1974, Surface waves in a laterally varying layered structure: Geoph. J. R. astr. Soc., 37, 461-490

Wu, R.S., and Toksöz, M.N., 1987, Diffraction tomography and multi-source holography applied to seismic imaging: Geophysics 52, 11-25

[1]This is fundamentally different from the linearized approximations (3) and (16).