True Amplitude Migration Taking Fine-Layering into Account

C.P.A. Wapenaar and F.J. Herrmann, Delft University of Technology, Netherlands*

Summary

The matched filter approach to inverse wavefield extrapolation ignores the amplitude and phase distortions due to fine-layering. In this paper we introduce a modified matched filter that properly accounts for these effects. The correction term by which the matched filter is modified can be derived directly from the data. Using this modified matched filter in prestack migration will result in a non-dispersed image with correct AVA behaviour.

Theory

The underlying assumption of any migration scheme is that a macro model of the subsurface accurately accounts for the propagation effects from the acquisition surface to the target and vice versa. A macro model consists of a number of geologically oriented macro layers, separated by macro boundaries. In general, the velocity and density in each macro layer are chosen to be simple functions of position (for instance linear functions of depth, accounting for the depth dependent compaction properties). Recent studies on wave propagation through finely layered media have shown that internal multiple scattering effectively results in an angle-dependent dispersion of the wave field [Burridge and Chang, 1989]. In the following we refer to this dispersed wave field as the 'generalized primary'. Figure 1 (a) shows an example of a homogeneous macro model and the construction of the forward extrapolation operator $\mathbf{W}_p^+(z|z_0)$, where p denotes 'primary'. Figure 1 (b) shows a 1-D example of a model with fine layering and the construction of the forward extrapolation operator $\mathbf{W}_g^+(z|z_0)$, where g denotes 'generalized primary'. Current macro models do not account for this dispersion effect. Consequently, this effect is also ignored in migration, which may result in dispersed images and erroneous amplitude versus angle (AVA) effects. In Delft we are investigating how to parametrize the effects of fine layering in an ***extended*** macro model [Herrmann and Wapenaar, 1992]. Now the question arises, what are the implications of the fine layering effects for migration? Generally, the inverse wave field extrapolation operators required for migration are approximated by the matched filter approach:

$$\mathbf{F}_p^+(z_0|z) = \mathbf{W}_p^+(z|z_0)^H, \tag{1}$$

(Berkhout, 1982; H stands for complex conjugate transpose). It can be shown that this approach yields accurate results both for homogeneous as well as inhomogeneous macro models (provided the one-way wave fields are properly scaled). Does this approach also hold for the generalized primary extrapolation operator, defined in an extended macro model? Unfortunately the answer is negative: the dispersion effects in the generalized primary wave are accompanied with an amplitude loss which is not compensated for by the matched filter. Hence, just as is the case with anelastic losses, the matched filter fails to account for losses due to fine layering. There is an important difference, however, between anelastic losses and losses due to fine layering. Whereas anelastic losses represent a conversion of seismic energy into heat, the losses related to fine layering represent a conversion of 'forward propagating seismic energy' into 'back scattered seismic energy', see Figure 1 (b). The 'forward propagating seismic energy' at depth level z can be quantified by the multi-dimensional autocorrelation of the downgoing generalized primaries, i.e. $\mathbf{W}_g^+(z|z_0)^H\mathbf{W}_g^+(z|z_0)$. Similarly, the 'backscattered seismic energy' at the acquisition surface z_0 can be quantified by the multi-dimensional autocorrelation of the upgoing wave fields at z_0, i.e. $\mathbf{X}_0^{(z)}(z_0|z_0)^H\mathbf{X}_0^{(z)}(z_0|z_0)$, where $\mathbf{X}_0^{(z)}(z_0|z_0)$ contains the (deconvolved) reflection measurements at the surface, see Fig. 1 (b). By using the power reciprocity theorem for one-way wave fields [Wapenaar, 1993] it can be shown that the following relation holds

$$\mathbf{I} - \mathbf{X}_0^{(z)}(z_0|z_0)^H\mathbf{X}_0^{(z)}(z_0|z_0) = \mathbf{W}_g^+(z|z_0)^H\mathbf{W}_g^+(z|z_0), \tag{2}$$

which can be interpreted as: 'normalized incident energy' minus 'normalized backscattered energy' equals 'normalized forward propagating energy'. We use the quotes because energy is a scalar quantity wheras the quantities in equation (2) are all matrices. Equation (2) can be rewritten as

$$\mathbf{F}_g^+(z_0|z)\,\mathbf{W}_g^+(z|z_0) = \mathbf{I} \tag{3}$$

with

$$\mathbf{F}_g^+(z_0|z) = \left[\mathbf{I} - \mathbf{X}_0^{(z)}(z_0|z_0)^H\mathbf{X}_0^{(z)}(z_0|z_0)\right]^{-1}\mathbf{W}_g^+(z|z_0)^H \tag{4}$$

In the following we call $\mathbf{F}_g^+(z_0|z)$ the ***modified*** matched filter for inverse wave field extrapolation of the downgoing generalized primary. It contains the matched filter $\mathbf{W}_g^+(z|z_0)^H$ for the generalized primaries defined in an extended macro model, and a correction term that can be derived entirely from the reflection measurements (see Fig. 1 (b)). In practice, the matrix inversion in (4) is replaced by a Neumann series expansion. Note that this equation holds for 3-D inhomogeneous (anisotropic) acoustic or elastic media. Using reciprocity, the modified matched filter for the upgoing generalized primary follows immediately:

$$F_g^-(z|z_0) = F_g^+(z_0|z)^T . \quad (5)$$

The underlying assumption for this approach is that the propagation losses may be entirely ascribed to the fine layering (scattering losses only). In the presentation it will be indicated how this approach can be generalized when anelastic losses play a role as well.

Example

To illustrate the limitations of the matched filter and the potential of the modified matched filter we consider a simple example. Fig. 2 shows a 1-D acoustic medium consisting of 15,000 layers with a thickness of 10 cm each. The statistics of the fine-layering are described by fractal Brownian motion. The average velocity $\bar{c}$ equals 2500 m/s. We modelled the upgoing plane waves, propagating from the bottom to the top of the configuration. The rayparameter p ranges from 0.0 to $0.9/\bar{c}$, hence, the propagation angle ranges from 0^0 to 64^o. The lower frame in Fig. 2 shows the modelling input at the bottom of the configuration. It will serve as a reference for the inverse extrapolation output. The upper frame in Fig. 2 shows the modelling output at the top of the configuration. Note the angle dependent amplitude and phase distortions. This result will serve as the input for the inverse extrapolation experiments. Fig. 3 shows the results of four different inverse extrapolations from the top to the bottom of the configuration in Fig. 2. Fig. 3 (a) was obtained by applying the matched filter, as defined in equation (l), using a homogeneous macro model with $\bar{c}$ = 2500 m/s. The only effect is a time shift per plane wave experiment, hence, the angle dependent amplitude and phase distortions are not accounted for. Fig. 3 (b) was obtained with a matched filter defined in the true model. The results are zero-phase, but the AVA behaviour is worse than in Fig. 3 (a), because the amplitude decay occurred twice (during modelling and during inverse extrapolation). Next we consider the modified matched filter, as defined in equations (4) and (5), with the matrix inversion replaced by a truncated Neumann expansion. Fig. 3 (c) and Fig. 3 (d) show the results of taking, respectively, one and five terms of the Neumann series into account. Particularly the last result (Fig. 3 (d)) shows a very good amplitude recovery up to very high propagation angles (compare with the lower frame of Fig. 2).

Conclusions

We have discussed an inverse extrapolation operator that accounts for the scattering losses of a 'generalized primary' propagating through a finely layered medium. This operator is given by the matched filter (defined in an extended macro model), premultiplied by a correction term that can be entirely derived from the reflection measurements. We have illustrated with a simple example that this modified matched filter properly accounts for the angle dependent amplitude and phase distortions related to the scattering losses. By integrating the proposed operator in prestack migration, one may expect to obtain a non-dispersed image with correct AVA behaviour.

Acknowledgments

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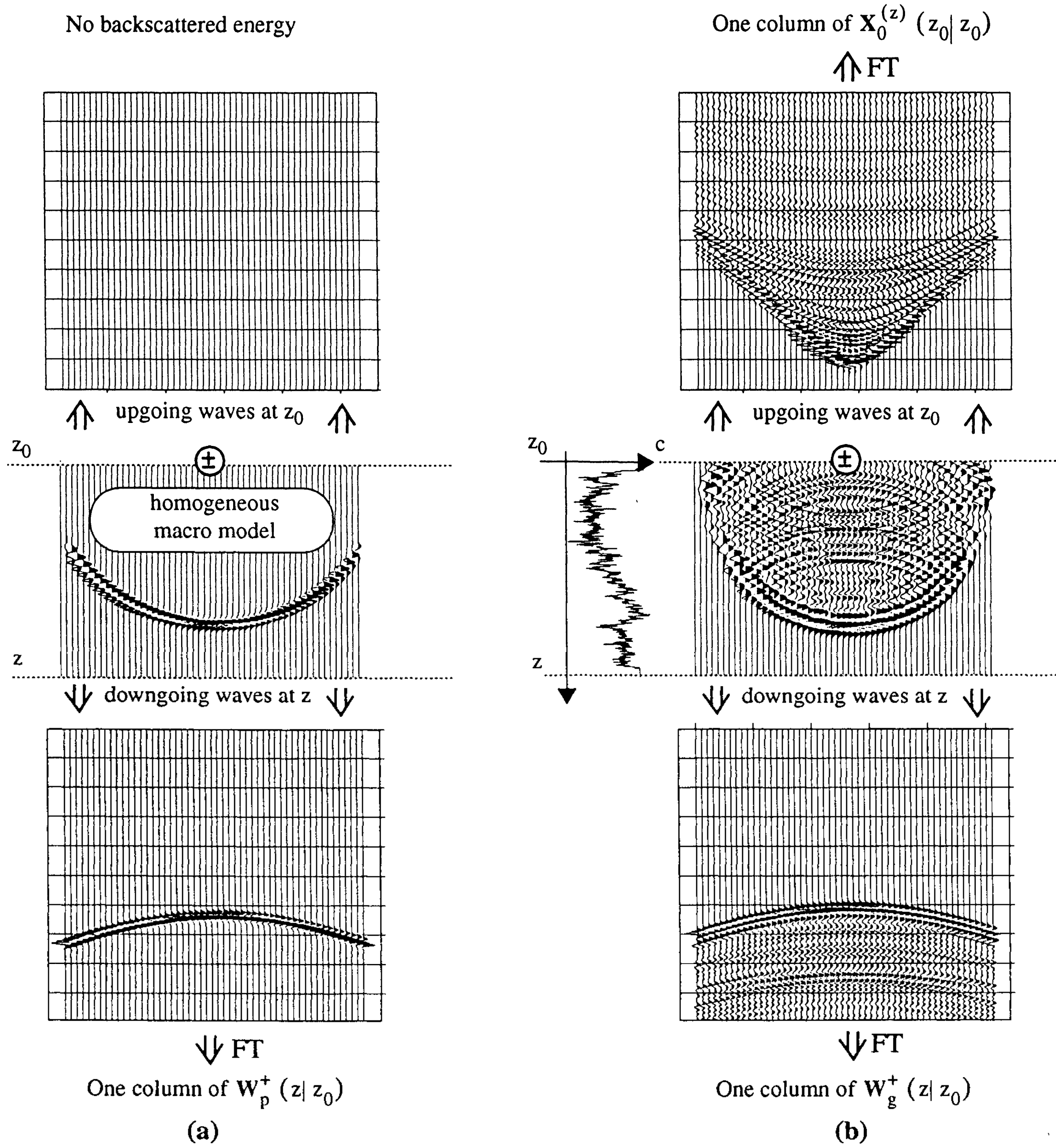

Fig 1. Construction of the forward operators in a homogeneous model (a) and in a model with fine layering (b). The subscripts p and g denote 'primary' and 'generalized primary', respectively.

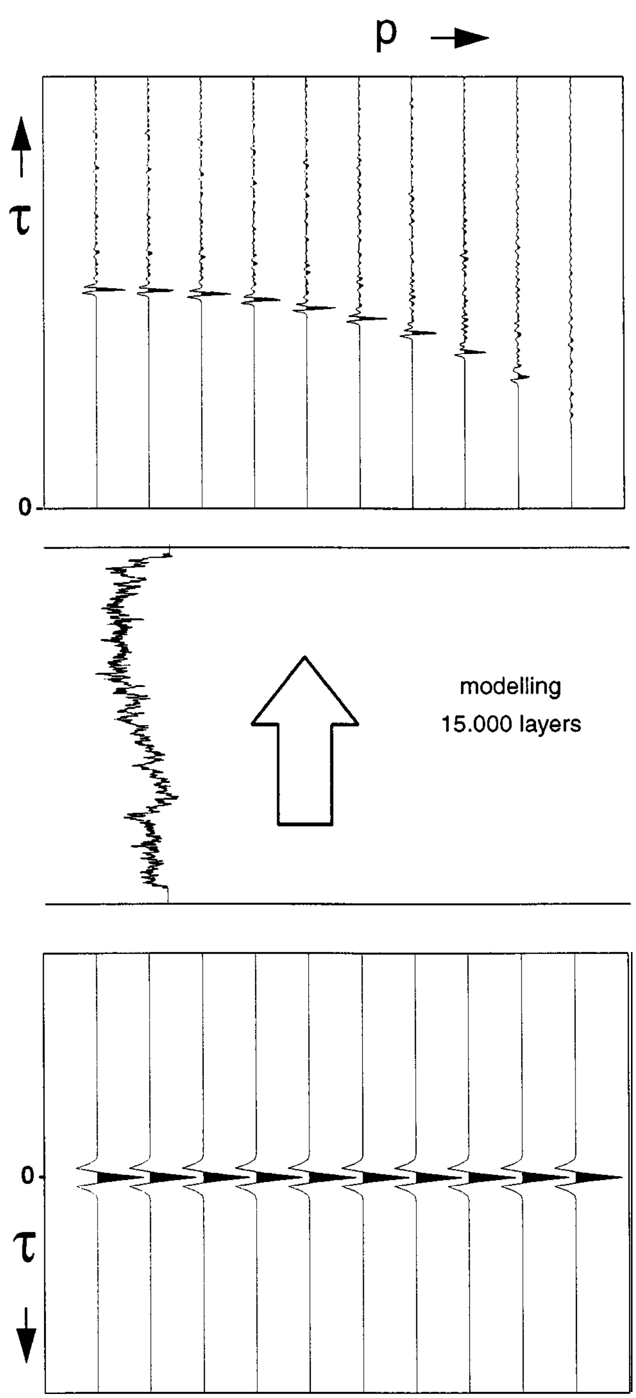

Fig. 2 Plane wave transmission response of a 1 - D acoustic medium (the time axes in both frames are different)

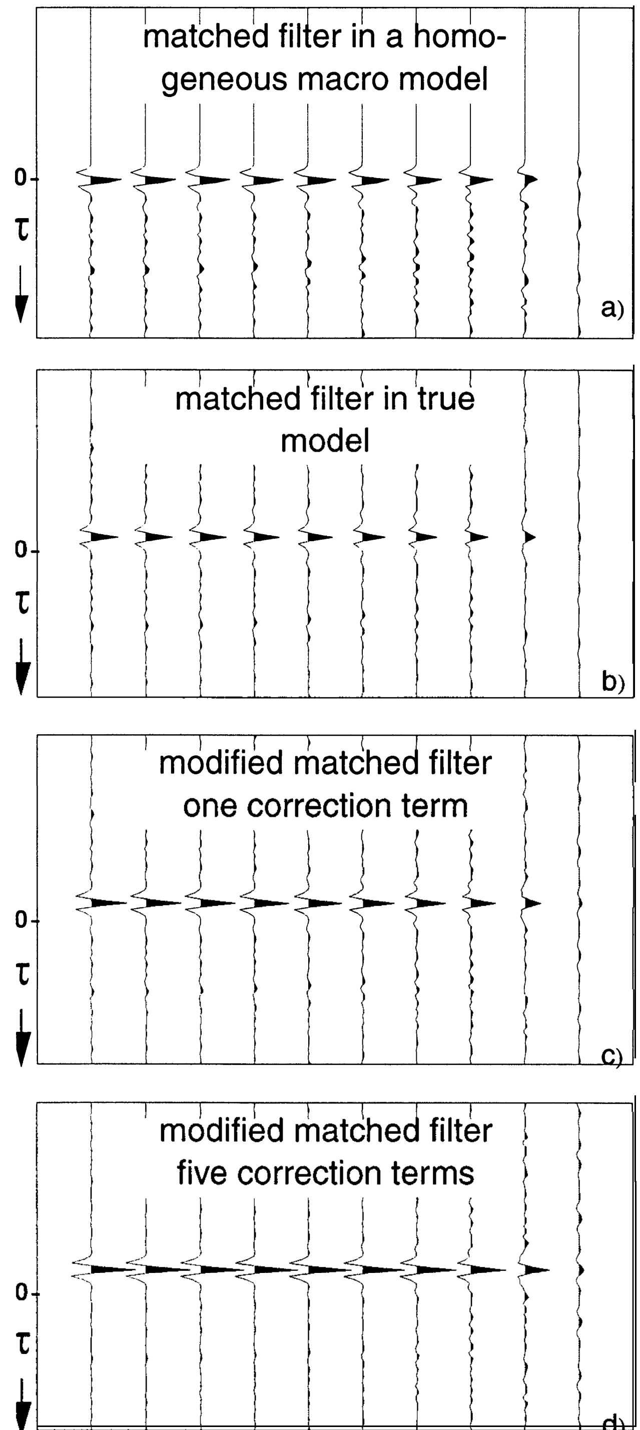

Fig. 3 Inverse extrapolation results, using (a) matched filter in homogeneous macro model, (b) matched filter in true model, (c) modified matched fllter with one correction term and (d) modifieed matched filter with five correction terms.