Macro Description of Fine Layering: Proposal for an Extended Macro Model

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Summary

Inspection of well-log measurements reveals that the medium parameters in the earth's subsurface behave irregularly. This irregularity tends to persist to the smallest length scales. Contemporary considerations on seismic wave propagation (down and up) limit themselves to the overall (smooth) trend in the subsurface (macro model). This paper addresses the study of the propagation effects related to the detail (fine layering), i.e. the rock and pore parameters of the individual layers within the resolution of the measurements.

As a wavefield propagates through the earth's subsurface its initial waveform gradually changes due to the presence of detail. This change in shape is not captured by the conventional macro model parameterisations which are based on the trend (compaction properties) in and throughout the stratigraphic sections. The main objective of this paper is to present an extended macro model which accounts for these dispersive propagation effects.

Introduction

The seismic response, measured at the surface, represents a mixture of propagation and reflection information. It is nowadays common practice to attribute the propagation effects to the macro layering in the subsurface and the reflection behaviour (within the seismic wavelength) to the detail (Berkhout 1985). Both theoretical and practical observations have indicated that the distortion of the pulse shape due to the detail can be significant (Resnick 1985; Herrmann 1991).

It will be shown that the quantitative effects acting on the wavefield can be accounted for by including additional information on the stratification (fine layering) to the conventional macro model. This information concerns only two parameters, the value of which depends on the geology of the macro layer.

In order to find a global parameterisation for the influence of detail on wave propagation a step by step approach will be followed. At first attention will be paid to the main features of acoustic wave propagation (normal incidence) in finely layered 1D media. It appears that the change in pulse shape is primarily determined by the second order statistics of the medium fluctuations (detail). So the second point of interest is to find a stochastic model for geologically related beds of thin layers. The combination of the proposed stochastic model (Fractal Brownian Motion FBM) with the expression for the wave propagation operator leads to the definition of an extended macro model. This model consists of the conventional macro quantities and two additional parameters. These parameters, the variance and the slope of the power spectrum of the reflection coefficients, depend on the detail (litho-stratigraphy) in each macro layer.

Wave propagation

Forward model for reflection data

The description of a seismic reflection event of arbitrary complexity can be divided in three main parts: *downward propagation* of the incident wavefield, *reflection* at an interface and a *propagation* of the reflected field back to the surface. Hence, one can write for the full (all interbed multiples included) acoustic 1D reflection response at the surface:

$$X(z_0,z_0;\omega) = \sum_{m=1}^{M} W^-(z_0,z_m;\omega)R^+(z_m)W^+(z_m,z_0;\omega), \qquad (1)$$

where $W^+(z_0,z_m;\omega)$ and $W^-(z_m,z_0;\omega)$ denote the "generalised" propagation operators, describing the propagation to the m^{th} interface and back. $R^+(z_m)$ constitutes the *local* reflection at the m^{th} interface and the total reflectivity is

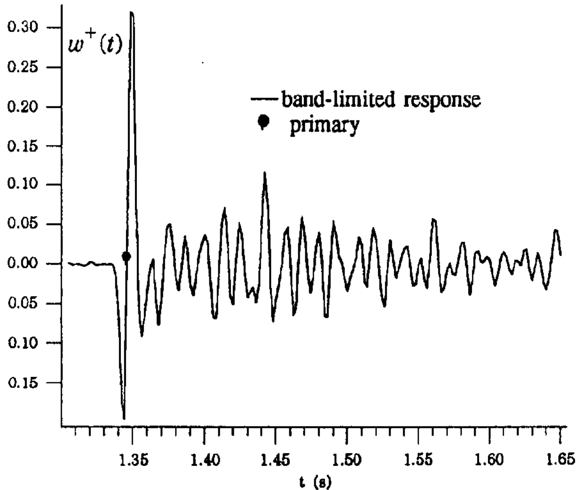

Fig. 1 Exact band-limited transmission response of a well-log with a large primary transmission loss, convolved with a Ricker wavelet (250Hz).

found after summation over all consecutive reflectors. Exact solutions for the generalised propagation operators can be found with a recurrence scheme which is driven by the local properties of the layers in the stack.

Quantitative effects

From the literature and several modelling examples (Fig. 1; Fig. 2) it can be seen that the signature of a plane wave, incident on a stack of many layers with layer thicknesses smaller than the prevailing wavelength, changes due to a complex system of internal multiple reflections. This process results in a frequency dependent amplitude and phase behaviour which arises from the gradual transformation of primary arrivals into a train of delayed internal multiples. These multiples interact with the vanishing primary (denoted by the dot in Fig. 1) and result in a backward shift of the "centre of mass" of the transmission response. This dispersion can easily be recognised in Fig. 2 where the transmission response is depicted at various depth levels for a medium defined in terms of realistic velocity and density functions obtained from well-log measurements. On the other hand it can be observed that the contribution of the multiples can lead to a substantial reinforcement of the "generalised primary" (Fig. 1), i.e. the ensemble of the primary and its first multiples (Burridge 1985; Resnick 1985). This reinforcement is the result of the constructive interference of the primary and its multiples. The amplitudes will, however, never exceed the amplitude yielded by the WKB-solution. Hence, there is always a *dispersive* effect compared to this solution but there can be an apparent *reinforcement* compared to the primary.

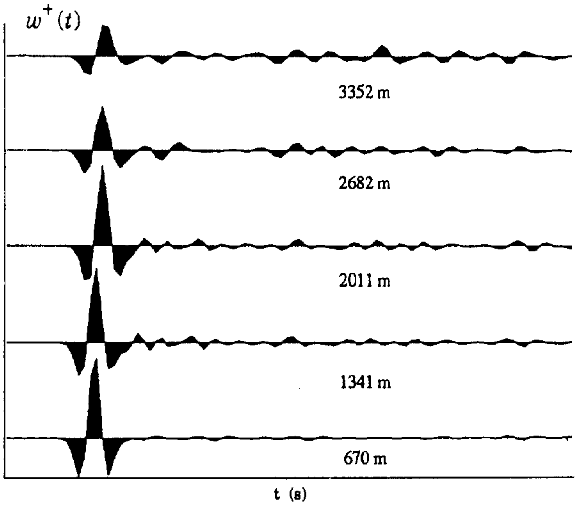

Fig. 2 Band-limited transmission responses; the primary travel times have been corrected for the various depth levels

O'Doherty-Anstey formula

The properties of the "generalised primary" are primarily determined by the mutual correlation of the reflection coefficients. Successive approximations (deterministic) on the forward model lead to a second order propagation operator which is explicitly defined in terms of the autocovariance function of the reflection coefficients. This comprehensive formulation reads

$$\widehat{W}^+(z_0,z_m;\omega) = \left\langle W^+(z_0,z_m;\omega)\right\rangle F_m(2\omega), \quad (2)$$

where $\left\langle W_m^+(z_0,z_m;\omega)\right\rangle$ is the WKB-solution (first order solution) and the second term in equation (2), the O'Doherty-Anstey formula (Anstey and O'Doherty 1971), equals

$$F_m(2\omega) = e^{-A_m(2\omega)}, \quad (3)$$

with $A_m(2\omega)$ representing the Fourier transform of the causal part of the autocovariance function of the reflection coefficients.

Stochastic subsurface model

The reflection coefficient sequence in each macro layer will be regarded as a realisation of a stochastic process. Fractal Gaussian Noise (FGN) yields a random process (Mandelbrot 1985; Walden and Hosken 1985; Herrmann et. al. 1991) of which the characteristics are very similar to the statistical properties of the reflection coefficients evidenced from well-logs. It provides an analytic description of the stochastic expectation for the related power spectra

$$S_r(\omega) \propto \omega^{\alpha} \qquad S_\zeta(\omega) \propto \omega^{-2} S_r(\omega) = \frac{1}{\omega^{\beta}} \qquad \beta = 2-\alpha, \quad (4)$$

of the reflection coefficients ($S_r(\omega)$ <->FGN) and of the acoustic impedances ($S_\zeta(\omega)$ <-> FBM) together with a tool to generate synthetic well-logs. The behaviour of the reflection coefficient sequence strongly depends on α, the slope of the power spectrum, and can be divided in correlated ($\alpha<0$); uncorrelated ($\alpha=0$) and anticorrelated ($\alpha>0$) sequences. FBM and FGN do not possess a characteristic size and have the same statistical properties on every scale range. This non-scaling behaviour, within certain physical bounds, is conform the characteristics of many geological features.

A spectral analysis conducted on well-log measurements shows that the reflection coefficients belong to the anticorrelated category, i.e. the reflection coefficients exhibit, in a statistical sense, a strongly alternating sign. This observation is substantiated by the linear fit, the slope of which determines the degree of anticorrelation (α) of the estimated log-log power spectrum (Fig. 4) yielded by the well-log of Fig. 3.

Fig. 5 emphasises the sensitivity of the "generalised primary" to the degree of anticorrelation of the reflection coefficients whilst the primary transmission loss is kept constant. This example clearly illustrates that the reinforcement increases due to an improved positive interference of the primary and its first multiples.

Parameterisation of detail

Analytic solution

It is known that the shape of the "generalised primary" gradually converges to a limiting solution (Burridge 1985), the form of which is determined by the second order statistics of the medium fluctuations. This convergence is not based on *ensemble* averaging but on a *spatial* averaging process along the wave path.

Given the fact that wave propagation is inherently an averaging process it is possible to find a formulation in terms of the *stochastic expectation* of the medium properties. Hence, the quantitative effects of detail can be captured by a *global* parameterisation without requiring detailed *local* information of the individual layers.

Taking the stochastic expectation of equation (3) yields

$$E[F(2\omega)] = e^{-A(2\omega)} \quad (5)$$

with

$$A(2\omega) = S_r(2\omega) + \hat{S}_r(2\omega) \quad (6)$$

where $S_r(\omega)$ is the expectation of the power spectrum of the reflection coefficients and $\hat{S}_r(\omega)$ its Hilbert transform.

Combination of the proposed stochastic model FGN with equation (6) leads to the *global* parameterisation where the limiting pulse shape is related to the *global* properties of the medium.

Application to a well-log

Comparison is made between the exact multiple response of the log depicted in Fig. 3 and the response yielded by the analytic solution with the global stochastic parameters set, according to the estimates computed from the complete log, see Fig. 4. The result of this procedure is depicted in Fig. 6 and it appears that the signature of the transmission response is remarkably well covered by the analytic solution.

Extended macro model

The subsurface can, on geological grounds, be subdivided into a few major stratigraphic sections. A macro model reflects this subdivision and focusses primarily on the traveltimes of the waves propagating through the earth.

Contemporary seismic processing techniques increasingly depend on true amplitudes. It is therefore of great importance to find a macro parameterisation for the amplitude and phase characteristics of the seismic signal.

The combination of the proposed stochastic model and the approximate propagation operators leads to the definition of an extended macro model. This model consists in the acoustic 1-D case of the conventional macro quantities (defining a piece-wise smooth function) and two additional parameters, namely the variance and slope (α) of the

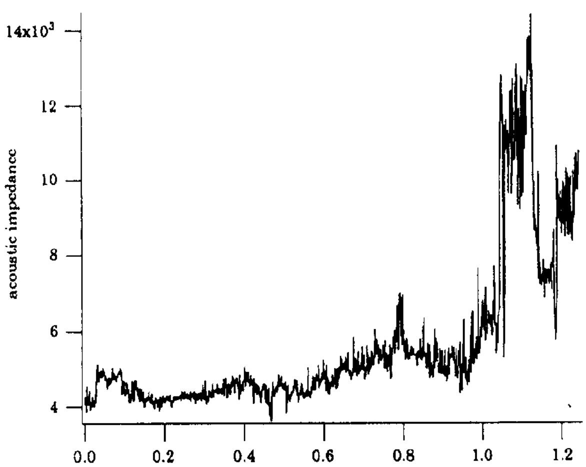

Fig. 3 Acoustic impedances of a real well-log (courtesy Faculty of Petroleum and Mining Engeneering)

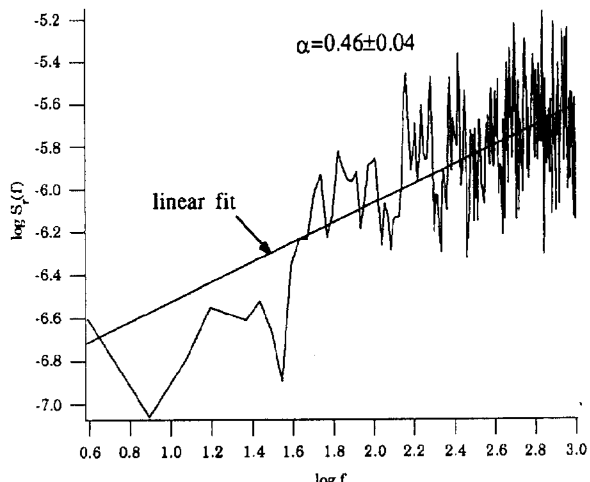

Fig. 4 Estimate of the log-log power spectrum of the reflection coefficients yielded by the log of Fig. 3, α=0.46±0.04

power spectrum of the detail in each macro interval (litho-stratigraphic interval).

A similar strategy can be followed for the effects of detail in 2-D or 3-D subsurface configurations. This generalisation increases the complexity of the forward model and more research has to be conducted on the definition of 2-D or 3-D variations of the stochastic model for the medium fluctuations.

Conclusions

From the analysis presented in this paper it can be concluded that the transparency of the earth's subsurface for seismic energy strongly depends on the degree of irregularity of the medium. So one has to include a priori information of the complexity of the subsurface in order to describe the seismic amplitudes and traveltimes accurately. To be more specific, this prior information consists of two additional *macro model parameters*, the variance and the degree of anticorrelation α, which depend on the geology in each macro layer. This information can be captured from well-logs or other geologic sources.

The proposed formulation also implies that the distorted wavefield carries information on the stratification of the medium and future investigations within our research project (DELPHI consortium) will be aimed at obtaining this information from seismic measurements.

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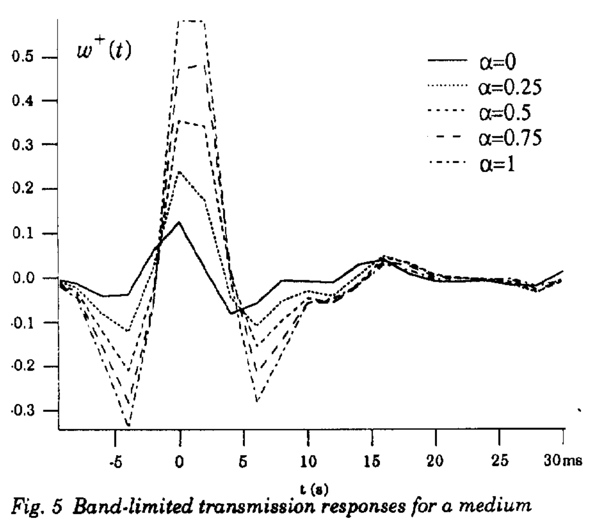

Fig. 5 Band-limited transmission responses for a medium defined by FGN with $0<\alpha<1$

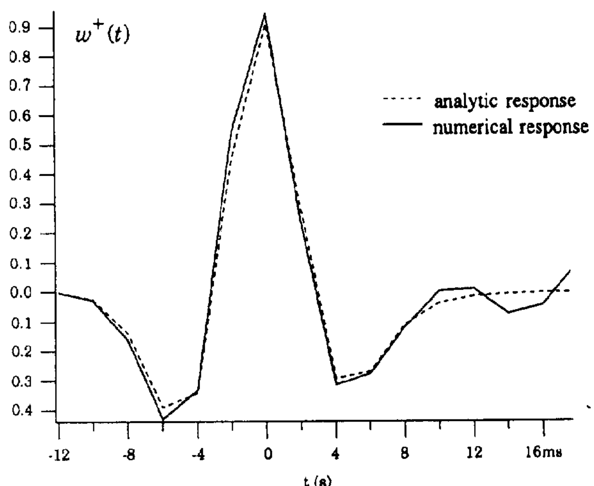

Fig. 6 Comparison of the exact transmission response and its analytic approximation eq. (5)