Elastic Wavefield Decomposition: Before or After Extrapolation?

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Summary

Multi-component seismic data is required to obtain information on the angle-dependent P-P, P-S, S-P and S-S reflectivity of the subsurface. It is generally accepted that wave field decomposition should form part of the multi-component processing sequence, aiming at retrieving this reflectivity information. However, the question whether decomposition should take place before or after downward extrapolation has not yet been unambiguously solved. In this paper we argue that, although both procedures are theoretically equivalent, decomposition before downward extrapolation has a number of practical advantages, one of the reasons being the robustness with respect to macro model errors.

Introduction

Wave field extrapolation of mutli-offset data has become one of the most important processes in modern seismic processing. Since amplitudes play an increasingly important role in seismic interpretation, it is nowadays well accepted that wave field extrapolation should be properly based on the wave equation. Generally the acoustic wave equation is chosen because of its simple *scalar* form. However, with the increasing interest for multi- component data acquisition, the concept of wave field extrapolation should be reconsidered, starting with the *vectorial* elastic wave equation. A straightforward derivation leads to full elastic wave field extrapolation algorithms that act on the total elastic wave field (see for instance Kuo and Dai, 1984 or Chang and McMechan, 1987). Hence, these algorithms simultaneously account for P- and S-waves. As an alternative, in Delft we have developed elastic operators for independent extrapolation of P- and S-waves (Wapenaar and Haimé, 1990). Hence, when the aim is to get undistorted (i.e. true amplitude) information on R_{pp}, R_{ps}, R_{sp} and R_{ss} in the subsurface, two different paths can be followed:

1. Downward extrapolation of the *vector* wave field, followed by decomposition into PP, PS, SP and SS data in the subsurface,

or,

2. Decomposition at the surface into PP, PS, SP and SS data, followed by downward extrapolation of the *scalar* wave fields into the subsurface.

In this paper we address the intriguing question which one of the two procedures should be preferred.

One-way or two-way extrapolation?

Experience with acoustic processing schemes has learned that one-way wave field extrapolation is to be preferred over two-way wave field extrapolation, the reason being that one-way schemes are robust with respect to small errors in the macro subsurface model. Berkhout and Wapenaar (1989) have presented a theoretical justification for the use of one-way extrapolation operators for downgoing or upgoing acoustic wave fields. Of course, true amplitude acoustic one-way wave field extrapolation should always be preceded by a decomposition *at the surface* into downgoing source waves and upgoing received waves, optionally combined with free surface multiple elimination (Verschuur et al, 1989).

Elastic decomposition comprises:

1. Decomposition into downgoing and upgoing waves and
2. Decomposition into P- and S-waves.

Similar as in the acoustic situation, we claim that elastic decomposition into downgoing and upgoing waves should be carried out at the surface, optionally followed by free surface multiple elimination.
Of course the question remains whether decomposition into P- and S-waves should be done at the surface, before elastic one-way extrapolation, or in the subsurface, after extrapolation.
In the following sections we briefly discuss elastic decomposition into downgoing and upgoing waves at the surface, as well as free surface multiple elimination. Next we address the question at which stage the decomposition into P- and S-waves should take place.

Decomposition into downgoing and upgoing waves

Throughout the discussion we consider for simplicity the 2-D situation. Moreover, we consider four-component data acquisition where horizontal and vertical vibrators impose shear and tensile stresses at the free surface and where horizontal and vertical geophones register the tangential and normal component of the particle velocity at the free surface. In the frequency domain, we denote the data by a matrix, according to

$$\mathbf{V}(z_0) = \begin{bmatrix} V_{x,x}(z_0) & V_{x,z}(z_0) \\ V_{z,x}(z_0) & V_{z,z}(z_0) \end{bmatrix} . \qquad (1)$$

The symbol $\mathbf{V}$ refers to the particle velocity; z_0 is the depth of the free surface. The first sub-script refers to the geophone-type (x: horizontal, z: vertical) and the second sub-script refers to the stress source type. Any of the four sub-matrices represents a data matrix, its columns containing monochromatic shot records (Berkhout, 1985). When designing a decomposition algorithm we are free to choose the physical quantity that represents the downgoing and upgoing wave fields. It appears to be attractive to express the decomposed wave fields in terms of stresses for the following three reasons:

1. The stresses imposed by the vibrators at the free surface z_0 are by definition the stresses of the downgoing wave fields at z_0, hence, no source decomposition is required.
2. The free surface reflectivity for upgoing waves expressed in terms of stresses is by definition minus one (-1). This simplifies the multiple elimination significantly (next section).
3. The decomposition of the registered velocities at the free surface into upgoing waves can be carried out independently for each component.

Hence, in the matrix notation of equation (1), the decomposition operator can be diagonalized, according to

$$T^-(z_0) = D(z_0)V(z_0), \tag{2a}$$

or

$$\begin{bmatrix} T^-_{x,x}(z_0) & T^-_{x,z}(z_0) \\ T^-_{z,x}(z_0) & T^-_{z,z}(z_0) \end{bmatrix} = \begin{bmatrix} D_s(z_0) & O \\ O & D_p(z_0) \end{bmatrix} \begin{bmatrix} V_{x,x}(z_0) & V_{x,z}(z_0) \\ V_{z,x}(z_0) & V_{z,z}(z_0) \end{bmatrix} \tag{2b}$$

The symbol T^- refers to the stresses in the upgoing wave fields. The first sub-script refers to the simulated upgoing stress-receiver type and the second sub-script refers to the downgoing stress source type.

A discussion of the decomposition operators $D_s(z_0)$ and $D_p(z_0)$ is beyond the scope of this paper.

We only remark that they largely depend on the S- and P-wave velocity, respectively, of the first layer. Hence, the full matrix $D(z_0)$ depends largely on the Poisson ratio of the first layer.

Free surface multiple elimination

Since the data contain strong surface related multiples they should be removed prior to downward extrapolation.

In analogy with Verschuur et al (1989) we write

$$T^-_0(z_0) = T^-(z_0)\left[I - \frac{1}{S(\omega)} T^-(z_0)\right]^{-1}, \tag{3}$$

where $T^-_0(z_0)$ denotes the multiple free data and where $S(\omega)$ denotes the frequency dependent source signature. The multiples are optimumly removed when the correct source signature is used in (3). Verschuur et al use a minimum energy criteria for estimating $S^{-1}(\omega)$. Upon substitution of (2a) in (3), according to

$$T^-_0(z_0) = D(z_0)V(z_0)\left[I - \left(\frac{1}{S(\omega)}D(z_0)\right)V(z_0)\right]^{-1}, \tag{4}$$

it becomes clear that a similar technique can be used for estimating $S^{-1}(\omega)D(z_0)$, i.e., the source signature and the Poisson ratio of the first layer.

Decomposition into P- and S-waves: before or after downward extrapolation?

The effect of the preprocessing described in the previous two sections, is that upgoing stress "measurements" related to downgoing stress sources are available and that the free surface effects have been removed completely. In this section we compare the following two procedures:

$$T^-_0(z_0) \xrightarrow[\text{extrapolation}]{\text{downward}} T^-_0(z_m) \xrightarrow{\text{decomposition}} X_0(z_m) \tag{5a}$$

and

$$T^-_0(z_0) \xrightarrow{\text{decomposition}} X_0(z_0) \xrightarrow[\text{extrapolation}]{\text{downward}} X_0(z_m) \tag{5b}$$

where z_m refers to a depth level in the subsurface.

In both expressions, matrix $X_0(z_i)$(i=o or i=m) is defined as

$$X_0(z_i) = \begin{bmatrix} X_{pp}(z_i) & X_{ps}(z_i) \\ X_{sp}(z_i) & X_{ss}(z_i) \end{bmatrix}. \tag{6}$$

Any of the four sub-matrices represents again a data-matrix, its columns containing *decomposed* monochromatic shot records. The first sub-script refers to the received upgoing wave type (p: compressional waves, s: shear waves) and the second sub-script refers to the downgoing source wave type. It can be shown theoretically that both procedures (5a) and (5b) yield identical results. However, procedure (5b) has some practical advantages that will now be discussed. In analogy with the acoutic situation (Berkhout, 1985), the downward extrapolation in (5b) can be written as

$$X_0(z_m) = F^-(z_m,z_0)X_0(z_0)F^+(z_0,z_m), \tag{7}$$

where operator $F^-(z_m,z_0)$ accounts for the decomposed upgoing waves and where operator $F^+(z_0,z_m)$ accounts for the decomposed downgoing waves (the integral versions of these operators are discussed in Wapenaar and Haimé, 1990).

Both $F^-(z_m,z_0)$ and $F^+(z_0,z_m)$ contain four sub-matrices, similarly as $X_0(z_i)$ in equation (6). Hence, from equation (7) we may derive

$$X_{pp}(z_m) = F^-_{pp}X_{pp}F^+_{pp} + F^-_{pp}X_{ps}F^+_{sp} + F^-_{ps}X_{sp}F^+_{pp} + F^-_{ps}X_{ss}F^+_{sp} \tag{8a}$$

and

$$X_{ss}(z_m)=F^-_{sp}X_{pp}F^+_{ps}+F^-_{sp}X_{ps}F^+_{ss}+F^-_{ss}X_{sp}F^+_{ps}+F^-_{ss}X_{ss}F^+_{ss} \quad (8b)$$

(for brevity z_0 and z_m are suppressed in the right-hand sides).

Note the interesting property that these equations may be approximated by

$$X_{pp}(z_m) \approx F^-_{pp}(z_m,z_0)X_{pp}(z_0)F^+_{pp}(z_0,z_m) \quad (9a)$$

and

$$X_{ss}(z_m) \approx F^-_{ss}(z_m,z_0)X_{ss}(z_0)F^+_{ss}(z_0,z_m). \quad (9b)$$

In both approximations, the neglected terms are of second order since they all contain a *product* of two converted terms (PS or SP). Similar approximations cannot be derived for the downward extrapolation algorithm in equation (5a).

Equations (9a) and (9b) for downward extrapolation after decomposition have three major practical advantages:

1. Both equations describe independent downward extrapolation of *scalar* wave fields, similar as in the acoustic situation. Hence, after some minor modifications "acoustic software" can be used.
2. The operators in equations (9a) and (9b) are determined by the P-wave and S-wave macro models, respectively. Hence, these equations are the basis for, respectively, P-wave and S-wave macro model estimation.
3. Equations (9a) and (9b) are robust with respect to small errors in the macro subsurface model. Taking the extra terms of equations (8a) and (8b) into account is useful only when the P- and S-wave macro models are fully consistent. In the case of macro model errors, typically encountered in seismic practice, equations (8a) and (8b) involve an uncontrolled "out of phase" superposition of the different contributions. This may lead to less accurate results than those obtained by the approximate equations (9a) and (9b)!
   Note that in equation (5a), where the downward extrapolation is done before decomposition, the "out of phase" superposition occurs implicitly when the macro model is in error.

Decomposition at the surface, followed by downward extrapolation according to equations (9a) and (9b), is illustrated in Figures 1 to 4.

Conclusions

In the acoustic as well as in the elastic situation, one-way wave field extrapolation is preferred over two-way wave field extrapolation because one-way schemes are robust with respect to errors in the macro subsurface model.

Hence, both in the acoustic and in the elastic situation downward extrapolation should be preceded by decomposition into one-way wave fields and, optionally, elimination of the free surface multiples (Verschuur et al (1989)). From a theoretical point of view, further decomposition of the elastic one-way wave fields into P- and S-wave fields can be done either before or after downward extrapolation. From a practical point of view, however, also this second decomposition should be done at the surface because:

1. It allows independent extrapolation of scalar wave fields.
2. It facilitates independent estimation of P- and S-wave macro models.
3. It may give more accurate results than decomposition after downward extrapolation in the case of P- and S-macro model inconsistencies.

Since both decomposition steps (two-way → one-way → P and S) should be done at the surface they may just as well be combined in one algorithm (Wapenaar et al, 1990).

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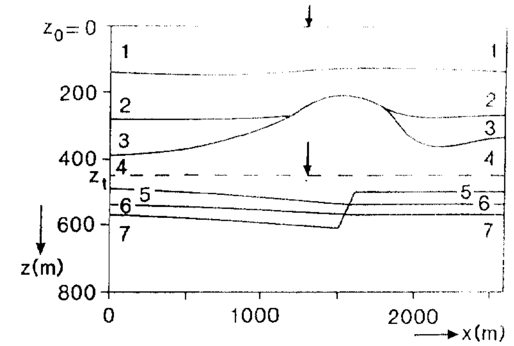

Layer	Cp(m/s)	Cs(m/s)	ρ(kg/m³)
1	2400	1400	1000
2	3000	2000	1600
3	3000	2000	2100
4	4100	2200	2200
5	3700	2000	2300
6	4200	2400	2000
7	3500	2100	1800

Fig. 1: Inhomogeneous elastic medium.

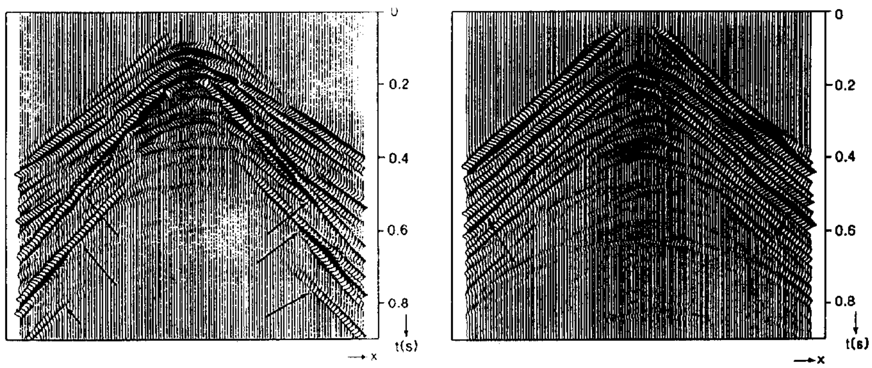

Fig. 2: Multi-component data (Left: $V_{zz}(z_o)$; Right: $V_{xx}(z_o)$).

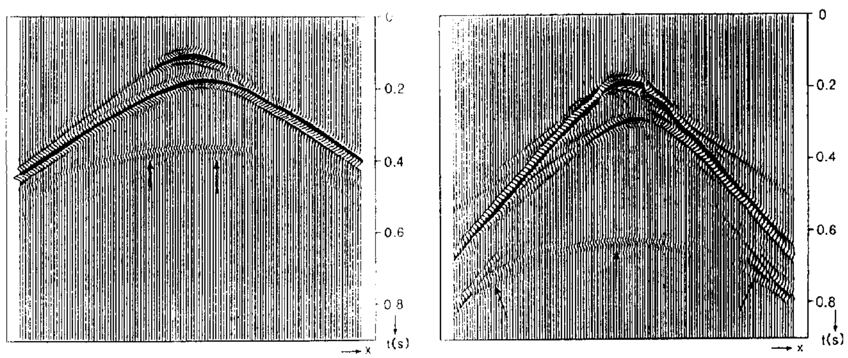

Fig. 3: Decomposed data (Left: $X_{pp}(z_o)$; Right $X_{ss}(z_o)$).

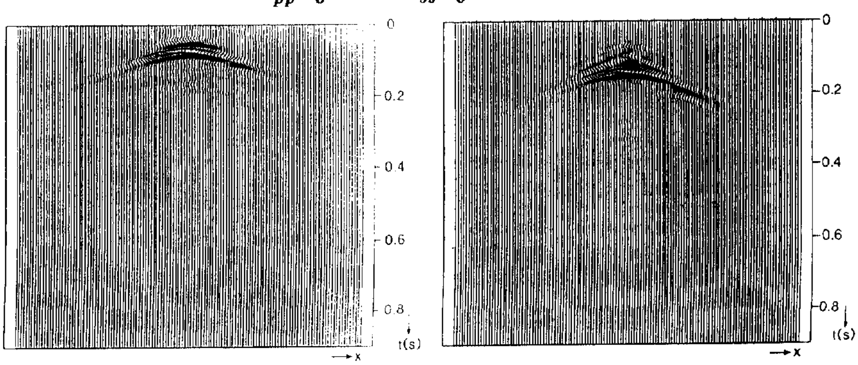

Fig. 4: Downward extrapolated data (Left: $X_{pp}(z_m)$; Right $X_{ss}(z_m)$).