A Systems Approach to Seismic Inversion

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Summary

Nowadays, it is well appreciated that the seismic inversion problem is seriously *ill-posed*. This means a.o. that parametric estimation by matching model responses with field data ('model fitting') has many pitfalls. To make seismic inversion feasible in practice, a hierarchical *subdivision* of the subsurface parameters is required; it leads to a *stepwise* approach to seismic inversion. We have proposed to the industry a stepwise inversion scheme with *three* hierarchical steps, i.e. *surface-related* pre-processing, reflectivity imaging and *target-related* post-inversion.
Using a systems formulation of the forward seismic model, the hierarchical stepwise approach can be easily explained.

Introduction

A simple and familiar way of representing the discretized output of a physical system is given by the matrix equation

$$P^- = XS^+ + N, \tag{1}$$

where S^+ represents the input matrix, P^- represents the output matrix, X defines the system transfer function and N refers to everything in the output that is not explained by XS^+. If the system is considered to be *time invariant* and *linear*, then (1) may be represented per frequency component.
For statistical filtering of seismic traces, the *scalar* version of (1) is a very familiar presentation, but in wave theory based processing a systems formulation in terms of *matrices* is not yet widely accepted.
In the following, systems formulation (1) is extended to a wave theory based model for multi-dimensional seismic data. The model is extremely well suited to design an inversion scheme in terms of pre-processing, imaging and post-imaging inversion.

Systems formulation of the forward model

In Berkhout (1982) a general forward model has been derived for *discrete* seismic reflection data:

$$P^-(z_0) = X(z_0,z_0)S^+(z_0) + N(z_0) \tag{2a}$$

with

$$X(z_0,z_0) = \sum_{m=1}^{M} W^-(z_0,z_m)R^+(z_m)W^+(z_m,z_0), \tag{2b}$$

$$S^+(z_0) = D^+(z_0)S(z_0), \tag{2c}$$

$$P(z_0) = D^-(z_0)P^-(z_0). \tag{2d}$$

In expressions (2a, b, c, d) two corresponding columns in the source matrix S (or S^+) and measurement matrix P (or P^-) refer to one experiment. Matrix operators D^+ and D^- are defined by the boundary conditions at the data acquisition surface ($z=z_0$) as well as the choice of sources and detectors (velocity and/or pressure). Propagation matrices W^+ and W^- define the propagation effects in the measurements; reflectivity matrix R^+ defines the (angle dependent) reflection effects in the measurements. All matrices refer to one Fourier component (frequency domain formulation). The choice of a *frequency* domain formulation has the important consequence that multi-dimensional forward model (2a) is relatively simple. To keep the notation simple as well, the frequency parameter ω has been omitted.

In media with one dimensional *macro* properties, the propagation matrices simplify to Toeplitz matrices and *matrix* equation (2b) can be rewritten as a *scalar* equation in the spatial Fourier domain:

$$\tilde{X}^-(\vec{k}_r,\vec{k}_s,z_0,\omega) = \sum_{m=1}^{M} \tilde{W}^-(\vec{k}_r,\Delta z_m,\omega)\tilde{R}^+(\vec{k}_r,\vec{k}_s,z_m,\omega)\tilde{W}^+(\vec{k}_s,\Delta z_m,\omega)$$

$$\tag{3}$$

with $\Delta z_m=(z_m-z_0)$, $\vec{k}=(k_x,k_y)$, subscript s referring to the source coordinate and subscript r referring to the receiver coordinate. Note that in one dimensional media (macro *and* detail), expression (3) can be further simplified by writing $\vec{k}_s=\vec{k}_r$.

Multiple scattering caused by the surface can be simply introduced by using in (2a):

$$P^+(z_0) = S^+(z_0) + R^-(z_0)P^-(z_0) \tag{4}$$

instead of $S^+(z_0)$, where $R^-(z_0)$ defines the surface reflectivity for upward travelling waves. Expression (4) can also be generalized for *internal* multiple scattering.

The systems formulation of the forward model is not for simulation purposes (there exist excellent finite difference and finite element algorithms of seismic data). The primary objective of our version of the forward model is the design of an applied seismic *imaging* and *inversion* philosophy.

Systems formulation of the inverse problem

In the *forward* problem all details about the data acquisition procedure are known, the elastic properties of the surface and medium (trend *and* detail) are available and the measurements need be computed ('numerical simulation'). In case we start with reflectivity, simulation means

$$\mathbf{R}^{+}(z) \;\rightarrow\; \mathbf{P}(z_0). \tag{5a}$$

In the *inverse* problem, all details about the data acquisition procedure should be known, the measurements are available and the medium parameters need be computed. If the spatial reflectivity distribution is aimed for (reflectivity imaging), inversion means

$$\mathbf{P}(z_0) \;\rightarrow\; \mathbf{R}^{+}(z). \tag{5b}$$

Generally, in reflectivity imaging the *diagonal* elements of $\mathbf{R}^{+}(z)$ are computed only, meaning that the *angle dependence* information of reflection is not aimed for (one reflection coefficient per medium grid point).
If *all* elements of the reflectivity matrix are computed, then for each grid point *angle dependent* reflection information is available as well and reflection imaging can be followed by the computation of the material parameters (post-imaging inversion):

$$\mathbf{R}^{+}(z) \;\rightarrow\; \rho(z), c_p(z), c_s(z), \tag{5c}$$

where $\rho(z)$, $c_p(z)$, $c_s(z)$ are diagonal matrices that represent respectively density, longitudinal velocity and shear velocity (if applicable) at each grid point of depth level z. In most imaging applications such as seismic migration, post-imaging inversion is not carried out. Hence, results are *not* in terms of material parameters but are in terms of local reflectivity.

Stepwise Inversion Scheme

In the first step the surface data $\mathbf{P}(z_0)$ is transformed to the subsurface transfer function $\mathbf{X}(z_0,z_0)$. This means that the first step consists of decomposition ($\mathbf{P}(z_0)$ is replaced by upward travelling reflected wavefields $\mathbf{P}^{-}(z_0)$ due to downward travelling source wave fields $\mathbf{P}^{+}(z_0)$) and surface related multiple elimination combined with source deconvolution ($\mathbf{P}^{+}(z_0)$ is replaced by a deconvolved version of $\mathbf{S}^{+}(z_0)$). Hence, after the first step the surface operators $\mathbf{D}^{+}(z_0)$, $\mathbf{D}^{-}(z_0)$, $\mathbf{S}^{+}(z_0)$ and $\mathbf{R}^{-}(z_0)$ have been estimated and their influence have been removed from the data. In case of land data, the first step is concluded by decomposing $\mathbf{X}(z_0,z_0)$ in separate P- and S- responses.

In the second step subsurface transfer function $\mathbf{X}(z_0,z_0)$ is transferred to 'redatumed' transfer function $\mathbf{X}(z_m,z_m)$ and, next, to subsurface reflectivity $\mathbf{R}^{+}(z_m)$. This means that in the second step the parameters of propagation operators $\mathbf{W}^{+}(z_m,z_0)$ and $\mathbf{W}^{-}(z_0,z_m)$ have been estimated and their influence have been removed from the data ('redatuming') yielding $\mathbf{X}(z_m,z_m)$. Next, by applying the generalized imaging principle, $\mathbf{X}(z_m,z_m)$ is transformed to a band limited version of $\mathbf{R}^{+}(z_m)$. Typically, the *diagonal* elements are chosen for display (standard pre-stack migration result).

Finally, in the third step the parameters of a target zone are estimated by applying a 'model fitting' process on $\mathbf{X}(z_m,z_m)$, the target zone being situated just below $z=z_m$. During this model fitting process the knowledge of $\mathbf{R}^{+}(z_m)$, augmented with geological knowledge, is taken into account.

Significance of the first inversion step

To illustrate the importance of step 1, consider the multi-component response of a hydrocarbon reservoir (fig. 1). Figure 2 shows the effect of inversion step 1 by removing the influence of the surface; Figure 3 shows the effect of the subsequent decomposition into P and S waves. The results speak for themselves. Note that the output of these steps (figure 3) is obtained *without any knowledge* of the *sub*surface.

Conclusions

1. In practical situations, seismic inversion should be based on a *hierarchical, stepwise* approach.

2. Based on a systems formulation of the seismic forward model a three-step inversion scheme has been proposed, consisting of *surface-related* pre-processing, reflectivity imaging and *target-related* post-inversion.

3. The large influence of the physical processes, that occur at the surface, on the subsurface response is not always appreciated. The enormous potential of the *first* inversion step (removal of the surface effects) has been illustrated on elastic finite difference data (compare fig. 1 with fig. 3).

References

Berkhout, A.J., 1982, Seismic migration: Imaging of acoustic energy by wave field extrapolation. A. Theoretical Aspects, 2nd edition: Elsevier Science Publ. Co., Inc.
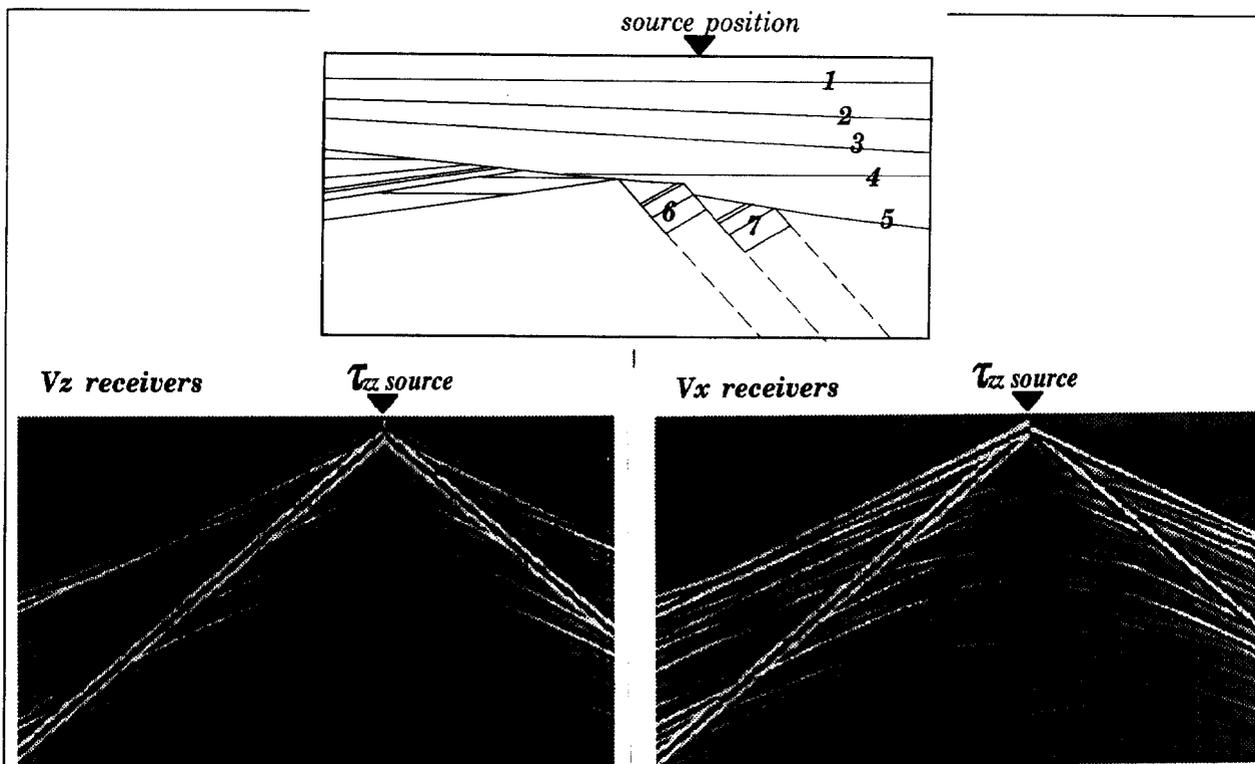
source position

1
2
3
4
6   7
5

Vz receivers τ_{zz} source   Vx receivers τ_{zz} source

Fig. 1: Shot record with free surface.

Vz receivers τ_{zz} source   Vx receivers τ_{zz} source

Fig. 2: Shot record without free surface.

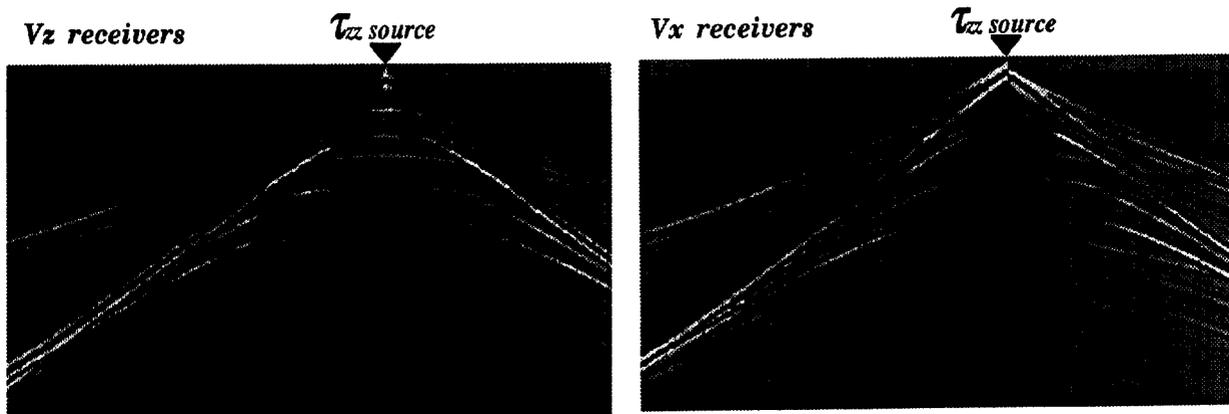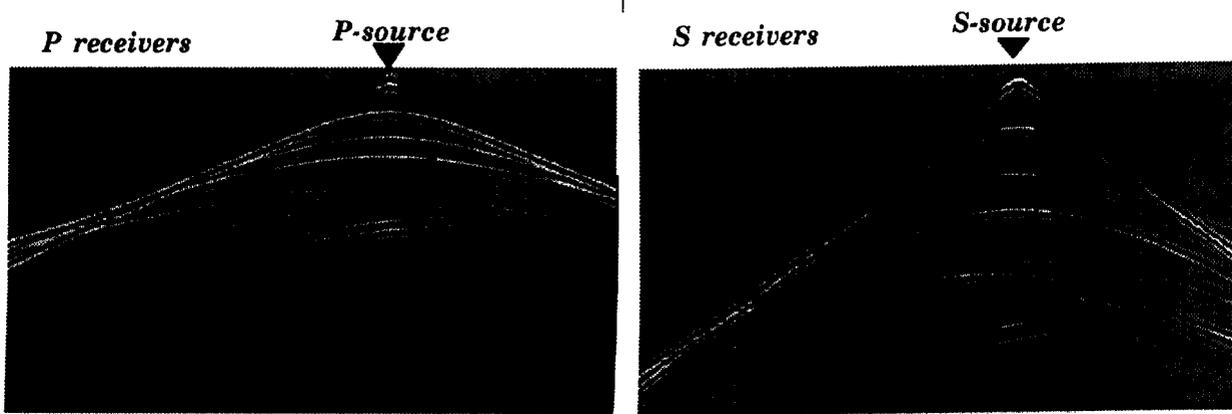P receivers   P-source   S receivers   S-source

Fig. 3: Shot record without free surface and after decomposition into P and S waves.