

**SUMMARY**

The amplitudes in 2-D seismic data depend largely on the 3-D expansion of seismic waves during propagation. In 2-D seismic processing, however, the amplitudes are treated as if they depend on 2-D rather than 3-D expansion of seismic waves. In this paper we introduce an amplitude pre-processing procedure that transforms point source responses into line source responses. By applying this procedure the amplitude handling of any existing 2-D processing technique may be validated. We show the principle for single-component data; the generalization for multi-component data will be discussed during the presentation.

**INTRODUCTION**

By nature, amplitudes in seismic data depend largely on the *three-dimensional* (3-D) expansion of seismic waves during propagation. As a consequence, 3-D seismic processing is a prerequisite if one fully wants to exploit the amplitude information contained in the seismic data. Although in the eighties there has been an important shift towards 3-D seismic data acquisition and processing, even in 1990 a major part of the seismic data acquisition and processing is two-dimensional (2-D). The shortcomings of 2-D seismic processing with respect to structural imaging (migration) are well known and are not discussed here. This paper deals with the shortcomings of 2-D seismic processing with respect to amplitude handling. The underlying assumption of any 2-D seismic processing technique is that the subsurface parameters as well as the seismic wave field are 2-D functions of the horizontal coordinate (x) and the depth coordinate (z). This implies that the seismic waves are assumed to be generated by *line sources* (along the y-axis) rather than *point sources*. Hence, in 2-D processing the amplitudes in the seismic data are treated as if they depend on 2-D rather than 3-D expansion of seismic waves. Needless to say that this yields erroneous amplitudes in the processed seismic data.

To deal with this problem, a time-dependent scaling is often applied to the data. This approach is justified only for acoustic point source data over a constant velocity medium. In this paper we introduce a more elegant amplitude pre-processing procedure that transforms point source responses into line source responses. By applying this procedure, the amplitude handling of any existing 2-D seismic processing technique may be validated. This is not only true for

inversion techniques like pre-stack migration, but also for advanced pre-processing techniques like elastic wave field decomposition and multiple elimination as well as for post-processing techniques like stratigraphic elastic and lithologic inversion.

**FROM POINT SOURCES TO LINE SOURCES**

The principle of transforming a point source response into a line source response is simple: as a line source may be seen as a distribution of point sources along a line (Figure 1a), a line source response is nothing but a superposition of point source responses. The only assumption is that the point source responses are described by a linear wave equation (acoustic or elastic). For a continuous distribution of point sources along the y-axis, the superposition principle may be mathematically formulated as

$$\hat{p}(x_r, z_r; x_s, z_s; t) = \int_{-\infty}^{\infty} p(x_r, y_r=0, z_r; x_s, y_s, z_s; t) dy_s, \quad (1)$$

where  $p(x_r, y_r, z_r; x_s, y_s, z_s; t)$  is a point source response as a function of time (t) at receiver point  $(x_r, y_r, z_r)$  for a point source at  $(x_s, y_s, z_s)$  and where  $\hat{p}(x_r, z_r; x_s, z_s; t)$  is a line source response at receiver point  $(x_r, z_r)$  for a line source at  $(x_s, z_s)$ . When the medium parameters are independent of the y-coordinate, then the response of a point source at  $(x_s, y_s, z_s)$  is just a shifted version of the response of a point source at  $(x_s, 0, z_s)$ :

$$p(x_r, y_r, z_r; x_s, y_s, z_s; t) = p(x_r, y_r - y_s, z_r; x_s, 0, z_s; t). \quad (2)$$

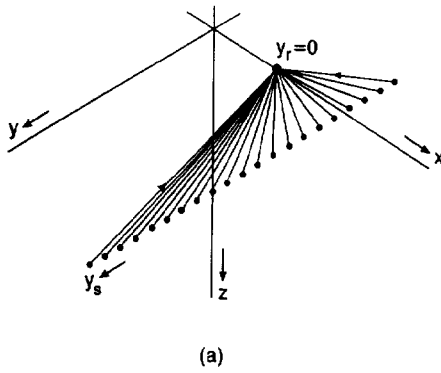
Substitution in equation (1) yields

$$\hat{p}(x_r, z_r; x_s, z_s; t) = \int_{-\infty}^{\infty} p(x_r, -y_s, z_r; x_s, 0, z_s; t) dy_s, \quad (3a)$$

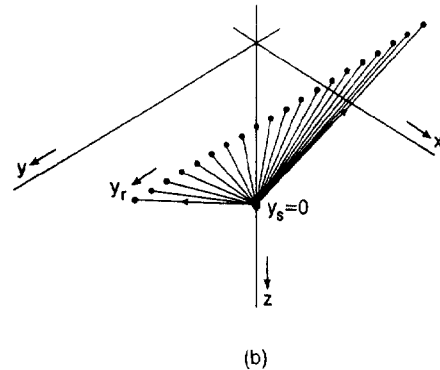
or, renaming the integration variable,

$$\hat{p}(x_r, z_r; x_s, z_s; t) = \int_{-\infty}^{\infty} p(x_r, y_r, z_r; x_s, 0, z_s; t) dy_r. \quad (3b)$$

The latter equation states that the line source response  $\hat{p}(x_r, z_r; x_s, z_s; t)$  may be synthesized from a single point source response by carrying out an integration along the receiver coordinate  $y_r$ . The principle is visualized in Figure 1b. Bear in mind that equation (3b) holds for



(a)



(b)

Figure 1a: A line source response is obtained from many point source responses by integration along the source coordinate  $y_s$ .

Figure 1b: For any 2-D inhomogeneous medium, a line source response may be obtained from one point source response by integration along the receiver coordinate  $y_r$ .

any 2-D inhomogeneous acoustic or elastic medium. However, equation (3b) is not yet suited for practical situations where the response is measured for  $y_r=0$  only. When the response satisfies certain symmetry properties in the  $x_r, y_r$ -plane, then the integral (3b) along the receiver coordinate  $y_r$  may be replaced by an integral along the receiver coordinate  $x_r$ . The most simple situation occurs when the response exhibits cylindrical symmetry with respect to the vertical axis through the source point. This situation is studied in the next section. More complicated forms of symmetry occurring for multi-component seismic data will be discussed during the presentation.

**HORIZONTALLY LAYERED MEDIUM**

Consider a horizontally layered medium bounded by a free surface at  $z=z_0$  with a cylindrically symmetric source at  $(x=0, y=0, z=z_s)$ . Let  $p(x, y, z_r, t)$  denote the response at receiver depth level  $z_r$  as a function of  $x, y$  and  $t$  (for notational convenience the source point coordinates are omitted). With this simplified notation, equation (3b) reads

$$\hat{p}(x, z_r, t) = \int_{-\infty}^{\infty} p(x, y, z_r, t) dy. \tag{4}$$

Throughout this section,  $p$  may either represent the pressure below the free surface or the vertical or radial component of the particle velocity at (or below) the free surface. For both situations the response exhibits cylindrical symmetry with respect to the  $z$ -axis. Hence, at the receiver depth level  $z_r$  at a fixed time  $t$ ,  $p(x, y, z_r, t)$  is constant on circles described by

$$\sqrt{x^2 + y^2} = \text{constant}. \tag{5}$$

Hence, if the response is measured only for  $y=0$ , the unknown response at  $(x, y, z_r)$  should be expressed in terms of the measured response at  $(\rho, 0, z_r)$ , according to

$$p(x, y, z_r, t) = p(\rho, 0, z_r, t), \tag{6a}$$

where

$$\rho = \sqrt{x^2 + y^2}. \tag{6b}$$

Let  $\rho$  be the new integration variable. Then

$$d\rho = \frac{\sqrt{\rho^2 - x^2}}{\rho} dy. \tag{6c}$$

The integration along the  $y$ -axis in equation (4) may thus be replaced by an integration along the  $\rho$ -axis, according to

$$\hat{p}(x, z_r, t) = 2 \int_{|x|}^{\infty} p(\rho, 0, z_r, t) \frac{\rho}{\sqrt{\rho^2 - x^2}} d\rho, \tag{7}$$

see Figure 2. Equation (7) is the basis for transforming the point source response  $p(\rho, 0, z_r, t)$  into a line source response  $\hat{p}(x, z_r, t)$ . The principle is visualized in Figure 3. Some aspects of the numerical evaluation of the integral will be discussed during the presentation.

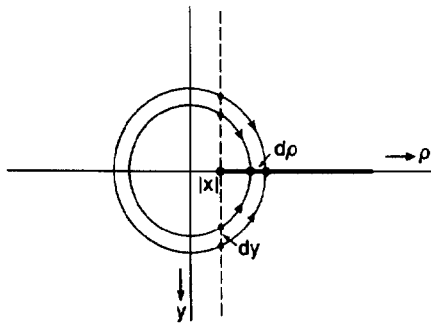


Figure 2: Plan view of Figure 1b. Assuming cylindrical symmetry, the integration along the y-axis for fixed x (dotted line) may be transformed into an integration along the rho-axis, starting at |x| (solid line). The scaling factor  $\rho/\sqrt{\rho^2-x^2}$  in equation (7) accounts for the ratio  $dy/d\rho$ .

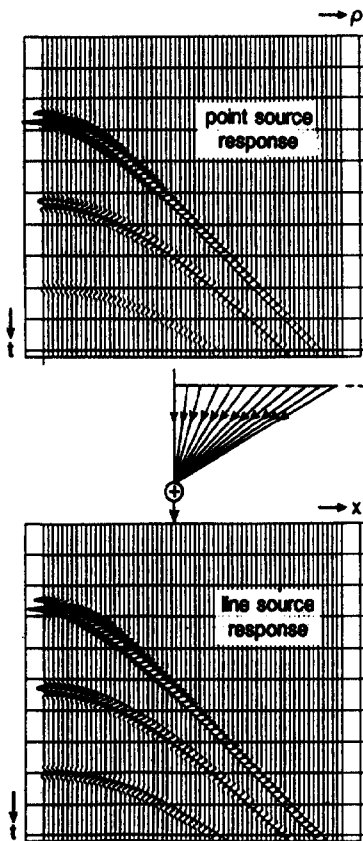


Figure 3: According to equation (7), one trace of a line source response is obtained as a weighted addition of traces of a point source response.

2-D INHOMOGENEOUS MEDIUM

The underlying assumption of equation (7) is that the response is cylindrically symmetric with respect to the z-axis. For an arbitrary 2-D inhomogeneous medium this assumption is violated. A partial remedy is resorting the common source point (CSP) data into common midpoint (CMP) gathers. On basis of reciprocity a CMP gather is symmetric in rho, where rho now stands for source-receiver offset. Of course the cylindrical symmetry is still not fulfilled. However, it can be made plausible that equation (7), applied to CMP gathers, gives accurate results. The main reason is that only a small number of input traces actually contributes to one output trace. This is most easily seen if we consider equation (7) for x=0,

$$\hat{p}(0, z_r, t) = \int_{-\infty}^{\infty} p(\rho, 0, z_r, t) d\rho. \tag{8}$$

Hence, the zero-offset line source response  $\hat{p}(0, z_r, t)$  is nothing but a plain stack of all traces of the point source response. Note that the reflections in  $\hat{p}(0, z_r, t)$  appear at the same times as the reflections in  $p(\rho, 0, z_r, t)$  for small offsets rho (see also Figure 3). Hence, only a few small offset traces contribute to the reflections in  $\hat{p}(0, z_r, t)$ . The information in the higher offset traces cancels during stacking. A direct consequence is that the cylindrical symmetry of  $p(x, y, z_r, t)$  is actually required only for a small range of azimuth angles and therefore CMP gathers are the ideal input for equation (7). Based on these arguments the following amplitude pre-processing procedure is proposed for 2-D single-component seismic data:

1. Resort the CSP gathers into CMP gathers.
2. Treat each CMP gather as a point source response and simulate line source responses by applying equation (7) to each CMP gather.
3. Resort the CMP gathers into CSP gathers.

For a horizontally layered medium this procedure is exact whereas for a 2-D inhomogeneous medium accurate results may be expected. Of course the accuracy will decrease for increasing complexity of the medium; for very complex media 3-D processing is the only acceptable approach.

#### CONCLUSIONS

When 3-D data are available, the best way to go is by full 3-D processing. However, whenever the data acquisition is restricted to line surveys rather than areal surveys, processing is necessarily in two dimensions. In this paper it is argued that 2-D processing is preferably applied *after* transforming the point source responses into line source responses. The effect of this transformation is a correction of the amplitudes in the data. A line source response is nothing but a superposition of point source responses. Hence, in principle a line source response can be simulated by integrating point source responses along the desired line source axis. In practice, however, this integration cannot be carried out due to the incompleteness of the data. In this paper it is shown that the integration along the source axis can be replaced by an integration along the receiver axis. The underlying assumption is that the wave fields exhibit cylindrical symmetry. For single-component data over horizontally layered media this assumption is satisfied. For multi-component data over horizontally layered media the weighting factors must be adjusted. Furthermore, we have found that different components of a point source response must be combined to obtain one component of a line source response. Obviously the conventional time-dependent scaling breaks down for multi-component data. For 2-D inhomogeneous (anisotropic) media the assumption of cylindrical symmetry is approximately satisfied, provided the data are sorted in CMP gathers. Having transformed the point source responses into line source responses, the results may be considered as "true amplitude" 2-D data. Hence, it is then justified to proceed with existing 2-D seismic processing techniques. This will be illustrated during the presentation with real data examples.

#### ACKNOWLEDGEMENTS

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