Inverse Elastic Wave Field Extrapolation

G. C. Haimé and C. P. A. Wapenaar, Delft University of Technology, Netherlands

Abstract

The elastic Kirchhoff-Helmholtz integral expresses the components of the monochromatic displacement vector at any point A in terms of the elastic wave field at any closed surface surrounding A. The volume inside and outside the closed surface may be arbitrarily inhomogeneous and anisotropic. By introducing Green's functions for P and S waves, it is shown that the elastic Kirchhoff-Helmholtz integral can be split into two separate integrals; the first one expresses the P wave potential at A and a second one expresses the S wave potential at A, both in terms of the elastic wave field at the closed surface surrounding A. These integrals are then transformed into one-way elastic Rayleigh-type integrals for forward and inverse extrapolation of downgoing and upgoing P and S waves. The *inverse* one-way elastic extrapolation operators, derived in this paper, are the basis for a new prestack migration scheme for elastic data (Berkhout and Wapenaar,1988).

Introduction

Berkhout and Wapenaar (1988) propose a new approach for processing of elastic seismic data which consists of the following steps:

1.Decomposition of the multi-component seismic data into one-way P^+-P^-, P^+-S^-, S^+-P^- and S^+-S^- data.

2.Elimination of the surface related multiple reflections and conversions.

3.Estimation of the elastic macro subsurface model from the P^+-P^- and S^+-S^- data.

4.Modeling of inverse one-way extrapolation operators for primary P and S waves.

5.Shot record migration of the P^+-P^-, P^+-S^-, S^+-P^- and S^+-S^- data, yielding the subsurface reflectivity in terms of R^+_{PP}, R^+_{PS}, R^+_{SP} and R^+_{SS}, optionally as a function of angle α.

6.Elastic inversion for the detailed velocity and density information (c_p,c_s,ρ).

7. Lithologic inversion for the rock and pore parameters.

It is interesting to note that the actual elastic migration (step 5) is not more complicated than acoustic shot record migration. Of course the quality of the migrated sections will depend largely on the accuracy of the extrapolation operators, which are modeled in step 4. The theory of the true amplitude inverse extrapolation operators for primary P and S waves is the subject of this paper.

The elastic Kirchhoff-Helmholtz integral for inhomogeneous, anisotropic solids

We consider a sub-volume V in an inhomogeneous, anisotropic solid see Figure 1. The elastic wave field at any point A within V (source free) can be calculated using the full elastic Kirchhoff-Helmholtz integral which reads (De Hoop, 1958)

$$U_m(\mathbf{r}_A,\omega) = \oint_S [\underline{\tau}\, G_m - \underline{\theta}_m\, U].n\ dS\ . \tag{1}$$

where $U_m=U_m(\mathbf{r}_A,\omega)$ for m = 1, 2, 3 represents one component of the monochromatic displacement vector $\mathbf{U}(\mathbf{r}_A,\omega)$ in A; $\mathbf{U}=\mathbf{U}(\mathbf{r},\omega)$ and $\underline{\tau}=\underline{\tau}(\mathbf{r},\omega)$ represent the elastic wave field on S, due to sources outside V, in terms of displacements and stresses, respectively; $\mathbf{G}_m=\mathbf{G}_m(\mathbf{r},\mathbf{r}_A,\omega)$ and $\underline{\theta}_m=\underline{\theta}_m(\mathbf{r},\mathbf{r}_A,\omega)$ represent the Green's functions on S, due to an impulse force in the m-direction in A, in terms of displacements and stresses, respectively; ω represents the radial frequency. In this paper we use equation (1) as the starting point for the derivation of forward and inverse one-way extrapolation operators for primary P and S waves.

One-way elastic Rayleigh integrals for forward wave field extrapolation

Consider now the situation depicted in Figure 2, in which sub-volume V is shaped as a pill-box. The (secondary) sources are supposed to be beneath surface S_1. If we assume that the medium is homogeneous and isotropic in a (infinitely) thin region around surface S_1, then, in this region the wave field can be separated into upgoing and downgoing waves. In addition, for the Green's functions we choose a homogeneous and isotropic lower half space. This choice is justified, because in the derivation of the Kirchhoff-Helmholtz integral we only considered the medium inside S. With this choice, the Green's functions at z_1 represent purely downgoing waves. If we choose surfaces S_0 and S_2 into infinity (this is allowed since the sources are supposed to be beneath S_1), then, equation (1) can be rewritten as an integral over S_1 only. By introducing P and S wave potentials in A we obtain two separate integral relations similar to the one described above. One describes the P wave in A due to a Green's P wave source in A and another one which describes the S wave in A due to a Green's S wave source in A. Furthermore, we define P and S wave potentials at z_1 for the wave field and for the Green's functions, according to

$$[U^\pm]_{z_1} \cong \frac{1}{\rho\omega^2}[\nabla\Phi^\pm + \nabla\times\Psi^\pm]_{z_1}\ , \tag{2a}$$

and

$$[G^+]_{z_1} \cong \frac{1}{\rho\omega^2}[\nabla\Gamma^+_\varphi + \nabla\times\Gamma^+_\psi]_{z_1}\ . \tag{2b}$$

Then, by applying one-way wave equations for the P and S wave potentials at z_1, the two separate P and S Kirchhoff-Helmholtz integrals can be approximated by (assuming P wave sources in the lower half-space)

$$\Phi(r_A,\omega) \approx -2\iint_{-\infty}^{+\infty} \frac{1}{\rho\omega^2}\left[\left(\frac{\partial\Gamma^+_{\varphi,\varphi}}{\partial z}\right)\Phi^-\right]_{z_1} dxdy. \tag{3a}$$

and (assuming S wave sources in the lower half-space)

$$\Psi_h(r_A,\omega) \approx -2\iint_{-\infty}^{+\infty} \frac{1}{\rho\omega^2}\left[\left(\frac{\partial\Gamma^+_{\psi,\psi_h}}{\partial z}\right).\Psi^-\right]_{z_1} dxdy. \tag{3b}$$

Note that these expressions have the same simple form as the acoustic one-way Rayleigh II integrals. Furthermore, it should be emphasized that these simple elastic expressions (3a) and (3b) hold for arbitrarily inhomogeneous, anisotropic solids. The errors are of the same order as the negligence of multiply converted waves.

The elastic Kirchhoff-Helmholtz integral with back-propagating Green's functions

The one-way elastic Rayleigh integrals discussed sofar describe *forward* wave field extrapolation: assuming the sources are in the lower half space $z > z_1$, the upgoing part of the elastic wave field at z_1 is extrapolated away from the sources towards a point $\mathbf{r}_A$ in the subsurface, $z < z_1$. In the following we derive expressions for elastic *inverse* wave field extrapolation, towards the sources. Therefore we introduce the back-propagating Green's functions $G^*_m(\mathbf{r},\mathbf{r}_A,\omega)$ and $\underline{\theta}^*_m(\mathbf{r},\mathbf{r}_A,\omega)$, where * denotes complex conjugation. Using the back-propagating Green's functions we may derive the following Kirchhoff-Helmholtz integral

$$U_m(\mathbf{r}_A,\omega) = \oint_S [\underline{\tau}\, G^*_m - \underline{\theta}^*_m\, U].n\ dS. \tag{4}$$

This elastic Kirchhoff-Helmholtz integral is exact and is equivalent to elastic Kirchhoff-Helmholtz integral (1). However, the back-propagating Green's functions in equation (4) appear to be a convenient choice when deriving inverse wave field extrapolation operators.

One-way elastic Rayleigh integrals for inverse wave field extrapolation

The seismic measurements $\mathbf{U}(\mathbf{r},\omega)$ and $\underline{\tau}(\mathbf{r},\omega)$ are only available at surface S_0. The contribution of the elastic Kirchhoff-Helmholtz integral over cylindrical surface S_2 to the wave field in A vanishes if R goes to infinity. However, for this situation it is not allowed to choose S_1 at infinite depth, analogous to the forward case, because, then, sub-volume V will no longer be source free, hence, the Kirchhoff-Helmholtz integral will no longer be valid. It can be shown, however, that the integral over S_1 is of the order of multiply reflected waves. So in the following we shall neglect this integral and only be concerned with the integral over S_0. If the Green's functions are due to a Green's P wave source at $\mathbf{r}_A$ and if the wave field is generated by P wave sources below z_1, then the upgoing P wave potential Φ^- in A, is described by (analogous to (3a))

$$\Phi^-(\mathbf{r}_A,\omega) \approx 2\int\int_{-\infty}^{+\infty} \frac{1}{\rho\omega^2}\left[\left(\frac{\partial\Gamma^-_{\varphi,\varphi}}{\partial z}\right)^* \Phi^-\right]_{z_0} dxdy. \tag{5a}$$

With similar arguments, for S wave sources below z_1, the upgoing S wave potential Ψ_h^- in A, is described by (analogous to (3b))

$$\Psi_h^-(\mathbf{r}_A,\omega) \approx 2\int\int_{-\infty}^{+\infty} \frac{1}{\rho\omega^2}\left[\left(\frac{\partial\Gamma^-_{\psi,\psi_h}}{\partial z}\right)^* .\boldsymbol{\Psi}^-\right]_{z_0} dxdy. \tag{5b}$$

In conclusion, assuming weak to moderate contrasts, integrals (5a) and (5b) describe non-recursive 'true amplitude' *inverse* extrapolation of primary P and S waves, respectively.

Examples of elastic inverse wave field extrapolation

We discuss the application of the inverse operator to the decomposed surface measurements with the aid of two numerical 2-D examples. In these examples we assume that the data is free from surface related multiples. Consider the model which is displayed in Figure 3a. A plane P wave source is buried at the depth of z = 2000 m. The response at z_0 is shown in Figure 3b and 3c. Using equation (5a) with elastic Green's functions and the decomposed P-data (Fig.3d) results in the data set depicted in Figure 3f. Note that the distorting propagation effects of the elastic overburden have been properly removed (compare with Figure 3a). The amplitude match with the exact result is very good (Figure 3h). Using the same model we now consider a buried plane SV wave source at z = 2000 m. The response at level z_0 is shown in Figure 4b and 4c. Using equation (5b), extrapolation of the decomposed SV-data (Fig.4d) results in the data set displayed in Figure 4e. The distorting propagation effects of the elastic overburden have been removed for the greater part (compare with Figure 4c). The amplitude match (Figure 4h) is less accurate then in the P wave example (Figure 3h). Apparently the assumption of 'moderate contrasts' is violated.

Conclusions

We derived one-way elastic Rayleigh II integrals for *inverse* extrapolation of P and S waves (relations (5a) and (5b) respectively. Assuming P wave sources below z_A, equation (5a) expresses the upgoing P wave at $\mathbf{r}_A$ in terms of the upgoing P wave at z_0 and the back-propagating Green's P wave at z_0. Assuming S wave sources below z_A, equation (5b) expresses the upgoing S wave at $\mathbf{r}_A$ in terms of the upgoing S wave at z_0 and the back-propagating Green's S wave at z_0. The underlying assumption is that the contrasts in the inhomogeneous, anisotropic sub-surface are weak to moderate. For the examples that we showed, this assumption appears to be more severe for S wave extrapolation than for P wave extrapolation.

We expect that the inverse one-way extrapolation operators for primary P and S waves will play a major role in the practice of prestack migration of decomposed elastic data (Berkhout and Wapenaar, 1988).

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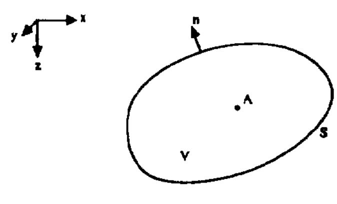

FIG. 1. Subvolume *V*, enclosed by surface *S*, in an inhomogeneous, anisotropic solid; *V* assumed to be source free. Full elastic Kirchhoff-Helmholtz integral (1) states elastic wave field at *A* in *V* can be computed when elastic wave field is known on *S*.

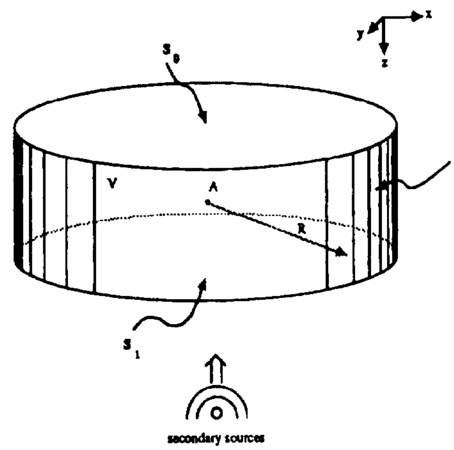

FIG. 2. Configuration in which closed surface integral (1) may be replaced by open surface integrals (3a) and (3b). Upper half-space assumed to be source free.

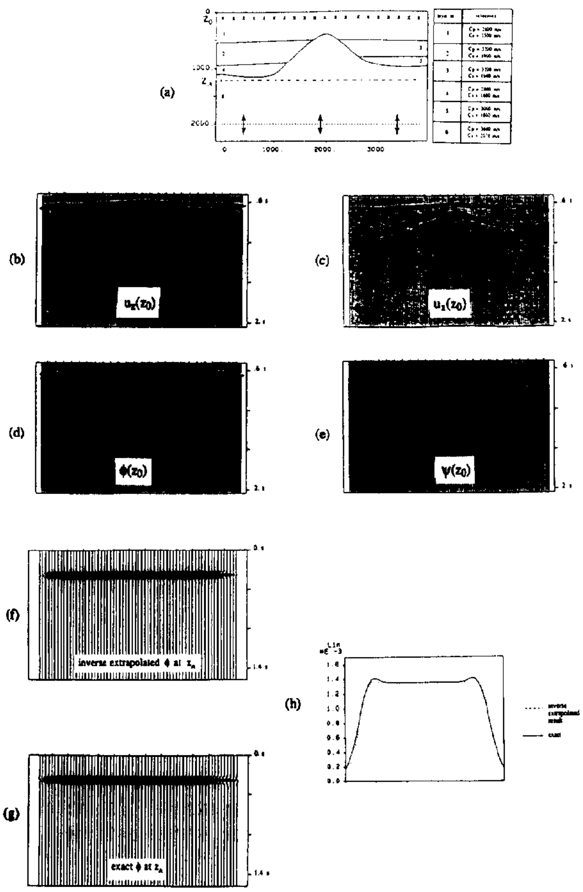

FIG. 3. (a) Complex overburden. Plane *P*-wave source buried at z = 2000 m. (b) Recorded vertical displacement at level z_0 (pseudo *P*-data). (c) Recorded horizontal displacement at level z_0 (pseudo *SV*-data). (d) Decomposed upgoing ϕ data at level z_0 (true *P*-data). (e) Decomposed upgoing ψ_y data at level z_0 (true *SV*-data). (f) Elastically inverse extrapolated upgoing *P*-data at z_A. (g) Exact upgoing *P*-wave at z_A. (h) Maximum amplitude per trace of (f) (dotted line) and exact result (g) (solid line).

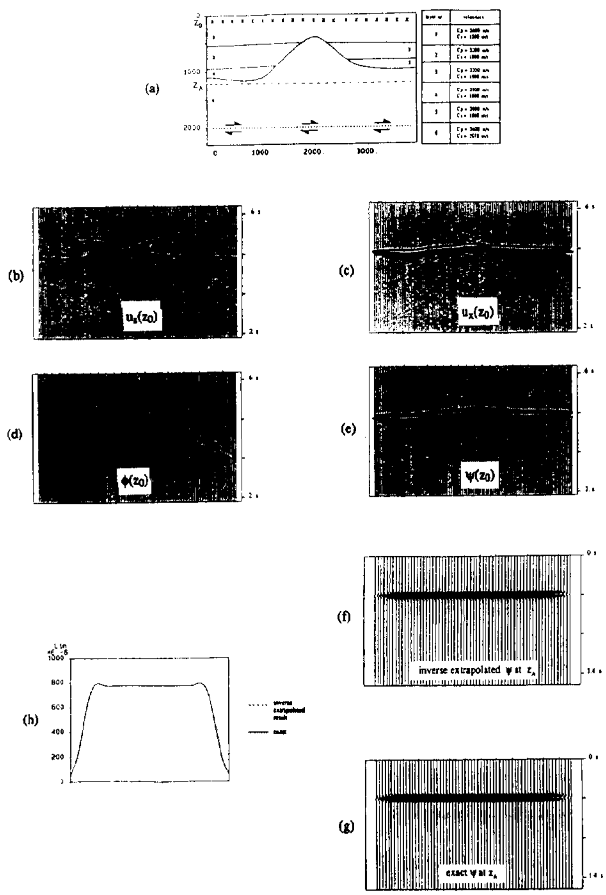

FIG. 4. (a) Complex overburden. Plane *SV*-wave source buried at z = 2000 m. (b) Recorded vertical displacement at level z_0 (pseudo *P*-data). (c) Recorded horizontal displacement at level z_0 (pseudo *SV*-data). (d) Decomposed upgoing ϕ data at level z_0 (true *P*-data). (e) Decomposed upgoing ψ_y data at level z_0 (true *SV*-data). (f) Elastically inverse extrapolated upgoing *SV*-data at z_A. (g) Exact upgoing *SV*-wave at z_A. (h) Maximum amplitude per trace of (f) (dotted line) and exact result (g) (solid line).