Elimination of Surface-Related Multiply Reflected and Converted Waves

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Summary

A new method is introduced for eliminating all surface related multiples in both acoustic and elastic media. This pre-stack multiple elimination procedure does not require any knowledge about the subsurface structure but only about the reflection characteristics at the surface and the source wave field. The procedure is carried out in the space-frequency domain and can therefore handle data from laterally inhomogeneous subsurfaces. The multiple elimination method is based on the wave equation. The main characteristic is the fact that the data itself is used as an multiple prediction operator. This in contrary to other wave equation based multiple suppressing schemes which assume a subsurface model and generate this operator in a synthetic way. The main operations in the algorithm are matrix multiplications, which makes it very suitable for vector computers.
In the case of elastic data (multi component recording) a decomposition is applied to acquire true P and S wave responses. After this decomposition converted multiples can be handled as well.
Applying this multiple elimination procedure on synthetic data, both acoustic and elastic, yields good results.

Introduction

Multiple reflections can give serious problems in seismic processing (e.g. velocity analysis, migration, lithologic inversion) and interpretation.
Especially marine data suffer from multiples generated by the sea surface in combination with the sea bottom. The water surface acts as a perfect mirror, reflecting all upgoing energy downward.
Several methods have already been proposed to suppress these multiples. Especially in more complicated subsurface structures with non-horizontal layers, a wave equation based method is most successful. In most recent multiple suppressing techniques the shot records are extrapolated up and down the first layer to predict the multiples and reverberations in this layer, after which they are subtracted from the data. This is done for example by Berryhill et al (1986).
A disadvantage of such a method is the fact that the position and shape of the first reflector together with its reflection characteristics should be known.
Here a method is introduced which was originally described by Berkhout (1982, chapter 7). It does not require any knowledge about the subsurface in order to eliminate multiples.

Theory of multiple elimination in the acoustic case

Firstly, an acoustic description of the multiple elimination procedure is given which after that will be extended to the elastic case.
Following the matrix notation for seismic data of Berkhout (1982, chapter 6), the full prestack monochromatic seismic data without any surface related multiples (in the two dimensional case given by a matrix in the x-ω domain) can be represented by:

$$\mathbf{P}_0^-(z_0) = \mathbf{Q}(z_0)\mathbf{S}^+(z_0), \tag{1}$$

with

$$\mathbf{Q}(z_0) = \sum_{m=1}^{M} \mathbf{W}^-(z_0,z_m)\mathbf{R}^+(z_m)\mathbf{W}^+(z_m,z_0) + \mathbf{M}^-_{int}(z_0)\,. \tag{2}$$

In equation (1) $\mathbf{P}_0^-(z_0)$ represents the monochromatic upgoing wave field at the surface ($z=z_0$) and $\mathbf{S}^+(z_0)$ the downgoing source wave field. In the data matrix $\mathbf{P}_0^-(z_0)$ the columns contain the monochromatic shot records and the diagonal elements the zero offset data. The right hand side of equation (2) consists of the summation of primary reflections of M reflectors and the internal multiples $\mathbf{M}^-_{int}(z_0)$, which are one order lower in amplitude compared with the surface related multiples. $\mathbf{W}^+(z_m,z_0)$ describes the propagation from depth level z_0 to z_m, $\mathbf{R}^+(z_m)$ the reflection at $z=z_m$ and $\mathbf{W}^-(z_0,z_m)$ describes the propagation upwards to the surface again.
So $\mathbf{Q}(z_0)$ represents the impulse response of the earth with a reflection-free surface. This $\mathbf{Q}(z_0)$ is the response which can be used as input for further seismic processing.
In reality the surface reflects all upgoing energy. This downgoing reflected energy acts as a new illuminating source field. So the total upgoing wave field can be described as the sum of the responses of the source wave field and the total upgoing wave field after reflection at the surface. This gives an implicit expression for the total upgoing wave field $\mathbf{P}^-(z_0)$ at the surface containing all multiples:

$$\mathbf{P}^-(z_0) = \mathbf{Q}(z_0)\,[\,\mathbf{S}^+(z_0) + \mathbf{R}^-(z_0)\mathbf{P}^-(z_0)]\,. \tag{3}$$

with $\mathbf{R}^-(z_0)$ the reflectivity matrix for upgoing waves at the surface.
Now equation (3) can be transformed to write the total upgoing wave field $\mathbf{P}^-(z_0)$ explicitly:

$$P^-(z_0) = [\, I - Q(z_0)R^-(z_0)]^{-1}\, Q(z_0)S^+(z_0)\,. \qquad (4)$$

The first term at the right hand side generates all surface related multiples. This term can be expanded into a Taylor series:

$$P^-(z_0) = [I+Q(z_0)R^-(z_0)+\{Q(z_0)R^-(z_0)\}^2+..]Q(z_0)S^+(z_0) \qquad (5)$$

It can be understood easily that each term at the right hand side of equation (5) represents one order of multiples.
Now for eliminating the surface related multiples equation (3) has to be rewritten to get an explicit expression for $Q(z_0)$:

$$Q(z_0) = P^-(z_0)\,[\, S^+(z_0) + R^-(z_0)P^-(z_0)]^{-1}\,, \qquad (6)$$

or

$$Q(z_0) = P^-(z_0)S^{-1}(z_0)\,[\, I + R^-(z_0)P^-(z_0)S^{-1}(z_0)]^{-1}\,, \qquad (7)$$

in which $P^-(z_0)S^{-1}(z_0)$ represents the data deconvolved for the source wave field. If $P^-(z_0)S^{-1}(z_0)$ is named $P'(z_0)$ and equation (7) is expanded into a Taylor series again, the result is:

$$Q(z_0) = P'(z_0) - P'(z_0)R^-(z_0)P'(z_0) + P'(z_0)\{R^-(z_0)P'(z_0)\}^2 - .. \qquad (8)$$

Equation (8) now states the multiple elimination procedure without having to make the full matrix inversion like in equations (6) and (7). Therefore this method is always stable if only a restricted number of terms of equation (8) is taken into account. As was the case in the forward model, if only a limited number of multiples is present in the data (which is always true because of a limited registration time) only a limited number of terms in equation (8) has to be taken into account.

The procedure only consists of matrix multiplications in the x-ω domain which makes the algorithm very suitable for vector machines.

Again it should be noticed that apart from the source wave field and the reflection characteristics of the surface no other external information is used in the elimination procedure. Furthermore, in the acoustic case the free surface will be totally reflecting for all angles of incidence, which means that the reflectivity matrix $R(z_0)$ can be replaced by a single reflection coefficient r_0=-1.

The multiple elimination for elastic data

If elastic data is acquired with an ideal multi component recording the data consists of V_x and V_z measurements caused by source stresses τ_{zx} and τ_{zz} (in the two dimensional case).

To extend the acoustic multiple elimination scheme to the elastic case, the data must be available as separate PP, PS, SP and SS datasets (in which PS data means received upgoing P waves caused by a pure S wave source, etc.).

Therefore a decomposition of the multi component data has to be applied to get to P and S wave data. Firstly by a decomposition the V_x and V_z data are transformed into received P and S data, secondly a source-detector reordering is applied (reciprocity) and thirdly a second decomposition is done. This decomposition operator is described by Wapenaar et al (1987).

After this decomposition and a deconvolution for the source wave field, four data matrices can be acquired for each frequency in the x-ω domain. These can now be ordered into one new data matrix with P_{PP} in the upper left corner, P_{PS} in the upper right corner, P_{SP} in the lower left and P_{SS} in the lower right corner.

The reflection at the surface can no longer be described by a reflection coefficient because wave conversions occur. Therefore the full reflectivity matrices R^-_{PP}, R^-_{PS}, R^-_{SP} and R^-_{SS} (with R^-_{PS} representing the reflection at the surface from S to P waves, etc.) have to be taken. They can be ordered into one reflectivity matrix in the same way. With these new matrices equation (8) can be rewritten as:

$$\begin{bmatrix} Q_{PP}Q_{PS} \\ Q_{SP}Q_{SS} \end{bmatrix} = \begin{bmatrix} P_{PP}P_{PS} \\ P_{SP}P_{SS} \end{bmatrix} - \begin{bmatrix} P_{PP}P_{PS} \\ P_{SP}P_{SS} \end{bmatrix} \cdot \begin{bmatrix} R_{PP}R_{PS} \\ R_{SP}R_{SS} \end{bmatrix} \cdot \begin{bmatrix} P_{PP}P_{PS} \\ P_{SP}P_{SS} \end{bmatrix} + \begin{bmatrix} P_{PP}P_{PS} \\ P_{SP}P_{SS} \end{bmatrix} \cdot \{ \begin{bmatrix} R_{PP}R_{PS} \\ R_{SP}R_{SS} \end{bmatrix} \cdot \begin{bmatrix} P_{PP}P_{PS} \\ P_{SP}P_{SS} \end{bmatrix} \}^2 - .. \qquad (9)$$

With this algorithm <u>all</u> surface related multiply reflected and converted waves are eliminated. Hence, after applying this algorithm the data may be interpreted as if the free surface has been replaced by a reflection free homogeneous upper half space.

Examples

Two examples of the multiple elimination method will be shown now: one for an acoustic and one for an elastic dataset.

Figure 1 shows a subsurface model used for an acoustic finite difference modelling.

Figure 2a shows the shot record with the source position indicated with the arrow in figure 1. All multiples are present in this shot record. After applying the multiple elimination procedure on all shot records, the result for the shot record at the same source position is shown in figure 2b. All surface related

multiples have been eliminated even for this strongly laterally inhomogeneous medium.
To check this the same shot record is modelled without a reflecting surface. Figure 2c shows this multiple free shot record and gives very good resemblance with the multiple eliminated shot record of figure 2b.
For an elastic data example the subsurface model of figure 3 has been taken. It contains three macro layers and a 'target zone' containing four thin layers.
First full elastic data is modelled with τ_{zx} and τ_{zz} sources (horizontal and vertical vibrators) and V_x and V_z detectors (horizontal and vertical geophones) resulting in four datasets. For this laterally homogeneous medium all shot records are equal.
Figure 4 shows the four different shot records of this elastic medium. First a decomposition procedure is applied to these datasets. After this decomposition the result is a PP, PS, SP and SS dataset, which is shown in figure 5. Next the full elastic multiple elimination procedure is applied on these datasets and the results are plotted in figure 6. As can be seen in figure 6 all surface related and converted multiples have been eliminated. The remaining events are primaries and internal multiples only.

Conclusions

A method has been introduced to eliminate all surface related multiples without requiring any knowledge about the subsurface structure. This scheme has been elaborated for the acoustic and for the elastic case. For the latter a decomposition of the multi component elastic data is required to get true P and S wave responses separately. After this decomposition all surface related multiples, even if wave conversions have occurred, can be eliminated.
Two examples have been shown for the acoustic and elastic case. The method showed very good results.

References

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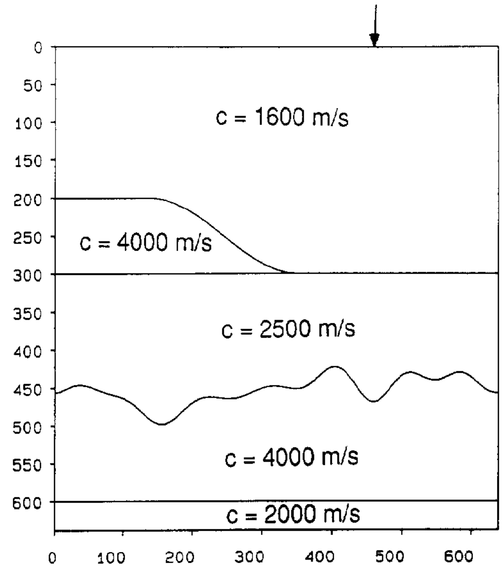

FIG. 1. Subsurface model used as input for acoustic finite-difference modeling.

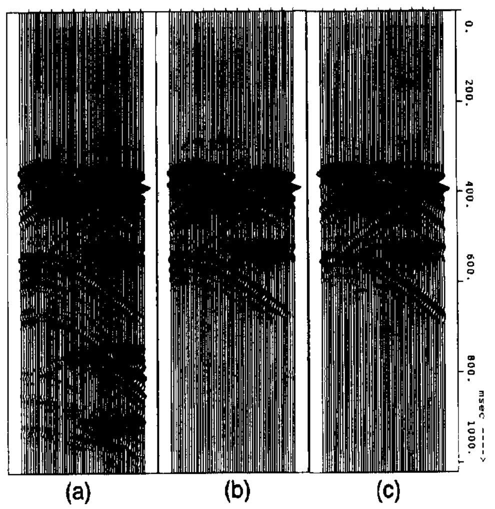

FIG. 2. (a) Shot record after finite-difference modeling with source at position indicated with arrow in Figure 1. (b) Same shot record as (a) after elimination of surface related multiples. (c) Shot record at same shot position as (a) modeled without surface-related multiples.

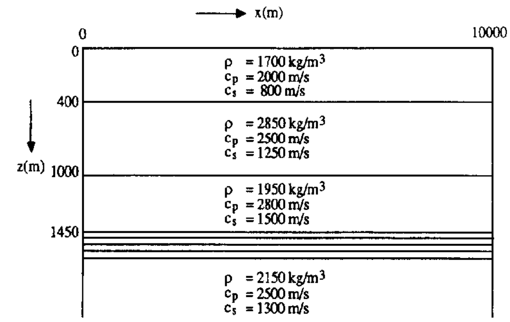

FIG. 3. Subsurface model used as input for elastic modeling.

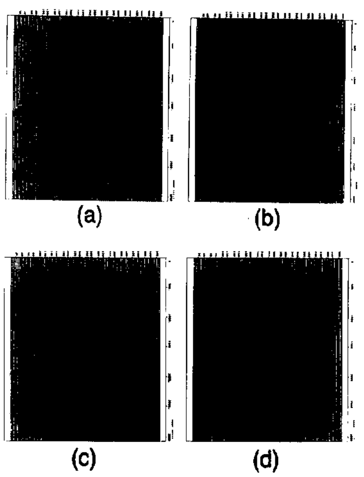

FIG. 4. Multicomponent shot record for subsurface model of Figure (3). (a) τ_{zz} source and V_z detectors. (b) τ_{zx} source and V_z detectors. (c) τ_{zz} source and V_x detectors. (d) τ_{zx} source and V_x detectors.

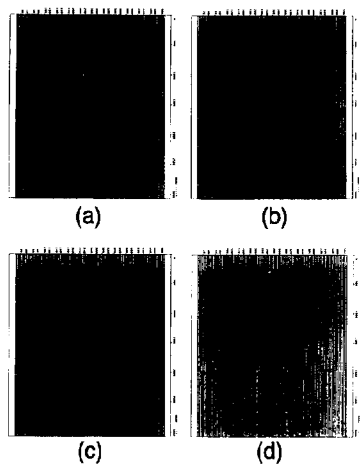

FIG. 5. Results after decomposition of shot record shown in Figure 4. (a) *PP* data, (b) *PS* data, (c) *SP* data, and (d) *SS* data.

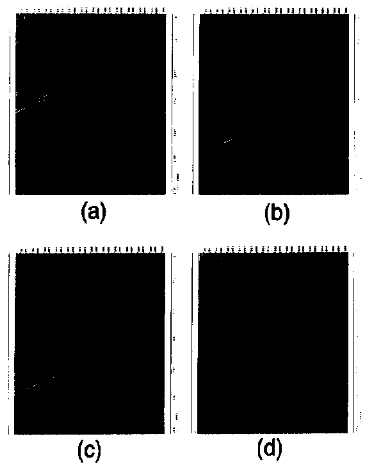

FIG. 6. Results after full elastic elimination of surface related and converted multiples. (a) *PP* data, (b) *PS* data, (c) *SP* data, and (d) *SS* data.