Verification of Macro Subsurface Models Using a Shot Record Approach S 9.6

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Summary

It is nowadays well known that a so-called macro subsurface velocity model is needed to perform an accurate depth migration. The macro subsurface model lacks detail and contains all the trend information, i.e. it determines the propagation effects of the subsurface. In this paper a new method is presented to update and verify the macro subsurface model. The method uses intermediate results from a shot record migration scheme.

Depth migration can be subdivided into two main steps. The first step is the **extrapolation** of the data recorded at the surface. The extrapolation operators are derived from the macro subsurface model and the wave equation. The second step in migration is the **imaging** step. After imaging, which generally involves taking the zero time components from the extrapolated data, a migrated depth section will be the result.

It will be shown that with prestack migration techniques it is possible to construct true Zero Offset traces at any depth point in the subsurface before imaging. This property is used to construct VSP-like sections of true Zero Offset data. From such (z,t)-panels a focussing depth analysis can tell whether the macro model was correct and, if not, how the parameters in the model have to be updated.

The reason that shot record migration is preferred over a full prestack depth migration scheme (such as S-G migration) is twofold. First, with the shot record approach the constructed Zero Offset data can be studied before and after true Common Depth Point stacking has been applied (true CDP-analysis and focussing depth analysis respectively). Second, the shot record oriented technique is still feasible in 3-D because of the easier data handling. During the presentation the method will be illustrated by applying it to realistic synthetic data from 2-D subsurfaces. Also 3-D extensions will be discussed.

Principle of shot record migration

Shot record migration as described by Berkhout (1985) and Wapenaar (1986) is a One-Way Prestack Depth migration method which uses single shot records as input. In our model verification method the shot record approach is followed as well.

One-way wave propagation can be elegantly described by the matrix multiplication

$$\vec{P}^-(z_0) = \sum_i \mathbf{W}^-(z_0,z_i)\,\mathbf{R}(z_i)\,\mathbf{W}^+(z_i,z_0)\,\vec{S}^+(z_0) \quad (1),$$

where vector $\vec{S}^+(z_0)$ denotes the source wave field at the surface z_0, vector $\vec{P}^-(z_0)$ denotes the reflected wave field that propagated back to the surface, matrices $\mathbf{W}^+(z_i,z_0)$ and $\mathbf{W}^-(z_0,z_i)$ describe propagation of the waves from depth levels z_0 to z_i and from z_i to z_0 respectively and matrix $\mathbf{R}(z_i)$ describes the reflection properties of level z_i.

This is called the forward problem (the notation is a monochromatic one until stated otherwise). From this forward problem the shot record migration scheme is derived. The purpose of migration is to extract the **reflectivity** properties of the medium without the distortion by **propagation** effects. So, the aim is to eliminate these propagation effects described by the operators **W** through inverse wave field extrapolation. In other words, the propagation operators will have to be inverted for. In order to obtain the operators a model is assumed, which describes the propagation effects in the medium. This model is called the **Macro Subsurface Model.**

The shot record migration scheme involves the following steps

- forward propagation of the source wave field from the surface z_0 to depth level z_i

$$\hat{\vec{S}}^+(z_i) = \hat{\mathbf{W}}^+(z_i,z_0)\,\vec{S}^+(z_0) \quad (2),$$

where vector $\vec{S}^+(z_0)$ is the downgoing source wave field at the surface, vector $\hat{\vec{S}}^+(z_i)$ is the calculated downgoing source wave field at depth level z_i, and matrix $\mathbf{W}^+(z_i,z_0)$ describes the forward propagation operator from the surface z_0 to depth level z_i, the hat denoting that a model estimate is used to construct the operator.

- The next step is the inverse propagation of the detected upgoing wave field from the surface z_0 to depth level z_i

$$\hat{\vec{P}}^-(z_i) = \hat{\mathbf{F}}^-(z_i,z_0)\,\hat{\vec{P}}^-(z_0) \qquad (3),$$

where matrix $\mathbf{F}^-(z_i,z_0)$ describes the (monochromatic) inverse wave field extrapolator from the surface z_0 to depth level z_i. This inverse operator is defined by

$$\hat{\mathbf{F}}^-(z_i,z_0) = \left[\hat{\mathbf{W}}^-(z_0,z_i)\right]^{-1} \qquad (4).$$

Here too, the hat means that the operator is derived using an estimate of the macro subsurface model. Ideally, the macro model describes the true propagation properties of the subsurface and the operators $\hat{\mathbf{W}}^+(z_i,z_0)$ and $\hat{\mathbf{F}}^-(z_i,z_0)$ in equations (2) and (3) equal the true propagation operators $\mathbf{W}^+(z_i,z_0)$ and $\{\mathbf{W}^-(z_0,z_i)\}^{-1}$, respectively. In migration, the reflectivity matrix $\mathbf{R}(z_i)$ should now be resolved from

$$\hat{\vec{P}}^-(z_i) = \hat{\mathbf{X}}(z_i)\,\hat{\vec{S}}^+(z_i) \qquad (5),$$

where

$$\hat{\mathbf{X}}(z_i) = \mathbf{R}(z_i) + \sum_{j\neq i} \hat{\mathbf{F}}^-(z_i,z_0)\,\mathbf{W}^-(z_0,z_j)\,\mathbf{R}(z_j)\,\mathbf{W}^+(z_j,z_0)\,\hat{\mathbf{F}}^+(z_0,z_i) \qquad (6),$$

with $\hat{\mathbf{F}}^+(z_0,z_i)$ being defined as the inverse of matrix $\hat{\mathbf{W}}^+(z_0,z_i)$. The inversion of equation (5) is an ill–posed problem, since only one shot record was used. However, a particular (non–unique) solution is obtained by

$$\langle\hat{\mathbf{X}}(z_i)\rangle_m = \frac{1}{s^2}\,\hat{\vec{P}}^-(z_i)\left[\hat{\vec{S}}^+(z_i)\right]^{*T} \qquad (7),$$

where $s^2 = \|\hat{\vec{S}}^+(z_i)\|^2$, a frequency dependent scaling factor. The notation $\langle..\rangle_m$ denotes that the non–unique solution is obtained from shot record m.

In the proposed macro model verification method it is the aim to construct Zero Offset data at depth points in the subsurface. In our mathematical model the Zero Offset elements are on the diagonal of matrix **X**. It is therefore sufficient to calculate only the diagonal elements $\langle\hat{X}_{ZO}(x,z_i)\rangle_m$ from

$$\langle\hat{X}_{ZO}(x,z_i)\rangle_m = \frac{1}{s^2}\,\hat{P}^-(x,z_i)\left[\hat{S}^+(x,z_i)\right]^* \qquad (8),$$

where s^2 as defined in equation (7), $\hat{P}^-(x,z_i)$ and $\hat{S}^+(x,z_i)$ represent the complex elements of vectors $\hat{\vec{P}}^-(z_i)$ and $\hat{\vec{S}}^+(z_i)$ and were $\langle\hat{X}_{ZO}(x,z_i)\rangle_m$ is a (complex) scalar function denoting the single fold monochromatic Zero Offset response at depth z_i as a function of the lateral position x.

If we incorporate all other frequencies a two dimensional function $\langle\hat{X}_{ZO}(x,z_i)\rangle_m$ is obtained, which is a function of x and ω, describing a single fold Zero Offset section (in the frequency domain) that would have been recorded at positions (x,z_i). We call this a single fold section because only one shot record was used to construct it. A different shot record would result in a different single fold Zero Offset section (even in the noise free case), because the illumination of the subsurface is from a different direction (see Figure 1). By applying the scheme to all depth levels a 3–D data volume is obtained which is a function of the depth point coordinates x and z and of time t (or frequency ω).So, we may conclude that with this approach single fold Zero Offset traces can be constructed at any depth point (x,z) in the subsurface! Note that in migration the imaging step is done (extraction of the zero time components of **X**. This yields a migrated shot record (in terms of the Zero Offset reflectivity),

$$\langle\hat{R}_{ZO}(x,z)\rangle_m = \langle\hat{X}_{ZO}(x,z;t=0)\rangle_m \qquad (9).$$

For our focussing analysis, however, we skip this imaging step. So far the results were obtained for shot record m only. Repeating the procedure for all shot records yields a number of single fold Zero Offset sections. These sections can be rearranged per depth point (x,z), yielding true Common Depth Point gathers. Common Depth Point stacking of these single fold results yields multi fold Zero Offset traces at any depth point (x,z)

$$\hat{X}_{ZO}(x,z;t) = \sum_m \langle\hat{X}_{ZO}(x,z;t)\rangle_m \qquad (10).$$

Since we now have genuine Zero Offset <u>traces</u> available at any depth point in the subsurface we can (for our purpose) construct Zero Offset panels $\hat{X}_{ZO}(x=\text{constant},z;t)$ by selecting those traces that belong to depth points on a **vertical 'datum'**.

Focussing analysis

Synthetic shot records were generated from the true subsurface (fig 1a). The operators for the inverse extrapolation were derived from an erroneous subsurface model (fig. 1b). By applying the operators to the prestack data a multi fold Zero Offset panel was obtained at depth points on a vertical datum. Such a multi fold Zero Offset panel has some nice properties (fig. 2). At a reflector a Zero Offset trace contains an event at zero time, provided that the macro model is correct. Also, the amplitude of the data is highest at zero time. This can be easily seen by inspection of the CDP-gather at this level (fig. 3a). The CDP–gather contains single fold Zero Offset traces obtained from various shot records. It is clear that for all shot positions the data is perfectly aligned at t=0, so a Common Depth Point stack will result in a high amplitude.

If the model is incorrect, the highest amplitude (focus) will no longer occur at zero time neither at the depth of the reflector (fig. 2, second focus and fig. 3b). In a migrated section this will result in an reflector imaged at the wrong depth and having a poor amplitude.

From the focus position it is possible to tell what the errors in the model were and thus how the model will have to be updated. Assuming for simplicity a horizontally layered macro model the updating formulas are given by (assuming small errors)

$$T_I = \bar{T}_I \; (1 + \varepsilon_t) \qquad (11),$$

with $$\varepsilon_t = \frac{t_f}{\bar{T}_I} \qquad (12),$$

where T_I is the true two way traveltime from the surface to reflector I, t_f is the time at which the focus occurs in the Zero Offset panel and $\bar{T}_I$ is the two way traveltime from the surface to the focus depth (using the model parameters).

The second updating formula is given by

$$C_I^2 = \bar{C}_I^2 \; (1 + \varepsilon_c) \qquad (13),$$

with $$\varepsilon_c = \frac{-\varepsilon_t}{1+\varepsilon_t} \qquad (14),$$

where C_I^2 is the true root mean square velocity, and $\bar{C}_I^2$ is the root mean square velocity in the model

The layer parameters of the I^{th} layer can be found using

$$c_I = \frac{C_I^2 T_I - C_{I-1}^2 T_{I-1}}{T_I - T_{I-1}} \qquad (15),$$

and

$$z_I = z_{I-1} + \frac{c_I}{2} (T_I - T_{I-1}) \qquad (16).$$

For the first reflector equations (15) and (16) can be rewritten as

$$z_1 = \sqrt{z_m z_f} \qquad (17),$$

where z_m is the depth at which the reflector is imaged, z_f is the depth at which focussing occurs, z_1 is the true depth of the first reflector and

$$c_1 = c_e \sqrt{\frac{z_f}{z_m}} \qquad (18),$$

where c_e is the velocity used in extrapolation c_1 is the true interval velocity of the first layer.
These equations were also derived by Faye and Jeannot (1986).

During the presentation the method will be illustrated by applying it to realistic synthetic data from 2–D subsurfaces. Also 3–D extensions will be discussed

References

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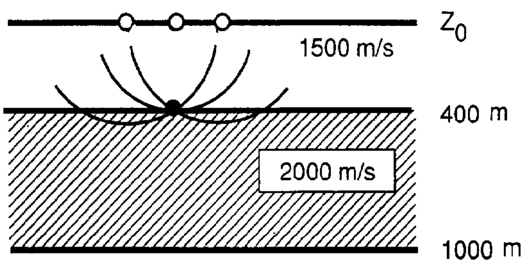

FIG. 1a. For each shot position, subsurface is illuminated differently.

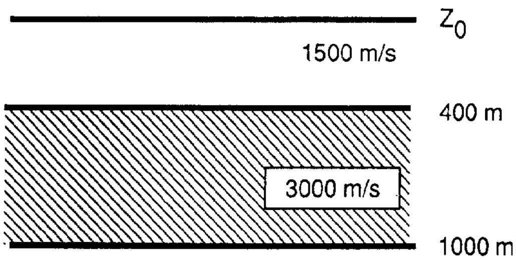

FIG. 1b. Erroneous subsurface model is used for generation of operators.

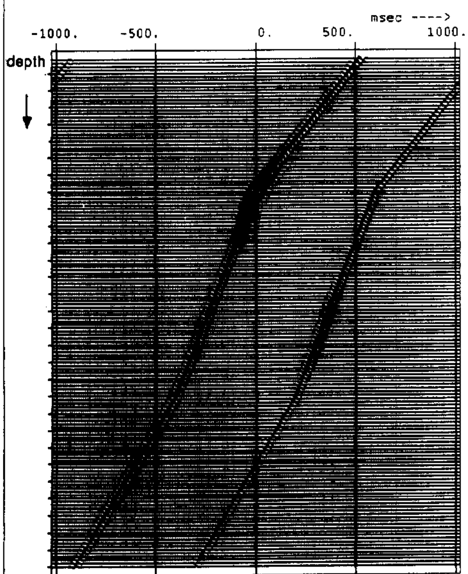

FIG. 2a. Multi-fold zero-offset panel for depth points situated on a vertical line. Using correct velocity for first layer yields focus at zero time and at reflector depth (400 m). Incorrect velocity in second layer results in focus off the zero time axis and not at reflector depth.

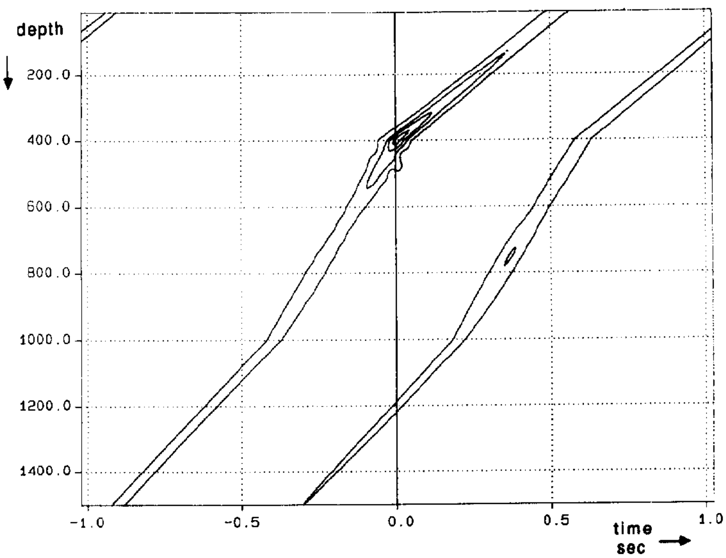

FIG. 2b. Contours of data of Figure 2a.

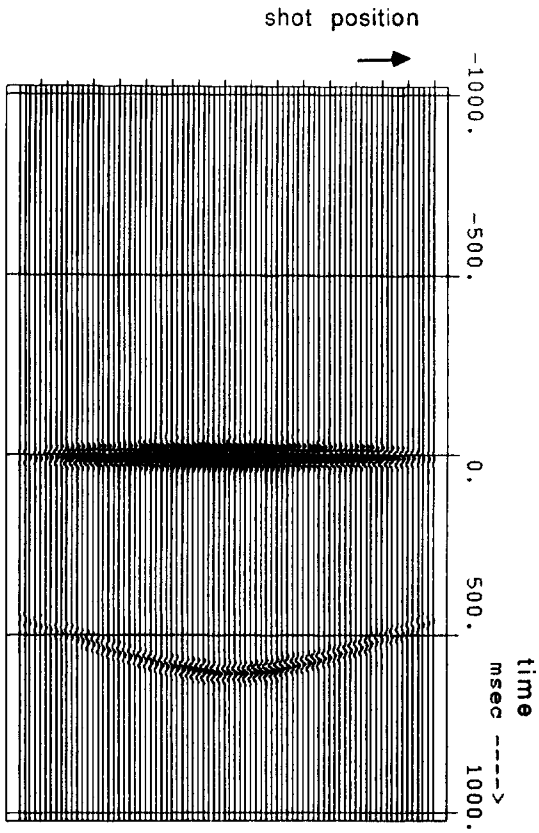

FIG. 3. Common-depth-point gather of a depth point at 400 m.