Full Prestack Versus Shot Record Migration

C. P. A. Wapenaar and A. J. Berkhout, Delft University of Technology, Netherlands

SUMMARY

Full pre-stack migration can be replaced by shot record migration plus stacking without loss of accuracy. To prove this, it is most convenient to describe forward seismic modeling in terms of matrix multiplications. Then full pre-stack migration can be described in terms of matrix inversions. By using some simple rules of matrix algebra it can then be shown that the full pre-stack migration result can be written as the sum of shot record migration results. This conclusion is particularly important for the 3-D case because the data management in 3-D shot record migration is dramatically more convenient than in 3-D full pre-stack migration.

INTRODUCTION

In seismic literature there is much confusion concerning the similarities and differences between full pre-stack migration (or double downward continuation migration) on one hand and migration per shot record (or profile migration) on the other hand. It is commonly accepted that full pre-stack (depth) migration of seismic data is the proper way to obtain a well resolved and accurately positioned true amplitude image of a geologically complicated subsurface. Unfortunately, full pre-stack migration is not very attractive from a practical point of view because it requires cumbersome data reordering (from common shot records into common detector records or vice versa) at each depth step. Migration per shot record, followed by stacking, is commonly considered as a practical alternative with good resolution and positioning capacities but with incorrect amplitude handling. For many existing shot record migration schemes this is indeed true. However, it is shown in this paper that a properly designed shot record migration scheme yields exactly the same results as a full pre-stack migration scheme. The practical significance of this observation is that true amplitude depth sections may be obtained by shot record migration plus stacking, thus avoiding cumbersome data reordering at each depth step. This conclusion is particularly important for 3-D applications.

MATRIX REPRESENTATION OF WAVE PROPAGATION

We introduce a matrix representation of wave field extrapolation. For convenience we consider the 2-D case. With reference to Figure 1, we describe monochromatic one-way wave field extrapolation for the following four cases.

i. Forward extrapolation of downgoing waves (Figure 1a):

$$\mathbf{P}^+(z_i)=\underset{\sim}{\mathbf{W}}^+(z_i,z_0)\,\mathbf{P}^+(z_0). \qquad (1a)$$

ii. Forward extrapolation of upgoing waves (Figure 1b):

$$\mathbf{P}^-(z_0)=\underset{\sim}{\mathbf{W}}^-(z_0,z_i)\,\mathbf{P}^-(z_i). \qquad (1b)$$

iii. Inverse extrapolation of downgoing waves (Figure 1c):

$$\mathbf{P}^+(z_0)=\underset{\sim}{\mathbf{F}}^+(z_0,z_i)\,\mathbf{P}^+(z_i). \qquad (2a)$$

iv. Inverse extrapolation of upgoing waves (Figure 1d):

$$\mathbf{P}^-(z_i)=\underset{\sim}{\mathbf{F}}^-(z_i,z_0)\,\mathbf{P}^-(z_0). \qquad (2b)$$

Here the data vector $\mathbf{P}^{\pm}(z_n)$ contains the pressure values of a downgoing (+) or upgoing (-) wave at all lateral positions at depth level z_n. The forward and inverse extrapolation matrices $\underset{\sim}{\mathbf{W}}^{\pm}$ and $\underset{\sim}{\mathbf{F}}^{\pm}$ contain Green's functions. We define a Green's matrix $\underset{\sim}{\mathbf{G}}(z_0,z_i)$, the ℓ'th column of which contains the monochromatic pressure response at all lateral positions at the surface z_0, due to a monopole in the subsurface at (x_ℓ, z_i), see also Figure 2. In this matrix notation, the principle of reciprocity reads

$$\underset{\sim}{\mathbf{G}}(z_i,z_0) = \underset{\sim}{\mathbf{G}}^T(z_0,z_i), \tag{3}$$

where T denotes that rows and columns should be interchanged. If we consider an acoustic loss-less, variable velocity, constant density medium between z_0 and z_i, then the forward wave field extrapolation operators are related to this Green's matrix, according to

$$\underset{\sim}{\mathbf{W}}^+(z_i,z_0) = \partial_z \underset{\sim}{\mathbf{G}}(z_i,z=z_0) \tag{4a}$$

and

$$\underset{\sim}{\mathbf{W}}^-(z_0,z_i) = -\partial_z \underset{\sim}{\mathbf{G}}(z_0,z=z_i). \tag{4b}$$

Furthermore, the spatially band-limited inverse wave field extrapolation operators are given by

$$\underset{\sim}{\mathbf{F}}^+(z_0,z_i) \triangleq [\underset{\sim}{\mathbf{W}}^+(z_i,z_0)]^{-1} \approx -\partial_z \underset{\sim}{\mathbf{G}}^*(z_0,z=z_i) \tag{5a}$$

and

$$\underset{\sim}{\mathbf{F}}^-(z_i,z_0) \triangleq [\underset{\sim}{\mathbf{W}}^-(z_0,z_i)]^{-1} \approx \partial_z \underset{\sim}{\mathbf{G}}^*(z_i,z=z_0), \tag{5b}$$

where * denotes that the complex conjugated should be taken. A series of papers dealing with the generalization of these relations for acoustic or full elastic, lossy, arbitrarily inhomogeneous media is currently being prepared by the authors.

THE FORWARD MODEL OF A SEISMIC DATASET

With the forward extrapolation matrices, defined in the previous section, the (monochromatic) response of a single reflector at depth z_i can be written as

$$\mathbf{P}_m^-(z_0) = \underset{\sim}{\mathbf{W}}^-(z_0,z_i)\,\underset{\sim}{\mathbf{R}}(z_i)\,\underset{\sim}{\mathbf{W}}^+(z_i,z_0)\,\mathbf{S}_m^+(z_0), \tag{6}$$

see also Figure 3. Here $\mathbf{S}_m^+(z_0)$ represents one frequency component of the downgoing source wave field at z_0. In case of a single source at lateral position x_m this vector contains one non-zero element only. Matrix $\underset{\sim}{\mathbf{R}}(z_i)$ describes the reflectivity properties of the reflector at z_i. With the Born-approximation it would be a diagonal matrix, each diagonal element representing a reflection coefficient at one lateral position at depth z_i. In the following we do not make use of the Born approximation, hence, in general matrix $\underset{\sim}{\mathbf{R}}$ may have a band-structure. Finally, vector $\mathbf{P}_m^-(z_0)$ represents one frequency component of the detected upgoing wave at z_0 (one shot record), each element representing the response at one detector position. Based on relation (6), the forward model of a seismic line reads

$$\underset{\sim}{\mathbf{P}}^-(z_0) = \underset{\sim}{\mathbf{D}}(z_0)[\sum_i \underset{\sim}{\mathbf{W}}^-(z_0,z_i)\,\underset{\sim}{\mathbf{R}}(z_i)\,\underset{\sim}{\mathbf{W}}^+(z_i,z_0)]\underset{\sim}{\mathbf{S}}^+(z_0). \tag{7}$$

Source matrix $\underset{\sim}{\mathbf{S}}^+(z_0)$ contains all downgoing source waves for the different shot records. The m'th column contains source vector $\mathbf{S}_m^+(z_0)$. We included an optional detector matrix $\underset{\sim}{\mathbf{D}}(z_0)$ which represents field patterns. Finally, matrix $\underset{\sim}{\mathbf{P}}^-(z_0)$ represents one frequency component of all detected upgoing waves in a complete seismic dataset (one seismic line). The m'th column contains shot record $\mathbf{P}_m^-(z_0)$.

FULL PRE-STACK MIGRATION

Based on forward model (7) we define a matrix $\underset{\sim}{\mathbf{X}}(z_j)$ according to

$$\underset{\sim}{\mathbf{X}}(z_j) = [\underset{\sim}{\mathbf{D}}(z_0)\underset{\sim}{\mathbf{W}}^-(z_0,z_j)]^{-1}\underset{\sim}{\mathbf{P}}^-(z_0)[\underset{\sim}{\mathbf{W}}^+(z_j,z_0)\underset{\sim}{\mathbf{S}}^+(z_0)]^{-1}. \tag{8}$$

Note that

$$\underset{\sim}{\mathbf{X}}(z_0) = [\underset{\sim}{\mathbf{D}}(z_0)]^{-1}\,\underset{\sim}{\mathbf{P}}^-(z_0)[\,\underset{\sim}{\mathbf{S}}^+(z_0)]^{-1} \tag{9a}$$

represents the seismic dataset, deconvolved for the source and detector properties. Similarly,

$$\underset{\sim}{\mathbf{X}}(z_j) = \underset{\sim}{\mathbf{F}}^-(z_j,z_0)\,\underset{\sim}{\mathbf{X}}(z_0)\,\underset{\sim}{\mathbf{F}}^+(z_0,z_j), \tag{9b}$$

[with $\underset{\sim}{\mathbf{F}}^+$ and $\underset{\sim}{\mathbf{F}}^-$ defined in (5)], represents the deconvolved seismic dataset, redatumed to depth level z_j. By substituting relation (7) into (9) we find that $\underset{\sim}{\mathbf{X}}(z_j)$ consists of two terms

$$\underset{\sim}{\mathbf{X}}(z_j) = \underset{\sim}{\mathbf{R}}(z_j) + \sum_{i\neq j} [\underset{\sim}{\mathbf{F}}^-(z_j,z_0)\underset{\sim}{\mathbf{W}}^-(z_0,z_i)\,\underset{\sim}{\mathbf{R}}(z_i)\underset{\sim}{\mathbf{W}}^+(z_i,z_0)\,\underset{\sim}{\mathbf{F}}^+(z_0,z_j)], \quad (10)$$

hence, the redatumed dataset $\underset{\sim}{\mathbf{X}}(z_j)$ contains the reflectivity matrix $\underset{\sim}{\mathbf{R}}(z_j)$ plus phase-distorted contributions from the reflectivity matrices $\underset{\sim}{\mathbf{R}}(z_{i\neq j})$ at other depth levels. Imaging takes place by summing $\underset{\sim}{\mathbf{X}}(z_j)$ for all frequencies ω in the seismic bandwidth, thus suppressing the phase distorted term $\sum_{i\neq j}[\ldots]$, according to

$$<\underset{\sim}{\mathbf{R}}(z_j)> = \frac{\Delta\omega}{\pi} \text{ Real} \left[\sum_{\omega_{min}}^{\omega_{max}} \underset{\sim}{\mathbf{X}}(z_j)\right]. \quad (11)$$

Optionally only the diagonal elements are computed. In practical applications (9b) should be carried out recursively in the following way

$$\underset{\sim}{\mathbf{Y}}(z_j) = \underset{\sim}{\mathbf{F}}^-(z_j,z_{j-1})\,\underset{\sim}{\mathbf{X}}(z_{j-1}), \quad (12a)$$

$$\underset{\sim}{\mathbf{X}}^T(z_j) = [\,\underset{\sim}{\mathbf{F}}^+(z_{j-1},z_j)]^T\,\underset{\sim}{\mathbf{Y}}^T(z_j). \quad (12b)$$

Relation (12a) describes compensation for upgoing propagation by spatially deconvolving common shot records along the detector-coordinates. After reordering common shot records into common detector records ($\underset{\sim}{\mathbf{Y}} \rightarrow \underset{\sim}{\mathbf{Y}}^T$), compensation for downgoing propagation takes place according to relation (12b), by spatially deconvolving common detector records along the shot-coordinates.

MIGRATION PER SHOT RECORD

According to relation (8) we may write for the redatumed data set

$$\underset{\sim}{\mathbf{X}}(z_j) = \underset{\sim}{\mathbf{P}}^-(z_j)[\,\underset{\sim}{\mathbf{S}}^+(z_j)]^{-1}, \quad (13a)$$

where

$$\underset{\sim}{\mathbf{P}}^-(z_j) = [\,\underset{\sim}{\mathbf{D}}(z_0)\underset{\sim}{\mathbf{W}}^-(z_0,z_j)]^{-1}\,\underset{\sim}{\mathbf{P}}^-(z_0) \quad (13b)$$

and

$$\underset{\sim}{\mathbf{S}}^+(z_j) = \underset{\sim}{\mathbf{W}}^+(z_j,z_0)\,\underset{\sim}{\mathbf{S}}^+(z_0). \quad (13c)$$

Relation (13a) states that at z_j the 'spatial impulse response' of the medium is defined as the ratio of upgoing and downgoing waves at z_j. Now we define vectors $\mathbf{P}_m^-(z_j)$ for m=1...M, which represent the columns of matrix $\underset{\sim}{\mathbf{P}}^-(z_j)$. Futhermore, we define vectors $\mathbf{Z}_m^+(z_j)$ for m=1...M, which represents the rows of inverse matrix $[\underset{\sim}{\mathbf{S}}^+(z_j)]^{-1}$. With these definitions, relation (13a) can be rewritten as

$$\underset{\sim}{\mathbf{X}}(z_j) = \sum_{m=1}^{M} \mathbf{P}_m^-(z_j)[\,\mathbf{Z}_m^+(z_j)]^T. \quad (14a)$$

The 'proof' for this relation is visualized in Figure 4. Note that, according to (5b) and (13b), vectors $\mathbf{P}_m^-(z_j)$ are related to the measured shot records, according to

$$\mathbf{P}_m^-(z_j) = \underset{\sim}{\mathbf{F}}^-(z_j,z_0)\frac{[\underset{\sim}{\mathbf{D}}(z_0)]^{*T}}{||D(\omega)||^2}\,\mathbf{P}_m^-(z_0), \quad (14b)$$

where we assumed for simplicity that $\underset{\sim}{\mathbf{D}}(z_0) = D(\omega)\underset{\sim}{\mathbf{I}}$. Similarly, according to (5a) and (13c), vectors $\mathbf{Z}_m^+(z_j)$ are related to the source vectors, according to

$$[\,\mathbf{Z}_m^+(z_j)]^T = \frac{[\,\mathbf{S}_m^+(z_0)]^{*T}}{||S(\omega)||^2}\,\underset{\sim}{\mathbf{F}}^+(z_0,z_j), \quad (14c)$$

where we assumed for simplicity that $\underset{\sim}{\mathbf{S}}^+(z_0) = S(\omega)\underset{\sim}{\mathbf{I}}$. Hence, full pre-stack redatuming, as described by (13), may be replaced by wave field extrapolation per shot record (14b,c), followed by 'stacking' (14a), without loss of accuracy. Of course the same remark is true for pre-stack migration (which involves pre-stack redatuming plus imaging).

CONCLUSIONS

It has been shown for 2-D media that full pre-stack migration may be replaced by migration per shot record (profile migration), followed by stacking, without loss of accuracy. Hence, both amplitude and phase may be correctly handled by shot record migration plus stacking. A similar proof can be given for 3-D media. Particularly in the 3-D case it is of great practical importance that pre-stack migration (or redatuming) can be properly applied per shot record, thus avoiding cumbersome data handling.

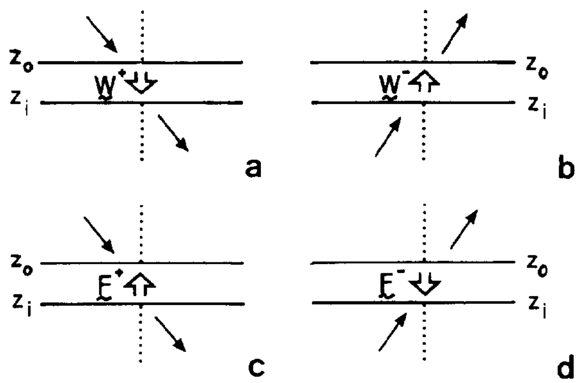

FIG. 1. Forward and inverse extrapolation of downgoing and upgoing waves.

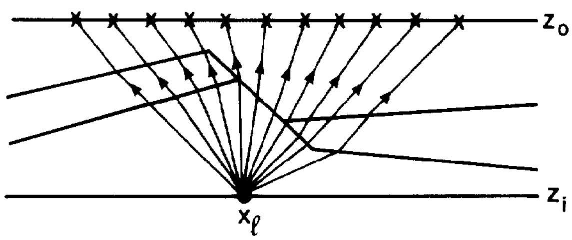

FIG. 2. The ℓth column of the Green's matrix.

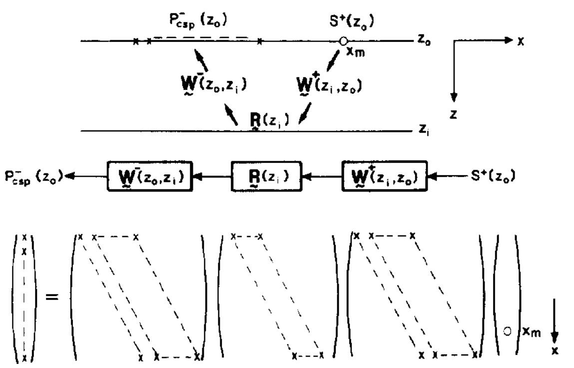

FIG. 3. Basic model for the seismic response from depth level z_i.

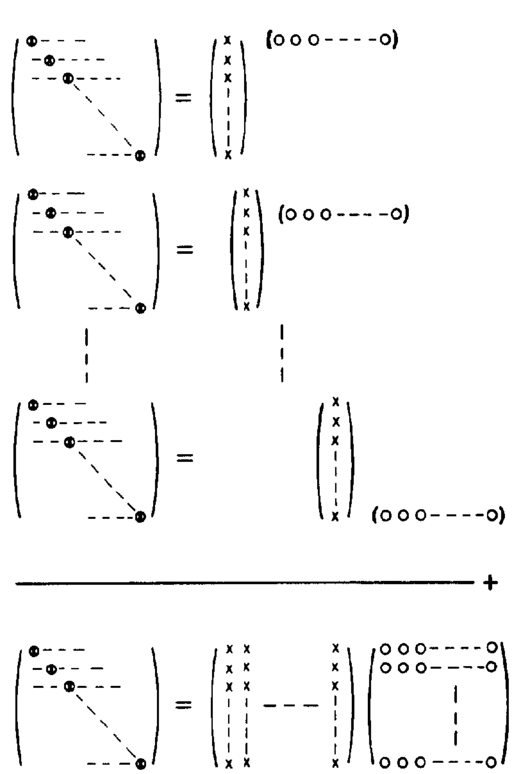

FIG. 4. Visualization of relation (14a), top, and (13a) bottom.