migration results can be summed. By this summation procedure true common-depth-point (CDP) stacking is accomplished.

Discussion

In comparison with conventional acoustic one-way migration schemes, the full elastic prestack two-way migration scheme, as introduced in this paper, has the following advantages: use of the square root operator is avoided, true CDP-stacking is accomplished, multiple reflected waves are properly handled, and wave conversion is properly handled. For a proper handling of multiple reflections and wave conversion, accurate knowledge of the subsurface model is required. Therefore the scheme should preferably be applied iteratively. We implemented a full elastic two-way wave equation migration scheme for 1-D inhomogeneous media, using an imaging principle in the ray parameter domain. It will be shown with the aid of numerical examples that the angle dependent *P-P* and *P-SV* reflectivities can be recovered independently.

Acknowledgments

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Three-Dimensional Prestack Migration by Single Shot Record Inversion S2.5

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This paper starts with a review of the principle of prestack migration by single shot record inversion (SSRI). Prestack migration by SSRI is very laborious when applied in three dimensions. Therefore, an alternative procedure is discussed which can be implemented very efficiently on vector computers, particularly when use is made of a fast mixed-domain operator, as introduced in this paper. Some preliminary results will be shown and a benchmark on a Cray vector computer will be discussed.

Introduction

In areas with complicated structures, 2-D as well as 3-D *poststack* migration techniques give a poor image of the subsurface because diffraction energy and conflicting dips are not treated correctly by common-midpoint (CMP) stacking. Therefore, in the past few years much research has been done to the development of *prestack* migration techniques. Because of the enormous amount of data to be processed in full 3-D prestack migration, serious attention has been paid so far only to 2-D prestack migration. Various techniques have been developed which are satisfactory when a 2-D subsurface may be assumed. However, for complicated 3-D inhomogeneous structures, 3-D prestack migration becomes necessary. Even with the new generation of fast vector computers full 3-D prestack migration is still unthinkable, so compromises need be made. An interesting solution appears to be the 3-D version of the dip moveout technique. Although simple velocity models must be assumed, this solution should certainly be explored because of its efficiency, particularly when applied in the Fourier domain (Hale, 1984). In this paper a more general and rigorous approach to 3-D prestack migration will be presented, namely the 3-D version of the single shot record inversion (SSRI) technique. In principle this method can handle any 3-D inhomogeneous velocity model. It will be shown step by step how the efficiency can be improved such that the procedure is computationally manageable with vector computers.

Principle of the SSRI technique

In this section we briefly review the principle of prestack migration by SSRI, as proposed by Berkhout (1984). For each single shot record, this migration procedure basically consists of the following two steps in the frequency domain:

(1) Downward extrapolation by means of forward extrapolation of the downgoing source wave, yielding $S^+(x, y, z_i, \omega)$ and inverse extrapolation of the upgoing detected wave, yielding $P^-(x, y, z_i, \omega)$. These extrapolation steps can be carried out independently, using the one-way wave equations for downgoing and upgoing waves (De Graaff, 1984). Alternatively, S^+ and P^- can be downward extrapolated simultaneously, using the two-way wave equation for the total wave field, thus properly handling primary as well as multiple reflected energy (Wapenaar and Berkhout, 1984).

(2) Imaging by means of integration of the zero offset (*ZO*) impulse response Y over all frequencies ω, in order to resolve the *ZO* reflectivity R at the current level z_i (downgoing and upgoing waves are time-coincident), according to

$$< r(x, y, z_i)> = \frac{1}{2\pi}\int_{\omega} Y(x, y, z_i, \omega)d\omega, \qquad (1a)$$

where

$$Y(x, y, z_i, \omega) = P^-(x, y, z_i, \omega)/S^+(x, y, z_i, \omega), \qquad (1b)$$

in some stable sense. Stabilization is assured when relation (1b) is replaced by

$$y(x, y, z_i, \omega) = [S^+(x, y, z_i, \omega)]^* P^-(x, y, z_i, \omega), \qquad (1c)$$

where the symbol * denotes complex conjugation. Notice that relation (1c) is the frequency-domain representation for the correlation of the downgoing source wave S^+ and the upgoing reflected waves P^-.

The above two steps are repeated (recursively or nonrecursively) for all depths z_i, thus yielding one migrated shot in terms of the *ZO* reflectivity R. When all shots have been migrated, then the individual migration results can be summed. Optionally a residual NMO-correction can be applied if the input velocity model is in error. By the summation procedure, true common-depth-point (CDP) stacking is accomplished. The whole procedure can be applied in two or three dimensions. For the 2-D case the processing sequence is visualized in Figure 1.

Generation of multifold zero-offset data by SSRI

Prestack migration by SSRI, as described in the previous section, is very laborious when applied in three dimensions. Here

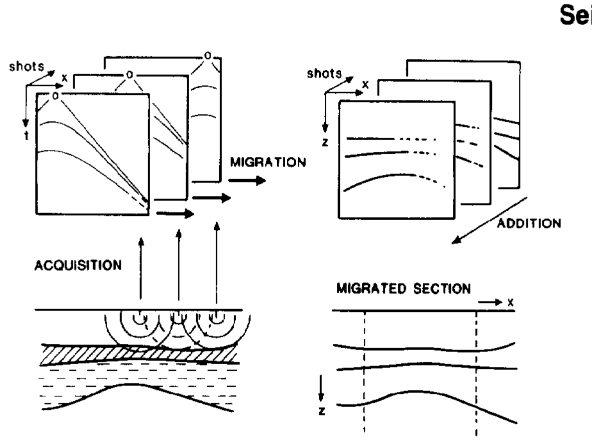

FIG. 1. Processing sequence of prestack migration by SSRI.

we present an interesting alternative procedure. For each single shot record the following two steps should be applied: (1) *Nonrecursive* downward extrapolation to a fixed reference plane z_i (for instance the sea bottom), yielding $S^+(x, y, z_i, \omega)$ and $P^-(x, y, z_i, \omega)$. (2) Computation of the *ZO* impulse response $Y(x, y, z_i, \omega)$, according to relation (1c). Note that the imaging step [relation (1a)] is deleted. Above two steps are applied for one depth z_i only, thus yielding one downward extrapolated shot in terms of the *ZO* impulse response *Y*. When all shots have been downward extrapolated, then the individual *ZO* impulse responses *Y* can be summed (true CDP-stacking). Optionally a residual NMO correction can be applied if the input velocity model is in error. Following this procedure, high quality multifold "true" *ZO*-data are obtained at the stacking plane z_i. It is interesting to note that diffraction energy and conflicting dips are fully preserved in the stacked data. These data can be further processed by a 3-D poststack migration algorithm in order to obtain a well resolved depth section. The procedure, suggested in this section, is much more efficient than the general procedure, as described in the previous section.

Practical aspects of SSRI with respect to 3-D pre-stack migration

The SSRI technique is computationally handy because individual seismic experiments (which obey the wave equation) are migrated. Therefore this approach is very well suited for arbitrary data acquisition configurations which may appear in 2-D and 3-D surface seismic profiling as well as in vertical seismic profiling. In any case *one* experiment is treated at a time and no regular grid spacing is required, which means that the binning problem is evaded. Also the incompleteness of many 3-D data acquisition techniques does not involve any restriction with respect to the applicability of the SSRI technique. In particular we consider a widely applied acquisition technique in marine seismics, where the 3-D prestack data set is built up by many conventional 2-D seismic lines. In this configuration one single shot record in the frequency domain consists of a 1-D vector of complex numbers, representing the monochromatic response registered by a 1-D detector array. Nonrecursive 3-D inverse downward extrapolation (previous section, step 1) of a 1-D detector array is visualized in Figure 2. Notice that the 1-D array at the acquisition surface z_o is spread out over a 2-D area at depth z_i. Obviously the inversion result at z_i is badly resolved because the inverse extrapolation problem is underdetermined when a 1-D input array is considered. However, multiplicity at z_i is built up by

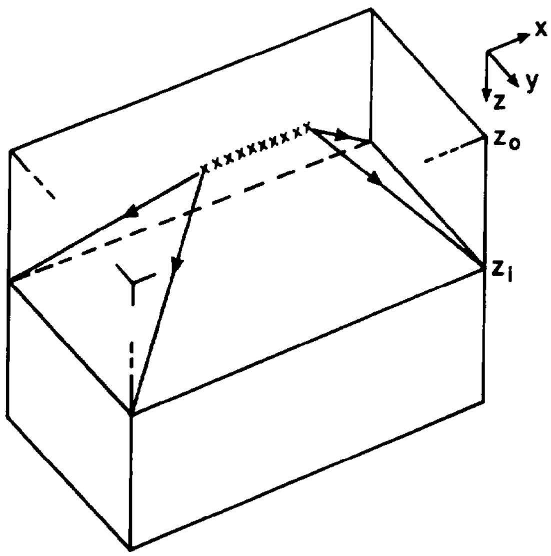

FIG. 2. 3-D downward extrapolation of a 1-D detector array.

CDP stacking (see previous section), which will improve the spatial resolution significantly.

For one seismic line (consisting of many 1-D detector arrays), the computation time needed for 3-D nonrecursive prestack downward extrapolation and CDP stacking is the same order as the computation time needed for full 2-D prestack migration.

A fast mixed domain operator

So far we have not made any assumptions concerning the propagation velocity model. In principle any velocity distribution can be handled if the appropriate extrapolation operator is chosen. Here we introduce a fast operator, assuming for the moment that the velocity is constant between the surface z_0 and the reference level z_i. We consider the acoustic wave equation in the mixed space-wavenumber-frequency domain:

$$\frac{\partial^2 \tilde{P}}{\partial y^2} + \frac{\partial^2 \tilde{P}}{\partial z^2} + \kappa^2 \tilde{P} = 0, \tag{2a}$$

where

$$\kappa^2 = (\omega/c)^2 - k_x^2, \tag{2b}$$

with $\tilde{P} = \tilde{P}(k_x, y, z, \omega)$ = acoustic pressure, ω = circular frequency, c = propagation velocity, k_x horizontal wavenumber, y = cross direction coordinate, and z = depth coordinate. Relation (2a) can be interpreted as a 2-D wave equation with k_x as a parameter, so a solution is given by the following 2-D Rayleigh II integral

$$\tilde{P}^+(k_x, y, z_i, \omega) = \int_{-\infty}^{\infty} \tilde{W}(k_x, y-y', \Delta z, \omega)\tilde{P}^+(k_x, y', z_0,\omega)dy', \tag{3a}$$

where

$$\tilde{W}(k_x, y, \Delta z, \omega) = -\frac{j\kappa}{2}\cos\phi H_1^{(2)}(\kappa\rho), \tag{3b}$$

with $\rho = (y^2+\Delta z^2)^{1/2}$, $\Delta z = z_i - z_0$, $\cos\phi = \Delta z/\rho$ and κ being given by (2b). $H_1^{(2)}$ represents the first-order Hankel function of the second kind. Notice that relation (3a) describes forward extrapolation of a downgoing wave. For inverse extrapolation of the detected upgoing waves $\tilde{P}^-$ the matched filter approach may be applied. Therefore $\tilde{W}$ should be replaced by inverse operator $\tilde{F}$, with

$$\tilde{F}(k_x, y, \Delta z, \omega) = \tilde{W}^*(k_x, y, \Delta z, \omega). \tag{4}$$

For the acquisition configuration discussed in the previous section (1-D detector arrays), each single shot record can be written as a scaled spatial delta function if an appropriate coordinate system is chosen, according to

$$\tilde{P}^-(k_x, y, z_0, \omega) = \tilde{P}^-(k_x, y_\ell, z_0, \omega)\delta(y - y_\ell), \tag{5}$$

where y_ℓ represents the cross-direction coordinate of the current single shot record. Hence, 3-D inverse extrapolation of a 1-D detector array can be written as

$$\tilde{P}^-(k_x, y, z_i, \omega) =$$

$$\tilde{P}^-(k_x, y_\ell, z_0, \omega) \int_{-\infty}^{\infty} \tilde{F}(k_x, y - y', \Delta z, \omega)\delta(y' - y_\ell)dy', \tag{6a}$$

or

$$\tilde{P}^-(k_x, y, z_i, \omega) = \tilde{F}(k_x, y - y_\ell, \Delta z, \omega)\tilde{P}^-(k_x, y_\ell, z_0, \omega). \tag{6b}$$

Notice that, according to relation (6b), the nonrecursive extrapolation algorithm involves 1-D spatial Fourier transforms and vectorized multiplications only. Vertical velocity variations between z_0 and z_i can be incorporated when the propagation velocity c is replaced by the effective velocity (Δz should then be replaced by the apparent depth). Cross-direction velocity variations can be incorporated because the operator $\tilde{F}$ is a function of the cross-direction coordinate y. In-line velocity variations can be incorporated by updating the operator $\tilde{F}$ for each subsequent single shot record.

Discussion

The general 3-D prestack migration scheme presented above is too laborious even for fast vector computers. We implemented the fast option for the marine data acquisition configuration. We used the fast mixed domain operator as described in the previous section. It will be shown with the aid of numerical examples that a very good spatial resolution is obtained when using the procedure suggested in this paper. Also a benchmark on a Cray vector computer will be discussed.

Acknowledgments

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Depth Migration of Seismic Data by Local Extrapolation Operators — S2.6

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The downward extrapolation of the wave field in laterally inhomogeneous media can be done by applying a matrix operator in the spatial-frequency domain. This matrix operator is derived either from the acoustic wave equation or a one-way wave equation by an analytical integration with respect to the depth. We use an eigenvalue technique to compute it. The matrix operators can be used to extrapolate the wave field, i.e., to integrate it in depth. We then derive local extrapolation operators by extracting vector operator from the matrix operator. The depth extrapolation procedure of the wave field consists of only one step, even in the case of a medium with lateral velocity variations. This step is simply the dot product of the wave field by the local extrapolation operator. The stability analysis is carried out through the computation of the matrix operators described previously, since we estimate the eigenvalues of the acoustic wave equation (or the one-way wave equation).

The local extrapolation operators' technique can migrate steeply dipping reflectors. The performance of the method is shown on synthetic examples in the cases of lateral velocity variations or dipping reflectors. The wave field extrapolation method described in this paper can be used either for the depth migration of 2-D stacked or unstacked seismic data.

Introduction

The aim of this paper is to describe a new depth migration method. This technique integrates either the full acoustic wave equation or a one-way wave equation. This method has no algorithm complication in the case of lateral velocity variations and no dip limitation. We focus our attention on the wave field depth extrapolation method, since migration programs use the same imaging concept. Three techniques based on the scalar wave equation were studied to solve the wave field extrapolation problem. The first is the finite-difference method which was used by Kjartanson (1978) to solve the lateral velocity variations problem. He used the 45-degree approximation of the square-root operator (Claerbout, 1976) to extrapolate the wave field in the *F-X* domain. This method was extended in the case of steeply dipping reflectors by the means of a continued fraction expansion of the square-root introduced in Geophysics by Muir. The second is the phase-shift method which was introduced by Gazdag (1978) to solve the dip problem. This method was extended to the case of inhomogeneous media by the mean of the phase-shift plus interpolation technique (Gazdag, 1982).

These two extrapolation methods solve a one-way wave equation and can be viewed as the integration of a square-root operator. The third method (Kosloff and Baysal, 1983) solves the scalar wave equation with no approximation, but both the wave field and its partial derivative with respect to the depth must be extrapolated. In this paper, we present a general method to solve either the scalar wave equation or a one-way wave equation in the *F-X* domain. We introduce the concept of local extrapolation operators, and demonstrate that it leads to a simple wave field extrapolation algorithm, with no dip problems and no algorithm complications in the case of a medium with lateral velocity variations. We first study the analytical integration of both the scalar wave equation and one-way wave equations. The next section describes the derivation of the local extrapolation operators. The