by migration for variable midpoint and zero offset (poststack migration). This artificial operator separation is actually only allowed for horizontally stratified media ($k_x^{(m)} = 0$), as can be easily seen from relation (7). When the data contain dip information, an alternative procedure must be followed. It was shown in the previous section from a wave theoretical point of view that an exact wave field extrapolation operator separation is *not* possible for the midpoint-offset space. However, in the literature several interesting procedures in the midpoint-offset space have been proposed which are based on *geometrical* considerations.

We mention offset continuation (Bolondi et al., 1984) and dip moveout (Hale, 1984). In both techniques the data are dip corrected for constant offset and variable midpoint, followed by stacking for constant midpoint and variable offset and post-stack migration. Particularly the dip moveout technique can be applied very efficiently by Fourier transforms. Several aspects will be illustrated with synthetic examples during the presentation.

Conclusions

Using a matrix expression for multishot prestack data, we discussed a full prestack migration scheme. This full prestack migration scheme operates in the field-coordinate system (x_s, x_d). It was emphasized that an important advantage of migration in the field coordinate system is that complicated subsurface models can be handled. Furthermore, we discussed an alternative formulation of the full prestack migration algorithm in the midpoint-offset coordinate system. It was shown that in this coordinate system midpoint and offset are coupled, so inverse wave field extrapolation involves approximations when carried out as independent deconvolutions in the midpoint-offset space. Finally we gave an overview of prestack partial migration algorithms in the midpoint-offset space. Though prestack partial migration schemes only allow one-dimensional velocity distributions they turn out to be very attractive from a point of view of efficiency.

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Principle of Prestack Migration Based on the Full Elastic Two-Way Wave Equation S2.4

C.P.A. Wapenaar and A. J. Berkhout Delft Univ. of Technology, Netherlands

This paper starts with a brief comparison of the one-way and the two-way modeling and migration approaches. Next, a full elastic two-way wave field extrapolation operator is discussed, which is accurate up to 90 degrees. This operator is used in a full elastic prestack migration scheme for 2-D inhomogeneous media. In this scheme multiple reflected and converted waves are properly taken into account. Finally, the algorithm is illustrated in synthetic examples for 1-D inhomogeneous media. It is shown that with the aid of the full elastic two-way wave equation migration scheme, the angle-dependent *P-P* and *P-SV* reflectivities can be recovered independently.

Introduction

It has been shown by Berkhout (1982) that a seismic experiment can be elegantly described by a sequence of independent one-way processes, which is schematically represented by

$$S \rightarrow W^+ \rightarrow R \rightarrow W^- \rightarrow D \rightarrow P. \tag{1}$$

A wave field, generated by sources S at the surface, propagates downward into the earth, which is described by one-way wave field extrapolation operator W^+. In the subsurface this wave field is reflected, described by R, and propagates upward to the surface again, described by one-way operator W^-. At the surface the wave field is registered by detectors D, resulting in a seismic registration P. Based on this model, Berkhout discusses prestack modeling as well as prestack migration schemes for subcritical primary data in inhomogeneous fluids. Optionally multiple reflections can be included in modeling as well as in migration schemes by applying a feed-back system at each major reflecting boundary. An alternative approach to prestack modeling and prestack migration is discussed by Wapenaar and Berkhout (1984). They propose methods which are based on two-way wave field extrapolation, which is schematically represented by

$$\begin{bmatrix} P \\ -j\omega V_z \end{bmatrix}_{z_1} \rightarrow \underset{\sim}{\mathbf{W}} \rightarrow \begin{bmatrix} P \\ -j\omega V_z \end{bmatrix}_{z_2}. \tag{2}$$

Here $[P, -j\omega V_z]^T$ describes the *total* wave field in terms of pressure P and particle velocity V_z (ω denotes circular frequency), while $\underset{\sim}{\mathbf{W}}$ describes the two-way wave propagation effects (downgoing and upgoing reflected waves) between two depth levels. Relation (2) holds for primary as well as multiple reflected longitudinal P-waves in inhomogeneous fluids. In this paper the two-way extrapolation algorithm, which is accurate up to 90 degrees, will be generalized for full elastic media, according to

$$\begin{bmatrix} j\omega V_z \\ Z_x \\ Z_z \\ j\omega V_x \end{bmatrix}_{z_1} \rightarrow \underset{\sim}{\mathbf{W}} \rightarrow \begin{bmatrix} j\omega V_z \\ Z_x \\ Z_z \\ j\omega V_x \end{bmatrix}_{z_2}, \tag{3}$$

where $[j\omega\ V_z,\ Z_x,\ Z_z,\ j\omega\ V_x]^T$ describes the *total* wave field in terms of stress vector $[Z_x, Z_z]^T$ and particle velocity vector $[V_x, V_z]^T$, while $\underset{\sim}{\mathbf{W}}$ describes the full elastic two-way propagation effects between two depth levels. Relation (3) holds for primary as well as multiple reflected longitudinal P-waves and transversal SV-waves in inhomogeneous solids. This two-way extrapolation algorithm, which is accurate up to 90 degrees, provides the basis for prestack full elastic two-way migration schemes, as presented in this paper.

Full elastic two-way wave field extrapolation

In this section we briefly review the matrix formulation of the full elastic two-way wave equation and its solution. Details are given by Wapenaar and Berkhout (1985). We consider the following first-order differential equation in the space-frequency domain (Ursin, 1983, discusses an equivalent equation in the wavenumber-frequency domain):

$$\frac{\partial \mathbf{Q}}{\partial z} = \mathbf{AQ}, \tag{4a}$$

where

$$\underset{\sim}{\mathbf{A}} = \begin{bmatrix} \underset{\sim}{\mathbf{O}} & \underset{\sim}{\mathbf{A}}_1 \\ \underset{\sim}{\mathbf{A}}_2 & \underset{\sim}{\mathbf{O}} \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} j\omega V_z \\ Z_x \\ Z_z \\ j\omega V_x \end{bmatrix}, \tag{4b,c}$$

$$\underset{\sim}{\mathbf{A}}_1 = \begin{bmatrix} -\left(\frac{\omega^2}{\lambda+2\mu}\right) & -\left(\frac{\lambda}{\lambda+2\mu}\right)d_1(x)* \\ -\left(\frac{\lambda}{\lambda+2\mu}\right)d_1(x)* & \rho + \frac{4\mu}{\omega^2}\left(\frac{\lambda+\mu}{\lambda+2\mu}\right)d_2(x)* \end{bmatrix} \quad (4d)$$

$$\underset{\sim}{\mathbf{A}}_2 = \begin{bmatrix} \rho & -d_1(x)* \\ -d_1(x)* & -\omega^2/\mu \end{bmatrix}, \underset{\sim}{\mathbf{O}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (4e,f)$$

with λ = Lamé constant, μ = Lamé constant (shear modulus), and ρ = mass density. The symbol * refers to a spatial convolution along the x coordinate. The operators $d_n(x)$ represent nth order band-limited spatial differentiation operators to x. Notice that operator $\underset{\sim}{\mathbf{A}}$ can be applied exactly within the seismic bandwidth because the square operator is avoided. The solution of (4a) is given by the following Taylor series summation

$$\mathbf{Q}(z) = \sum_{m=0}^{\infty} \frac{(z-z_0)^m}{m!}\left[\frac{\partial^m \mathbf{Q}}{\partial z^m}\right]_{z_0}, \quad (5a)$$

where (5b)

$$\partial^m \mathbf{Q}/\partial z^m = \partial^{m-1}(\underset{\sim}{\mathbf{A}}\mathbf{Q})/\partial z^{m-1}, \text{ for } m \geq 1.$$

Based on relation (5), a fast converging extrapolation operator has been derived which is accurate up to 90 degrees. In the following, expression (5a) is abbreviated to

$$\mathbf{Q}(z) = \underset{\sim}{\mathbf{W}}(z, z_0)\,\mathbf{Q}(z_0), \quad (5c)$$

where matrix $\underset{\sim}{\mathbf{W}}$ represents the extrapolation operator for the total wave field $\mathbf{Q}$. A further discussion of operator $\underset{\sim}{\mathbf{W}}$ is beyond the scope of this paper. For the computationally convenient subsurface model of Figure 1, recursive full elastic two-way wave field extrapolation can simply be described by

$$\mathbf{Q}(z_I) = \underset{\sim}{\mathbf{W}}(z_I, z_{I-1}), \ldots, \underset{\sim}{\mathbf{W}}(z_i, z_{i-1}), \ldots, \underset{\sim}{\mathbf{W}}(z_1, z_0)\,\mathbf{Q}(z_0), \quad (6)$$

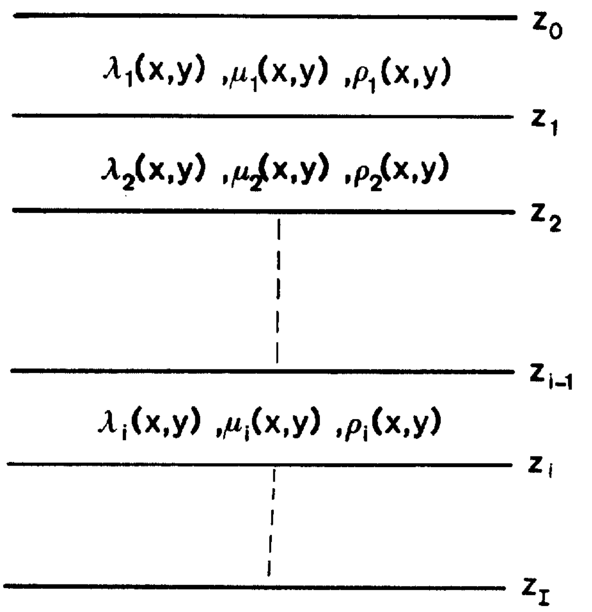

FIG. 1.

because the full elastic wave field $\mathbf{Q}$ is continuous for all depths. Multiple reflections and wave conversion are automatically incorporated.

Prestack migration scheme based on full elastic two-way wave field extrapolation

Berkhout (1982, 1984) discusses prestack migration by single shot record inversion (SSRI), based on the acoustic one-way model as described by relation (1). In this section this approach is generalized, based on the full elastic two-way model as described by relation (3). The scheme is based on recursive application of the following three steps for each single shot record.

Downward extrapolation. Given the total field $\mathbf{Q}(z_{i-1})$, then downward extrapolation can be applied according to

$$\mathbf{Q}(z_i) = \underset{\sim}{\mathbf{W}}(z_i, z_{i-1})\,\mathbf{Q}(z_{i-1}). \quad (7)$$

Decomposition. The total wave field can be decomposed into downgoing and upgoing waves according to

$$\mathbf{P}(z_i) = \underset{\sim}{\mathbf{L}}^{-1}(z_i)\,\mathbf{Q}(z_i), \quad (8a)$$

with (8b)

$$\mathbf{P} = [\phi^+, \psi^+, -\phi^-, \psi^-]^T,$$

where ϕ^+ and ψ^+ represent downgoing P and SV-waves, respectively. Similarly, ϕ^- and ψ^- represent upgoing P and SV-waves, respectively. A discussion of decomposition operator $\underset{\sim}{\mathbf{L}}^{-1}$ is beyond the scope of this paper. The longitudinal component (P-wave) of the downgoing source wave can be retrieved from the downgoing wave $\phi^+(z_i)$ by means of a "first arrival time window," yielding $\phi^+_{src}(z_i)$. This actually means that multiple reflected waves are excluded from the imaging procedure.

Imaging. Zero-offset (ZO) imaging can be applied by integrating the ZO impulse response Y over all frequencies, in order to resolve the ZO reflectivity at the current level (downgoing and upgoing waves are time-coincident), according to

$$< R_{p,\,p}(x, z_i)> = \int_\omega Y_{p,\,p}(x, z_i, \omega)\, d\omega, \quad (9a)$$

$$< R_{sv,\,p}(x, z_i)> = \int_\omega Y_{sv,\,p}(x, z_i, \omega) d\omega, \quad (9b)$$

where

$$Y_{p,\,p}(x, z_i, \omega) = \phi^-(x, z_i, \omega)/\phi^+_{src}(x, z_i, \omega), \quad (9c)$$

$$Y_{sv,\,p}(x, z_i, \omega) = \psi^-(x, z_i, \omega)/\phi^+_{src}(x, z_i, \omega), \quad (9d)$$

in some stable sense. Stabilization is assured when relations (9c) and (9d) are replaced by

$$Y_{p,\,p}(x, z_i, \omega) = [\phi^+_{src}(x, z_i, \omega)]^*\phi^-(x, z_i, \omega), \quad (9e)$$

$$Y_{sv,\,p}(x, z_i, \omega) = [\phi^+_{src}(x, z_i, \omega)]^*\psi^-(x, z_i, \omega), \quad (9f)$$

where the symbol * denotes complex conjugation. The above three steps (downward extrapolation, decomposition, and imaging) should be applied recursively, thus yielding one migrated shot. The procedure starts at the stress free surface z_0 where the initial condition $\mathbf{Q}(z_0)$ follows from

$$\mathbf{Q}(z_0) = [j\omega V_z, Z_{x,\,src}, Z_{z,\,src}, j\omega V_x]^T_{z_0}. \quad (10)$$

Here $\mathbf{Z}_{src}(z_0) = [Z_{x,src}(z_0), Z_{z,src}(z_0)]^T$ represents the stress source and $\mathbf{V}(z_0) = [V_x(z_0), V_z(z_0)]$ represents the detected particle velocity. When all shots have been migrated, then the individual migration results can be summed. By this summation procedure true common-depth-point (CDP) stacking is accomplished.

Discussion

In comparison with conventional acoustic one-way migration schemes, the full elastic prestack two-way migration scheme, as introduced in this paper, has the following advantages: use of the square root operator is avoided, true CDP-stacking is accomplished, multiple reflected waves are properly handled, and wave conversion is properly handled. For a proper handling of multiple reflections and wave conversion, accurate knowledge of the subsurface model is required. Therefore the scheme should preferably be applied iteratively. We implemented a full elastic two-way wave equation migration scheme for 1-D inhomogeneous media, using an imaging principle in the ray parameter domain. It will be shown with the aid of numerical examples that the angle dependent *P-P* and *P-SV* reflectivities can be recovered independently.

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Three-Dimensional Prestack Migration by Single Shot Record Inversion S2.5

C.P.A. Wapenaar and A. J. Berkhout, Delft Univ. of Technology, Netherlands

This paper starts with a review of the principle of prestack migration by single shot record inversion (SSRI). Prestack migration by SSRI is very laborious when applied in three dimensions. Therefore, an alternative procedure is discussed which can be implemented very efficiently on vector computers, particularly when use is made of a fast mixed-domain operator, as introduced in this paper. Some preliminary results will be shown and a benchmark on a Cray vector computer will be discussed.

Introduction

In areas with complicated structures, 2-D as well as 3-D *poststack* migration techniques give a poor image of the subsurface because diffraction energy and conflicting dips are not treated correctly by common-midpoint (CMP) stacking. Therefore, in the past few years much research has been done to the development of *prestack* migration techniques. Because of the enormous amount of data to be processed in full 3-D prestack migration, serious attention has been paid so far only to 2-D prestack migration. Various techniques have been developed which are satisfactory when a 2-D subsurface may be assumed. However, for complicated 3-D inhomogeneous structures, 3-D prestack migration becomes necessary. Even with the new generation of fast vector computers full 3-D prestack migration is still unthinkable, so compromises need be made. An interesting solution appears to be the 3-D version of the dip moveout technique. Although simple velocity models must be assumed, this solution should certainly be explored because of its efficiency, particularly when applied in the Fourier domain (Hale, 1984). In this paper a more general and rigorous approach to 3-D prestack migration will be presented, namely the 3-D version of the single shot record inversion (SSRI) technique. In principle this method can handle any 3-D inhomogeneous velocity model. It will be shown step by step how the efficiency can be improved such that the procedure is computationally manageable with vector computers.

Principle of the SSRI technique

In this section we briefly review the principle of prestack migration by SSRI, as proposed by Berkhout (1984). For each single shot record, this migration procedure basically consists of the following two steps in the frequency domain:

(1) Downward extrapolation by means of forward extrapolation of the downgoing source wave, yielding $S^+(x, y, z_i, \omega)$ and inverse extrapolation of the upgoing detected wave, yielding $P^-(x, y, z_i, \omega)$. These extrapolation steps can be carried out independently, using the one-way wave equations for downgoing and upgoing waves (De Graaff, 1984). Alternatively, S^+ and P^- can be downward extrapolated simultaneously, using the two-way wave equation for the total wave field, thus properly handling primary as well as multiple reflected energy (Wapenaar and Berkhout, 1984).

(2) Imaging by means of integration of the zero offset (*ZO*) impulse response Y over all frequencies ω, in order to resolve the *ZO* reflectivity R at the current level z_i (downgoing and upgoing waves are time-coincident), according to

$$< r(x, y, z_i)> = \frac{1}{2\pi}\int_\omega Y(x, y, z_i, \omega)d\omega, \qquad (1a)$$

where

$$Y(x, y, z_i, \omega) = P^-(x, y, z_i, \omega)/S^+(x, y, z_i, \omega), \qquad (1b)$$

in some stable sense. Stabilization is assured when relation (1b) is replaced by

$$y(x, y, z_i, \omega) = [S^+(x, y, z_i, \omega)]^* P^-(x, y, z_i, \omega), \qquad (1c)$$

where the symbol * denotes complex conjugation. Notice that relation (1c) is the frequency-domain representation for the correlation of the downgoing source wave S^+ and the upgoing reflected waves P^-.

The above two steps are repeated (recursively or nonrecursively) for all depths z_i, thus yielding one migrated shot in terms of the *ZO* reflectivity R. When all shots have been migrated, then the individual migration results can be summed. Optionally a residual NMO-correction can be applied if the input velocity model is in error. By the summation procedure, true common-depth-point (CDP) stacking is accomplished. The whole procedure can be applied in two or three dimensions. For the 2-D case the processing sequence is visualized in Figure 1.

Generation of multifold zero-offset data by SSRI

Prestack migration by SSRI, as described in the previous section, is very laborious when applied in three dimensions. Here