synthetic data are shown in Figure 1. As we expect the flat event focuses at the rms velocity while the dipping event focuses at the dip adjusted velocity (dip adjusted velocity = rms velocity divided by the cosine of the dip) which, for this example, equals approximately 14 000 ft/s. Although the two layers do not actually cross in the subsurface, we arranged the model in such a way that they do cross on the CDP stacked section (see Figure 1). Thus we have an excellent example of the classic problem associated with the CDP stacking method, i.e., using a single valued stacking velocity function we cannot simultaneously image events which are coincident in time but have different time dip. We refer to this as the multivalue stacking velocity problem. The PSPM procedure described above was applied to the model 1 synthetic data and the resulting processed traces were used to generate a second set of line constant velocity stacks, Figure 2. Poststack *F-K* migration of the single 11 000 ft/s constant velocity stack is shown in Figure 3. Figure 4 shows the results of applying the velocity independent DMO procedure to the line constant velocity stacks of Figure 1. In Figure 5 each of the ve-

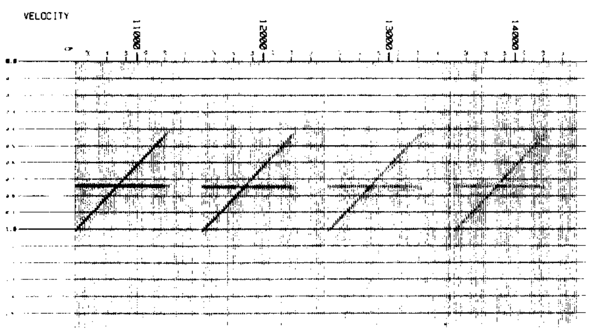

FIG. 2. Line constant velocity stacks after application of prestack partial migration, model 1.

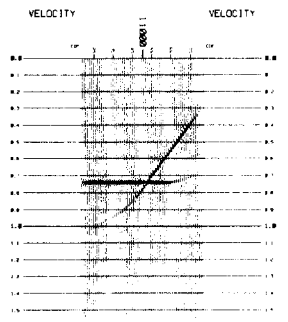

FIG. 3. *F-K* migration of 11 000 ft/s panel in Figure 2.

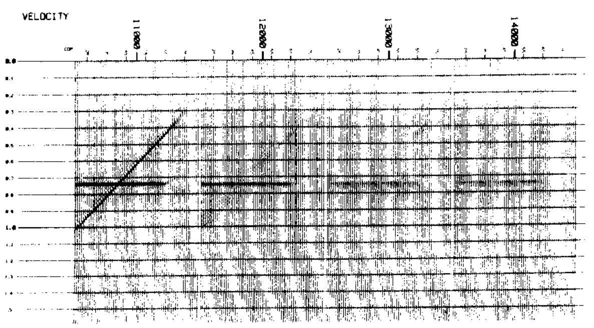

FIG. 4. Velocity independent DMO applied to constant velocity stacks of Figure 1.

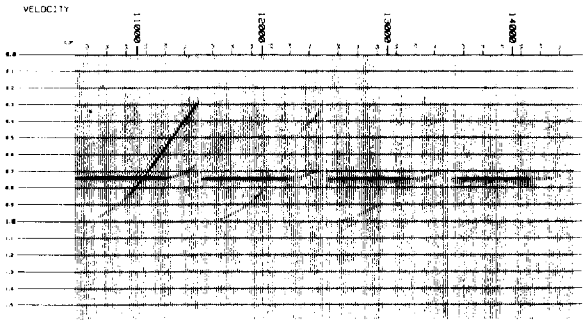

FIG. 5. Velocity independent DMO + *F-K* migration applied to constant velocity stacks of Figure 1.

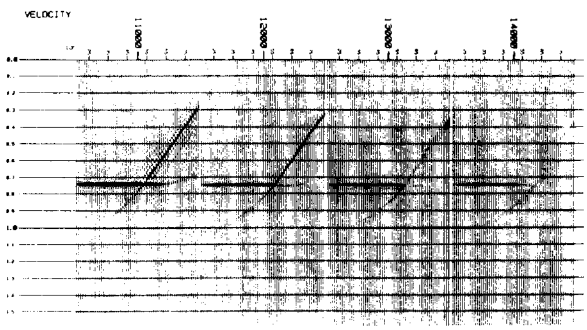

FIG. 6. *F-K* prestack migration, model 1.

locity independent DMO panels from Figure 4 has been *F-K* migrated using the appropriate velocity for each panel. Finally, the results of applying the *F-K* prestack migration procedure directly to the synthetic shot records is shown in Figure 6. Comparing Figures 3, 5, and 6, we see that all three procedures provide acceptable solutions to the multivalue stacking velocity problem. Unfortunately, model 1 cannot be used to analyze the problem of CRP smear nor is it large enough to provide a reliable comparison of costs. More complex models and field data examples examining these effects as well as the respective strategies for handling the velocity question are included in the talk and accompanying preprint.

References

Fowler, P., 1984, Velocity independent imaging of seismic reflectors: Presented at the 54th Annual SEG Meeting, Atlanta.

Hale, I. D., 1983, Dip-moveout by Fourier transform: Presented at the 53rd Annual SEG Meeting, Las Vegas.

Jakubowicz, H., 1984, A simple exact method of pre-stack partial migration: Presented at the 46th Meeting of the EAEG, London.

Larner, K. and Ng, P., 1984, 3-D marine seismic survey direction: Strike or dip?: Presented at the 54th Annual SEG Meeting, Atlanta.

Shurtleff, R. N., 1984, An F-K procedure for pre-stack migration and migration velocity analysis: Presented at the 46th Meeting of the EAEG, London.

A Critical Discussion on Prestack Migration Techniques S2.3

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Prestack migration schemes can be implemented in several ways. They may be classified by the coordinate system in which the data are migrated. In this paper several migration schemes are discussed based on different coordinate systems. It will be shown that *full* prestack migration can be described in terms of two orthogonal deconvolution procedures in the field coordinate system. On the other hand, *partial* prestack migration schemes can essentially be described in terms of two orthogonal deconvolution procedures in the midpoint-offset coordinate system. The former approach links up with the physical concept of seismic experiments and allows complicated subsurface models. The latter approach drops this physical concept, but allows a very efficient treatment in the Fourier domain.

Introduction

In any echo-acoustical technique, two basic processes are discerned, namely, the forward process and the inverse process. The forward process, also called modeling process, describes the downward propagation of a source wave field into a medium, the reflection and diffraction by inhomogeneities and finally the upward propagation and recording. The inverse process, also called seismic migration, aims at eliminating the propagation effects introduced by the forward process thus forming an acoustic image of the subsurface reflectivity. Both forward and inverse processes can be represented by physical models based on the acoustic wave theory. Propagation is described by the scalar acoustic wave equation, reflection and diffraction by the boundary conditions imposed to that equation. Based on the scalar wave equation and the well-known Huygens principle, one-way wave field extrapolation operators can be derived, which describe the forward propagation of sound waves.

Two-dimensional model of shot records

If we make use of the matrix notation (Berkhout, 1982), an elegant expression can be given for the measurements of one seismic experiment, i.e., the data of one shot record. The formulation is a monochromatic one and can be summarized as follows:

$$\mathbf{P}^-(z_0) = \mathbf{D}(z_0) \sum_i [\mathbf{W}(z_0, z_i)\, \mathbf{R}(z_i)\, \mathbf{W}(z_i, z_0)]\, \mathbf{P}^+(z_0). \quad (1)$$

Figure 1 gives a schematic illustration. It appeals very well to physical intuition and contains the essentials of any echo technique: illumination $\mathbf{P}^+(z_0)$, downward progation $\mathbf{W}(z_i,z_0)$, reflection $\mathbf{R}(z_i)$, back propagation $\tilde{\mathbf{W}}(z_0, z_i)$ and detection $\mathbf{D}(z_0)$.

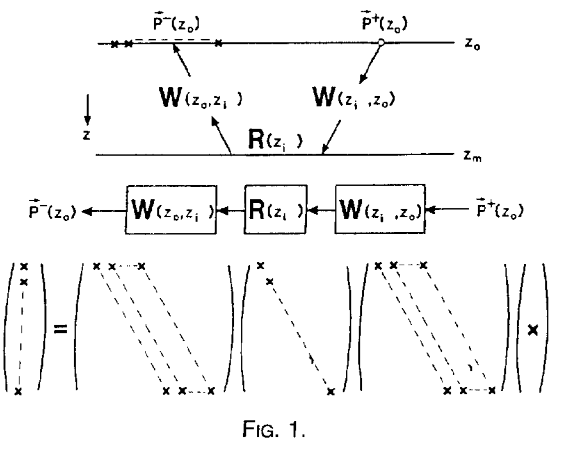

FIG. 1.

Principle of full prestack migration

Any inversion process, which aims at recovering reflectivity information from the subsurface, will be more successful if data are used from many different source positions. As a consequence, we would like to extend shot record model (1) to the model for a multirecord data set. This can be easily done by extending the source *vector* $\mathbf{P}^+(z_0)$ to a source *matrix* $\mathbf{p}(z_0)$. Each column of this matrix defines one source vector. If the source vector is replaced by a source matrix in expression (1) then response *vector* $\mathbf{P}^-(z_0)$ has to be replaced by a response *matrix* $\mathbf{p}^-(z_0)$ such that each column of this matrix defines one response vector, i.e., one common shot gather (CSG). Similarly each row of the "seismic data matrix" $\mathbf{P}^-(z_0)$ defines one common detector gather (CDG).

Migration may be considered as the inversion process which eliminates the propagation distortion from the reflectivity information taking into account the limitations due to data acquisition imperfections. Assuming for simplicity unity matrices for $\mathbf{P}^+(z_0)$ and $\mathbf{D}(z_0)$, the response from depth level z_i can be written as

$$\mathbf{P}^-(z_0) = \mathbf{W}(z_0, z_i)\, \mathbf{R}(z_i)\, \mathbf{W}(z_i, z_0). \quad (2)$$

Hence, an estimate of the reflectivity information can be obtained from data matrix $\mathbf{P}^-(z_0)$ by inverting for downward propagation matrix $\mathbf{W}\ (z_i, z_0)$ and upward propagation matrix $\tilde{\mathbf{W}}\ (z_0, z_i)$:

$$>\mathbf{R}(z_i)> = \mathbf{F}(z_i, z_0)\, \mathbf{P}^+(z_0)\, \mathbf{F}(z_0, z_i), \quad (3a)$$

or, using the matched filter approach,

$$<\mathbf{R}(z_i)> = \mathbf{W}^*(z_i, z_0)\, \mathbf{P}^-(z_0)\, \mathbf{W}^*(z_0, z_i), \quad (3b)$$

where * denotes complex conjugation. Expression (3b) describes two deconvolution processes (see Figure 2): (1) $\mathbf{W}^*(z_i, z_0)\, \mathbf{P}^-(z_0)$ describes a spatial deconvolution process along the *columns* of $\mathbf{P}^-(z_0)$, and (2) $\mathbf{P}^-(z_0)\, \mathbf{W}^*(z_0, z_i)$ describes a spatial deconvolution process along the *rows* of $\mathbf{P}^-(z_0)$. Hence, the upward propagation distortion is eliminated by deconvolving all shot records and the downward propagation distortion is eliminated by deconvolving all detector gathers.

In expression (2) we considered the response from one depth level only. However, from expression (1) it follows that data matrix $\mathbf{P}^-(z_0)$ contains reflectivity contributions from many depth levels. Therefore, inversion as described by (3b) is not complete, that is, $<\mathbf{R}(z_i)>$ still contains reflectivity information from many depth levels. The reflectivity at the current level can be resolved from $<\mathbf{R}(z_i)>$ by integrating over all frequencies. When zero offset (*ZO*) imaging is required, then the diagonal elements of $\int <\mathbf{R}(z_i)>\, d\omega$ should be selected. A further discussion of the imaging principle is beyond the scope of this paper.

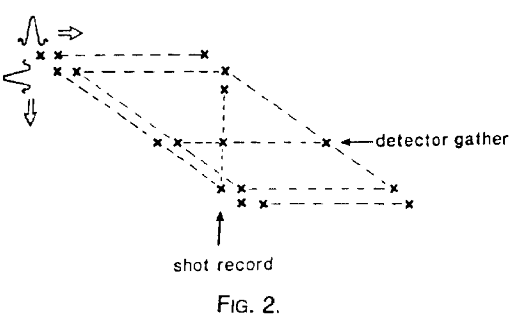

FIG. 2.

Remarks. We have seen that the output of prestack migration is chosen to be the zero-offset reflectivity distribution of the subsurface (diagonal elements of the reflectivity matrix). Hence it appears that too much work is done in the full prestack migration scheme if the zero-offset reflectivity is needed only. Therefore, Berkhout (1984) discusses a relatively simple single shot record inversion (SSRI) scheme. This scheme yields individually migrated shots in terms of the *ZO* reflectivity $\underline{\mathbf{R}}$. The individual migration results can be summed afterward. But this summation procedure, *true* common-depth-point (CDP) stacking is accomplished. Berkhout shows that migration by SSRI and CDP stacking yields exactly the same output as full prestack migration.

Prestack migration in alternative coordinate systems

The prestack migration approach we discussed above is carried out in the field coordinate system (x_s, x_d), where x_s and x_d represent the source and detector coordinates, respectively. A very important advantage of migration in the field coordinate system is that complicated subsurface velocity models can be handled when the appropriate extrapolation operators are chosen. For simple velocity models (vertical variations only), various attractive alternative prestack migration schemes have been proposed in literature. These alternative procedures are carried out in different coordinate systems. In this section some general considerations concerning coordinate transformations are presented.

First we consider inversion algorithm (3) for a constant velocity medium in the wavenumber-frequency domain:

$$< \bar{\underline{\mathbf{R}}}(k_x^{(s)}, k_x^{(d)}, z_i, \omega) > \; = \bar{\underline{\mathbf{F}}}(z_i, z_0)\, \bar{\underline{\mathbf{P}}}^-(k_x^{(s)}, k_x^{(d)}, z_0, \omega)\, \bar{\underline{\mathbf{F}}}(z_0, z_i), \quad (4a)$$

where

$$\bar{\underline{\mathbf{F}}}(z_0, z_i) = \exp\left[j\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(k_x^{(s)}\right)^2}\, \Delta z_i \right], \quad (4b)$$

$$\bar{\underline{\mathbf{F}}}(z_i, z_0) = \exp\left[j\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(k_x^{(d)}\right)^2}\, \Delta z_i \right], \quad (4c)$$

with ω = circular frequency, c = propagation velocity, $k_x^{(s)}$ = wavenumber related to x_s, $k_x^{(d)}$ = wavenumber related to x_d, $\Delta z_i = z_i - z_0$. The product $\bar{\underline{\mathbf{F}}}(z_i, z_0)$. $\bar{\underline{\mathbf{F}}}(z_0, z_i)$ is known in literature as the "double square root operator."

We now introduce midpoint (x_m) and half-offset (x_h) coordinates according to,

$$x_m = \frac{1}{2}(x_d + x_s), \quad (5a)$$

$$x_h = \frac{1}{2}(x_d - x_s). \quad (5b)$$

Following Yilmaz and Claerbout (1980), the wavenumbers $k_x^{(m)}$ and $k_x^{(h)}$, related to x_m and x_h, respectively, should satisfy the following set of equations

$$k_x^{(d)} = \frac{1}{2}(k_x^{(m)} + k_x^{(h)}), \quad (6a)$$

and

$$k_x^{(s)} = \frac{1}{2}(k_x^{(m)} - k_x^{(h)}). \quad (6b)$$

This means that for the new coordinate system, inversion algorithm (4) may be replaced by

$$<\bar{\underline{\mathbf{R}}}(k_x^{(m)}, k_x^{(h)}, z_i, \omega)> \; = \bar{\underline{\mathbf{F}}}'(z_i, z_0)\, \bar{\underline{\mathbf{P}}}^-(k_x^{(m)}, k_x^{(h)}, z_0, \omega)\, \bar{\underline{\mathbf{F}}}'(z_0, z_i), \quad (7a)$$

where

$$\bar{\underline{\mathbf{F}}}'(z_0, z_i) = \exp\left[j\sqrt{\left(\frac{\omega}{c}\right)^2 - \frac{1}{4}\left(k_x^{(m)} - k_x^{(h)}\right)^2}\, \Delta z_i \right], \quad (7b)$$

$$\bar{\underline{\mathbf{F}}}'(z_i, z_0) = \exp\left[j\sqrt{\left(\frac{\omega}{c}\right)^2 - \frac{1}{4}\left(k_x^{(m)} + k_x^{(h)}\right)^2}\, \Delta z_i \right]. \quad (7c)$$

Relation (7) provides the basis for migration of CMP slant stacks, as proposed by Ottolini and Claerbout (1984). Their method, which actually involves full prestack migration, is exact for 1-D inhomogeneous media. Further discussion of their approach is beyond the scope of this paper. Notice that in both operators (7b) and (7c) the wavenumbers $k_x^{(m)}$ and $k_x^{(h)}$ are coupled. For small wavenumbers (i.e., small dips both in midpoint and offset space), relation (7) can be approximated by

$$< \bar{\underline{\mathbf{R}}}(k_x^{(m)}, k_x^{(h)}, z_i, \omega) > \; \approx \bar{\underline{\mathbf{F}}}^{(m)}(z_i, z_0)\, \bar{\underline{\mathbf{P}}}^-(k_x^{(m)}, k_x^{(h)}, z_0, \omega)\, \bar{\underline{\mathbf{F}}}^{(h)}(z_0, z_i), \quad (8a)$$

where

$$\underline{\mathbf{F}}^{(m)}(z_i, z_0) = \exp\left[j\sqrt{\left(\frac{\omega}{c}\right)^2 - \frac{1}{2}(k_x^{(m)})^2}\, \Delta z_i \right], \quad (8b)$$

$$\underline{\mathbf{F}}^{(h)}(z_0, z_i) = \exp\left[j\sqrt{\left(\frac{\omega}{c}\right)^2 - \frac{1}{2}(k_x^{(h)})^2}\, \Delta z_i \right], \quad (8c)$$

which can be verified by expanding both relations (7) and (8) as a Taylor series. It is interesting to note that the equivalence of relation (8) in the midpoint-offset domain involves two independent orthogonal deconvolutions along the diagonals and anti-diagonals of data matrix $\underline{\mathbf{P}}^-(z_0)$, which is visualized in Figure 3. It is important to realize however that prestack inverse wave field extrapolation *always* involves approximations when carried out as independent deconvolutions in midpoint-offset space. This result follows directly from the wavenumber coupling in relation (7).

An overview of partial prestack migration schemes

In this section we give a brief overview of migration algorithms which are carried out in the midpoint-offset space. In conventional processing, common midpoint data are first stacked for constant midpoint and variable offset (CMP-stacking), followed

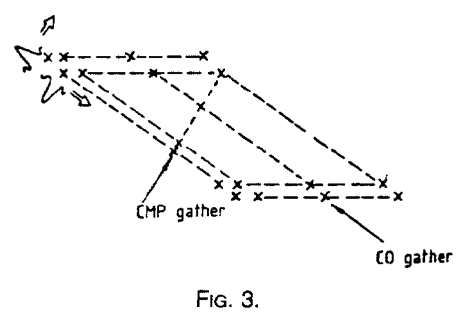

FIG. 3.

by migration for variable midpoint and zero offset (poststack migration). This artificial operator separation is actually only allowed for horizontally stratified media ($k_x^{(m)} = 0$), as can be easily seen from relation (7). When the data contain dip information, an alternative procedure must be followed. It was shown in the previous section from a wave theoretical point of view that an exact wave field extrapolation operator separation is *not* possible for the midpoint-offset space. However, in the literature several interesting procedures in the midpoint-offset space have been proposed which are based on *geometrical* considerations.

We mention offset continuation (Bolondi et al., 1984) and dip moveout (Hale, 1984). In both techniques the data are dip corrected for constant offset and variable midpoint, followed by stacking for constant midpoint and variable offset and post-stack migration. Particularly the dip moveout technique can be applied very efficiently by Fourier transforms. Several aspects will be illustrated with synthetic examples during the presentation.

Conclusions

Using a matrix expression for multishot prestack data, we discussed a full prestack migration scheme. This full prestack migration scheme operates in the field-coordinate system (x_s, x_d). It was emphasized that an important advantage of migration in the field coordinate system is that complicated subsurface models can be handled. Furthermore, we discussed an alternative formulation of the full prestack migration algorithm in the midpoint-offset coordinate system. It was shown that in this coordinate system midpoint and offset are coupled, so inverse wave field extrapolation involves approximations when carried out as independent deconvolutions in the midpoint-offset space. Finally we gave an overview of prestack partial migration algorithms in the midpoint-offset space. Though prestack partial migration schemes only allow one-dimensional velocity distributions they turn out to be very attractive from a point of view of efficiency.

References

Berkhout, A. J., 1982a, Seismic migration-imaging of acoustic energy by wave field extrapolation, v. 14a: Elsevier.

——— 1982b, Seismic migration-imaging of acoustic energy by wave field extrapolation, v. 14b: Elsevier.

Bolondi, G., Loinger, E., and Rocca, F., 1984, Offset Continuation in theory and practice: Geophys. Prosp. **32**.

Hale, D., 1984, Dip-moveout by Fourier transform: Geophysics, **49**.

Ottolini, R., and Claerbout, J. F., 1984, The migration of common midpoint slant stacks: Geophysics, **49**.

Yilmaz, O., and Claerbout, J. F., 1980, Prestack partial migration: Geophysics, **45**.

Principle of Prestack Migration Based on the Full Elastic Two-Way Wave Equation S2.4

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This paper starts with a brief comparison of the one-way and the two-way modeling and migration approaches. Next, a full elastic two-way wave field extrapolation operator is discussed, which is accurate up to 90 degrees. This operator is used in a full elastic prestack migration scheme for 2-D inhomogeneous media. In this scheme multiple reflected and converted waves are properly taken into account. Finally, the algorithm is illustrated in synthetic examples for 1-D inhomogeneous media. It is shown that with the aid of the full elastic two-way wave equation migration scheme, the angle-dependent *P-P* and *P-SV* reflectivities can be recovered independently.

Introduction

It has been shown by Berkhout (1982) that a seismic experiment can be elegantly described by a sequence of independent one-way processes, which is schematically represented by

$$S \rightarrow W^+ \rightarrow R \rightarrow W^- \rightarrow D \rightarrow P. \tag{1}$$

A wave field, generated by sources S at the surface, propagates downward into the earth, which is described by one-way wave field extrapolation operator W^+. In the subsurface this wave field is reflected, described by R, and propagates upward to the surface again, described by one-way operator W^-. At the surface the wave field is registered by detectors D, resulting in a seismic registration P. Based on this model, Berkhout discusses prestack modeling as well as prestack migration schemes for subcritical primary data in inhomogeneous fluids. Optionally multiple reflections can be included in modeling as well as in migration schemes by applying a feed-back system at each major reflecting boundary. An alternative approach to prestack modeling and prestack migration is discussed by Wapenaar and Berkhout (1984). They propose methods which are based on two-way wave field extrapolation, which is schematically represented by

$$\begin{bmatrix} P \\ -j\omega V_z \end{bmatrix}_{z_1} \rightarrow \underset{\sim}{\mathbf{W}} \rightarrow \begin{bmatrix} P \\ -j\omega V_z \end{bmatrix}_{z_2}. \tag{2}$$

Here $[P, -j\omega V_z]^T$ describes the *total* wave field in terms of pressure P and particle velocity V_z (ω denotes circular frequency), while $\underset{\sim}{\mathbf{W}}$ describes the two-way wave propagation effects (downgoing and upgoing reflected waves) between two depth levels. Relation (2) holds for primary as well as multiple reflected longitudinal P-waves in inhomogeneous fluids. In this paper the two-way extrapolation algorithm, which is accurate up to 90 degrees, will be generalized for full elastic media, according to

$$\begin{bmatrix} j\omega V_z \\ Z_x \\ Z_z \\ j\omega V_x \end{bmatrix}_{z_1} \rightarrow \underset{\sim}{\mathbf{W}} \rightarrow \begin{bmatrix} j\omega V_z \\ Z_x \\ Z_z \\ j\omega V_x \end{bmatrix}_{z_2}, \tag{3}$$

where $[j\omega\ V_z, Z_x, Z_z, j\omega\ V_x]^T$ describes the *total* wave field in terms of stress vector $[Z_x, Z_z]^T$ and particle velocity vector $[V_x, V_z]^T$, while $\underset{\sim}{\mathbf{W}}$ describes the full elastic two-way propagation effects between two depth levels. Relation (3) holds for primary as well as multiple reflected longitudinal P-waves and transversal SV-waves in inhomogeneous solids. This two-way extrapolation algorithm, which is accurate up to 90 degrees, provides the basis for prestack full elastic two-way migration schemes, as presented in this paper.

Full elastic two-way wave field extrapolation

In this section we briefly review the matrix formulation of the full elastic two-way wave equation and its solution. Details are given by Wapenaar and Berkhout (1985). We consider the following first-order differential equation in the space-frequency domain (Ursin, 1983, discusses an equivalent equation in the wavenumber-frequency domain):

$$\frac{\partial \mathbf{Q}}{\partial z} = \mathbf{AQ}, \tag{4a}$$

where

$$\underset{\sim}{\mathbf{A}} = \begin{bmatrix} \underset{\sim}{\mathbf{O}} & \underset{\sim}{\mathbf{A}}_1 \\ \underset{\sim}{\mathbf{A}}_2 & \underset{\sim}{\mathbf{O}} \end{bmatrix}, \qquad \mathbf{Q} = \begin{bmatrix} j\omega V_z \\ Z_x \\ Z_z \\ j\omega V_x \end{bmatrix}, \tag{4b,c}$$