Time-lapse data prediction by Marchenko-based reservoir transplantation

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SUMMARY

In a time-lapse experiment, changes in a reservoir cause changes in the reflection response. We discuss a method which predicts these changes from the baseline survey and a model of the changed reservoir. This method, which takes all multiple scattering into account, is significantly more efficient than modeling the response of the entire medium containing the changed reservoir. This can be particularly attractive for applications in time-lapse full wave form inversion, which requires repeated modelling of the reflection response.

INTRODUCTION

In a time-lapse experiment, changes in the medium often occur only locally, for example in a reservoir. Robertsson and Chapman (2000) devised a method to efficiently model the response of a medium after a local change of the medium parameters. They first model the wave field in the full original medium, define a boundary around the domain in which changes are going to take place, and evaluate the field at this boundary. Next, they numerically inject this field from the same boundary into a model of the changed domain (e.g. a reservoir after production). Because this domain usually covers only a small part of the full medium, this injection process takes only a fraction of the time that would be needed to model the time-lapse field in the full medium. This method is very well suited to model different time-lapse scenarios of, say, a producing reservoir in an efficient way. A limitation of the method is that multiple scattering between the changed domain and the embedding medium is not taken into account. The method was adapted by van Manen et al. (2007) to account for this type of multiple scattering, by modifying the field at the boundary around the changed domain at every time-step of the simulation. Wave field injection methods are not only useful for efficient numerical modelling of time-lapse wave fields, but can also be used to inject a field from a large numerical environment into a finite-size physical model (Vasmel et al., 2013).

Instead of numerically modelling the field at the boundary enclosing the changing domain, Elison et al. (2016) propose to use the Marchenko method to derive this field from reflection data at the surface. Hence, to obtain the time-lapse wave field in the changed domain (e.g. a reservoir), they need a measured reflection response at the surface of the original medium (the baseline survey) and a model of the changing domain. Their method exploits an attractive property of the Marchenko method, namely that redatumed reflection responses from above ($R^{\cup}$) and from below ($R^{\cap}$) can both be obtained from single-sided reflection data at the surface and an estimate of the direct arrivals (Wapenaar et al., 2014).

In all methods discussed above, the time-lapse fields are derived inside the changed domain. Here we discuss a method which predicts time-lapse data at the surface (monitor surveys) from reflection data at the surface (the baseline survey). The proposed method consists of two main steps. In the first step, which is analogous to the method proposed by Elison et al. (2016), we use the Marchenko method to surgically remove the response of the reservoir from the baseline survey. In the second step, we transplant the response of a new reservoir, yielding a monitor survey. Both steps fully account for multiple scattering. Note that, to predict different monitor surveys for different time-lapse scenarios, only the second step needs to be repeated. This procedure may be useful for example for time-lapse full wave form inversion.

A SIMPLE TIME-LAPSE EXPERIMENT

Before introducing the proposed methods, we discuss a simple time-lapse experiment. Figure 1 shows a horizontally layered medium. The velocities for the baseline survey are given in m/s, and the depth of the interfaces (denoted by the solid lines) in m. The layer between 1200 m and 1400 m is defined as the reservoir. To emphasise internal multiples, the mass densities are given the same numerical values as the propagation velocities. Figure 2(a) shows the plane-wave reflection response at $\mathbb{S}_0$ (which is a transparent surface), using a Ricker wavelet with a central frequency of 50 Hz. This is the baseline survey. The reflections from the top and bottom of the reservoir are indicated. Next, the velocity in the reservoir is changed from 4000 m/s to 3000 m/s (and a similar change is applied to the mass density). The plane-wave reflection response after this change, designated as the monitor survey, is shown Figure 2(b). The difference between the monitor and baseline surveys is shown in Figure 2(c). Note the significant multiple train following the difference response of the reservoir.

The aim of this paper is to show how a monitor survey can be predicted from the baseline survey by reservoir transplantation. The dotted lines in Figure 1 distinguish three different units. Unit *a* is the overburden, unit *b* contains the reservoir, and unit *c* is the underburden. Reservoir transplantation involves removing the response of unit *b* from the baseline survey and replacing it by the response of a new unit *b*. In the following, the theory will be discussed for the 3D situation, but the method will be applied to the 1D example of Figures 1 and 2.

REPRESENTATION OF THE REFLECTION RESPONSE

The starting point for the derivation of a suited representation of the reflection response is formed by flux-normalised one-way reciprocity theorems for down- and upgoing wave fields (Wapenaar and Grimbergen, 1996). We outline the main steps and leave the full derivation for a journal paper. Figure 3 shows six media which are used in the derivation. Media *a*, *b* and

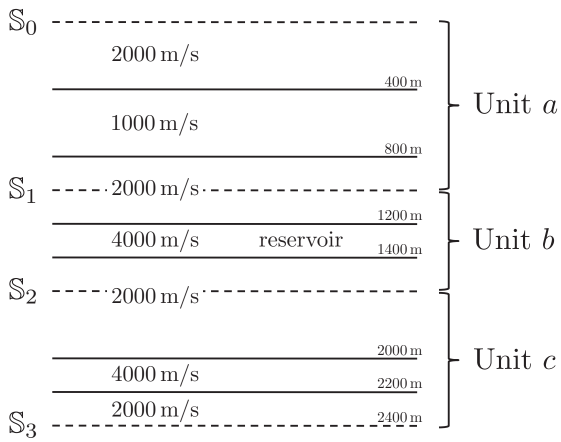

Figure 1: *Horizontally layered medium.*

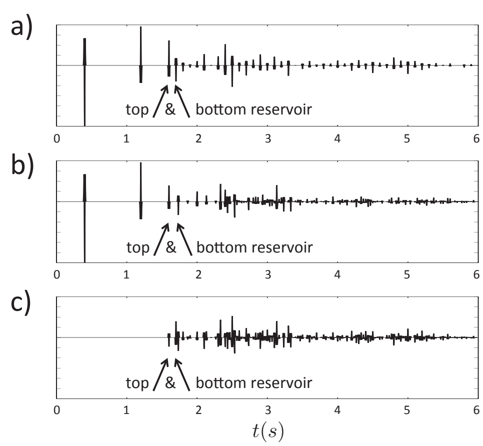

Figure 2: *Timelapse experiment. (a) Baseline survey. (b) Monitor survey. (c) Difference (monitor minus baseline).*

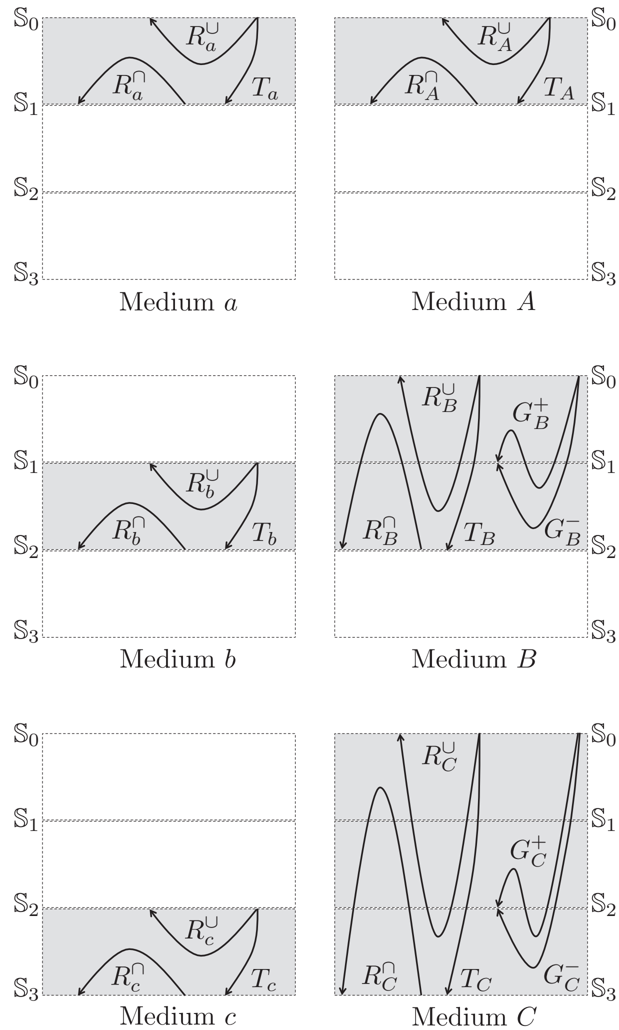

Figure 3: *Six media with their responses. Grey areas represent arbitrary inhomogeneous units. The rays stand for the full responses, including all orders of multiple scattering.*

c in the left column contain units a (the overburden), b (the reservoir) and c (the underburden), each embedded in a homogeneous background. The grey areas indicate arbitrary inhomogeneous units, whereas the white areas represent the homogenous embedding. Reflection responses from above and below are denoted by $R^{\cup}$ and $R^{\cap}$, respectively, and transmission responses by T. Because of the flux-normalisation, the transmission responses in the upward direction (not shown) are identical to those in the downward direction. The subscripts a, b and c refer to the units to which these responses belong. The rays are simplifications of the actual responses, which contain all orders of multiple scattering in the inhomogeneous units. Media A, B and C in the right column in Figure 3 consist of one to three units, as indicated. The reflection and transmission responses are indicated by capital subscripts A, B and C. In addition, the Green's functions G^{+} and G^{-} in these media represent the downgoing $(+)$ and upgoing $(-)$ responses at the top boundary of the deepest unit in response to a source at the upper boundary. Note that media a and A are identical.

The one-way reciprocity theorems relate down- and upgoing wave fields at the boundaries of two media. By choosing appropriate combinations of the media in Figure 3 for the one-way reciprocity theorems, we obtain a representation for $R_C^{\cup}$, being the reflection response from above of medium C (i.e., the total medium). This representation reads in the space-frequency domain

$$\begin{aligned} R_C^{\cup}(\mathbf{x}_R,\mathbf{x}_S,\omega) &= R_A^{\cup}(\mathbf{x}_R,\mathbf{x}_S,\omega) \\ &+ \int_{\mathbb{S}_1}\int_{\mathbb{S}_1} T_A(\mathbf{x}_R,\mathbf{x}',\omega) R_b^{\cup}(\mathbf{x}',\mathbf{x},\omega) G_B^{+}(\mathbf{x},\mathbf{x}_S,\omega)\mathrm{d}\mathbf{x}\mathrm{d}\mathbf{x}' \\ &+ \int_{\mathbb{S}_2}\int_{\mathbb{S}_2} T_B(\mathbf{x}_R,\mathbf{x}',\omega) R_c^{\cup}(\mathbf{x}',\mathbf{x},\omega) G_C^{+}(\mathbf{x},\mathbf{x}_S,\omega)\mathrm{d}\mathbf{x}\mathrm{d}\mathbf{x}'. \end{aligned} \tag{1}$$

Here $\mathbf{x}_S$ and $\mathbf{x}_R$ represent the source and receiver coordinate, respectively, at the upper boundary $\mathbb{S}_0$, and ω stands for angular frequency. The first term on the right-hand side is the reflection response of the overburden (Figure 3, medium A). The second and third terms on the right-hand side are visualised in Figure 4. When there are more units below unit c, equation 1 is trivially extended with additional terms on the right-hand side. On the other hand, when unit c includes all inhomogeneities below $\mathbb{S}_2$, then equation 1 is a complete representation of the reflection response at $\mathbb{S}_0$. Note that equation 1 is akin to the generalised primary representation (Wapenaar, 1996), in which the sum on the right-hand side is replaced by an integral along the depth coordinate, and the reflection responses under the integrals are replaced by local reflection operators.

For notational convenience we rewrite the representation of equation 1 with the following compact notation

$$R_C^{\cup} = R_A^{\cup} + \int_{\mathbb{S}_1}\int_{\mathbb{S}_1} T_A R_b^{\cup} G_B^{+} + \int_{\mathbb{S}_2}\int_{\mathbb{S}_2} T_B R_c^{\cup} G_C^{+}. \qquad (2)$$

This is the representation for the baseline survey. With the same notation, the representation for the monitor survey reads

$$\bar{R}_C^{\cup} = R_A^{\cup} + \int_{\mathbb{S}_1}\int_{\mathbb{S}_1} T_A \bar{R}_b^{\cup} \bar{G}_B^{+} + \int_{\mathbb{S}_2}\int_{\mathbb{S}_2} \bar{T}_B R_c^{\cup} \bar{G}_C^{+}. \qquad (3)$$

Here the bar in $\bar{R}_b^{\cup}$ denotes the reflection response of the new reservoir. $\bar{R}_C^{\cup}$ stands for the monitor survey. The bars on some of the other quantities indicate that these quantities are also effected by the new reservoir. In the following we discuss how all quantities needed for evaluating equation 3 are obtained from the baseline survey $R_C^{\cup}$ and a model of the new reservoir.

REMOVING THE RESERVOIR FROM THE BASELINE SURVEY

Using the baseline survey $R_C^{\cup}(\mathbf{x}_R, \mathbf{x}_S, \omega)$ as input, assuming in addition that an estimate of the direct arrival of the transmission response $T_A(\mathbf{x}_R, \mathbf{x}', \omega)$ is available, the Marchenko method yields the focusing functions $f_1^{+}(\mathbf{x}, \mathbf{x}', \omega)$ and $f_1^{-}(\mathbf{x}, \mathbf{x}', \omega)$ in medium A, for focal point $\mathbf{x}'$ at $\mathbb{S}_1$ and for $\mathbf{x}$ at $\mathbb{S}_0$ (Wapenaar et al., 2014; Slob et al., 2014). Subsequently, $R_A^{\cup}$ is resolved by inverting

$$f_1^{-} = \int_{\mathbb{S}_0} R_A^{\cup} f_1^{+} \qquad (4)$$

(we use again the compact notation of equation 2). In a similar way, $R_A^{\cap}$ is resolved by inverting

$$f_2^{+} = \int_{\mathbb{S}_1} R_A^{\cap} f_2^{-}, \qquad (5)$$

with $f_2^{-} = f_1^{+}$ and $f_2^{+} = -(f_1^{-})^{*}$, where the asterisk denotes complex conjugation. Finally, T_A is obtained from

$$\delta = T_A f_1^{+}, \qquad (6)$$

where δ is a spatial delta function. For the 1D example, the resolved response $R_A^{\cup}$ is shown in the time domain in Figure 5(b). Note that it contains the first two events of $R_C^{\cup}$ and a coda due to the internal multiples in the low-velocity layer in medium A.

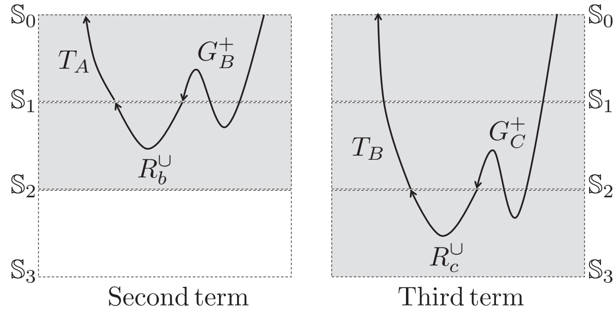

Figure 4: *Visualization of the second an third term in representation 1.*

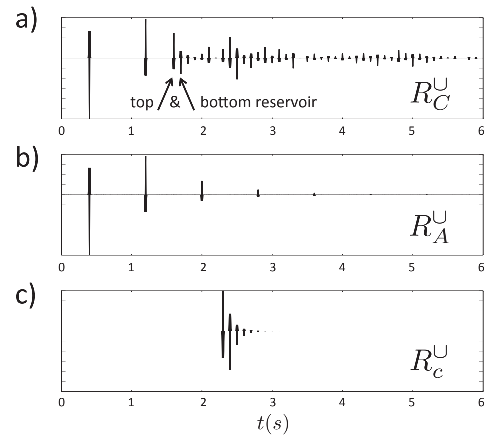

Figure 5: *Removal of the reservoir, using the Marchenko method. (a) Baseline survey (input). (b) Response of medium A (output). (c) Response of unit c (output).*

Next, the Marchenko method is used to estimate the Green's functions $G_C^{+}(\mathbf{x}, \mathbf{x}_S, \omega)$ and $G_C^{-}(\mathbf{x}, \mathbf{x}_S, \omega)$ in medium C, for receivers at $\mathbf{x}$ on $\mathbb{S}_2$. $R_c^{\cup}$ is then resolved by inverting

$$G_C^{-} = \int_{\mathbb{S}_2} R_c^{\cup} G_C^{+}. \qquad (7)$$

For the 1D example, $R_c^{\cup}$ is shown in Figure 5(c). For display purposes it has been shifted in time, so that the travel times correspond with those in Figure 5(a).

Summarizing, the method described here resolves the overburden responses $R_A^{\cup}$, $R_A^{\cap}$ and T_A, as well as the underburden response $R_c^{\cup}$ from the baseline survey $R_C^{\cup}$, see the left frame in Figure 6. The imprint of the reservoir response $R_b^{\cup}$ on $R_C^{\cup}$ (see equation 2) is absent in the resolved responses.

TRANSPLANTING A RESERVOIR INTO THE MONITOR SURVEY

Given a model of the reservoir in the monitor state, its reflection and transmission responses, $\bar{R}_b^{\cup}$ and $\bar{T}_b$ respectively, can be

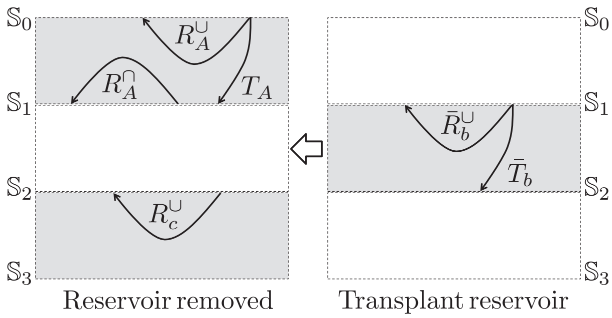

Figure 6: *Left: overburden and underburden responses, obtained from the baseline survey $R_C^{\cup}$, using the Marchenko method. Right: modelled responses of the new reservoir, to be transplanted between the overburden and underburden responses.*

obtained by numerical modelling, see the right frame in Figure 6. For the 1D example, $\bar{R}_b^{\cup}$ is shown in the time domain in Figure 7(a). For display it has been shifted in time, so that the travel time to the top of the reservoir corresponds with that in Figure 5(a).

To predict the monitor survey $\bar{R}_C^{\cup}$, equation 3 should be evaluated. Quantities that still need to be obtained are $\bar{G}_B^{+}$, $\bar{T}_B$ and $\bar{G}_C^{+}$. Applying again the one-way reciprocity theorems to appropriate combinations of the media in Figure 3, we obtain the relations that are needed to resolve these quantities. We outline only the main steps. To obtain $\bar{G}_B^{+}$ we need to invert the relation

$$\int_{\mathbb{S}_1}\left[\delta-\int_{\mathbb{S}_1}R_A^{\cap}\bar{R}_b^{\cup}\right]\bar{G}_B^{+}=T_A. \tag{8}$$

Next, $\bar{T}_B$ is obtained from

$$\bar{T}_B=\bar{T}_b\bar{G}_B^{+}. \tag{9}$$

Analogous to equation 8, $\bar{G}_C^{+}$ is obtained by inverting

$$\int_{\mathbb{S}_2}\left[\delta-\int_{\mathbb{S}_2}\bar{R}_B^{\cap}R_c^{\cup}\right]\bar{G}_C^{+}=\bar{T}_B. \tag{10}$$

To this end, we first need to resolve $\bar{R}_B^{\cap}$ from

$$\int_{\mathbb{S}_2}\bar{T}_B^{*}\bar{R}_B^{\cap}=-\int_{\mathbb{S}_0}(\bar{R}_B^{\cup})^{*}\bar{T}_B \tag{11}$$

(Wapenaar et al., 2004). Here $\bar{R}_B^{\cup}$ is the reflection response of medium B in the monitor state, which is represented by the first two terms of equation 3, hence

$$\bar{R}_B^{\cup}=R_A^{\cup}+\int_{\mathbb{S}_1}\int_{\mathbb{S}_1}T_A\bar{R}_b^{\cup}\bar{G}_B^{+}. \tag{12}$$

We now have all ingredients to predict the monitor survey with equation 3, or, since $\bar{R}_B^{\cup}$ is already available,

$$\bar{R}_C^{\cup}=\bar{R}_B^{\cup}+\int_{\mathbb{S}_2}\int_{\mathbb{S}_2}\bar{T}_BR_c^{\cup}\bar{G}_C^{+}. \tag{13}$$

For the 1D example, $\bar{R}_C^{\cup}$ obtained in this way is shown in the time domain in Figure 7(b). The difference with the directly

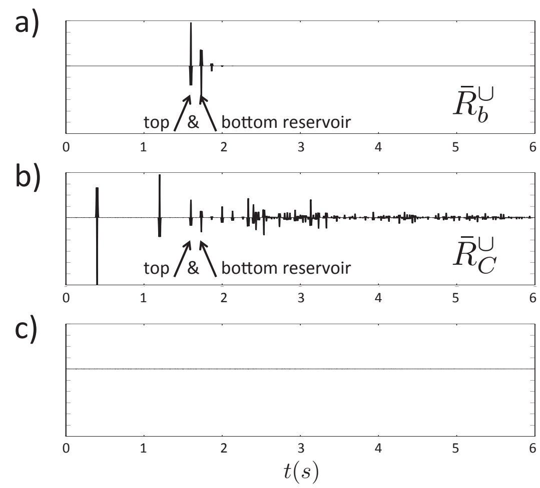

Figure 7: *Transplantation of the reservoir in the monitor state. (a) Numerically modelled response of the new reservoir. (b) Monitor survey, obtained with the representation of equations 12 and 13. (c) Difference between the monitor survey of Figure 7(b) and the directly modelled monitor survey of Figure 2(b).*

modelled monitor survey of Figure 2(b) is shown in Figure 7(c). This confirms that the monitor survey has been very accurately predicted by the proposed method.

CONCLUSIONS

We have proposed a two-step process to predict a time-lapse monitor survey from the baseline survey and a model of the reservoir in the monitor state. In the first step, the response of the original reservoir is surgically removed from the baseline survey, using the Marchenko method. In the second step, the modelled response of a new reservoir is transplanted between the overburden and underburden responses. The method fully accounts for multiple scattering. It can be employed to predict the monitor state for a range of time-lapse scenarios. In that case, the first step needs to be carried out only once. Only the second step needs to be repeated for each reservoir model. Since the reservoir model covers only a small part of the entire medium, repeated modelling of the reservoir response (and transplanting it each time between the same overburden and underburden responses) is a much more efficient process than repeated modelling of the entire response. This method may therefore find applications in time-lapse full wave form inversion. Because all multiples are taken into account, the coda following the response of the reservoir may be employed in the inversion. Because of the high sensitivity of the coda for changes in the medium (Snieder et al., 2002), this may ultimately improve the resolution of the inverted time-lapse changes. Finally, when parts of the overburden change as well during a time-lapse experiment, these can be transplanted in a similar way as the reservoir, but this will have a limiting effect on the efficiency gain.