Connection of scattering principles: focusing the wavefield without source or receiver

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Summary

Inverse scattering, seismic interferometry, and focusing are subjects usually studied as independent problems in different research areas. We speculate that a physical connection exists between them because the equations that rule these *scattering principles* have a similar functional form. With a visual explanation of the relationship between these principles, we describe the importance of the interaction between the causal and acausal Green's functions and provide physical insight to emphasize the connection between these principles. Finally, we show how to reconstruct the Green's function that radiates from a location where there is no source or receiver, going beyond seismic interferometry.

Introduction

Consider an incident plane wave, created by an array of sources, that is distorted due to the variations of the velocity inside the overburden (i.e., the portion of the subsurface that lies above the scatterer) as illustrated in Figure 1. Since we don't know the characteristics of the wavefield when it interacts with the region of the subsurface that includes the scatterer, it is difficult to reconstruct the properties of the scatterer without knowing the medium. We would like to create a special incident wave that, after interacting with the overburden (which is a scattering medium), collapses to a point in the subsurface creating a buried source, as illustrated in Figure 2. In this case, assuming that the medium around the scatterer is known, we would know the shape of the wavefield that probes the scatterer and partially remove the effect of the overburden, which would facilitate more accurate imaging of the scatterer. With such a special incident wave, we intend to compensate for the scattering as well as the propagation aspects of the overburden.

The construction of this special incident wave in two and three dimensions still needs further investigation (Wapenaar et al., 2011). In this work we show the insights gained by comparing several scattering principles for a one-dimensional problem.

Inverse scattering, seismic interferometry, and focusing are subjects usually studied in different research areas such as seismology (Aki and Richards, 2002), quantum mechanics (Rodberg and Thaler, 1967), optics (Born and Wolf, 1999), nondestructive evaluation of material (Shull, 2002), and medical diagnostics (Epstein, 2003).

Inverse scattering (Chadan and Sabatier, 1989; Gladwell, 1993; Colton and Kress, 1998) is the problem of determining the perturbation of a medium (e.g., of a constant velocity medium) from the field scattered by this perturbation. In other words, one wants to reconstruct the properties of the perturbation from a set of measured data. Inverse scattering takes into account the nonlinearity of the inverse problem, but it also presents some drawbacks: it may be improperly posed from the point of view of numerical computations (Dorren et al., 1994), and it requires data recorded at locations usually not accessible due to practical limitations.

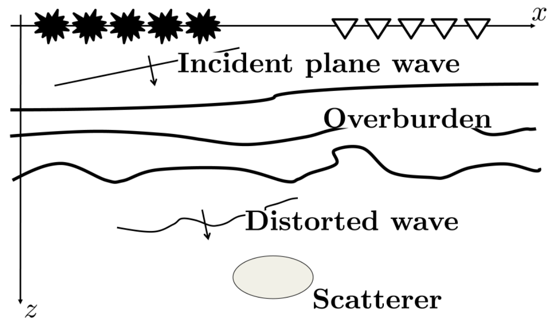

Figure 1: An incident plane wave created by an array of sources is injected in the subsurface. This plane wave is distorted due to the variations in the velocity inside the overburden (i.e., the portion of the subsurface that lies above the scatterer). When the wavefield arrives in the region that includes the scatterer, we do not know its shape.

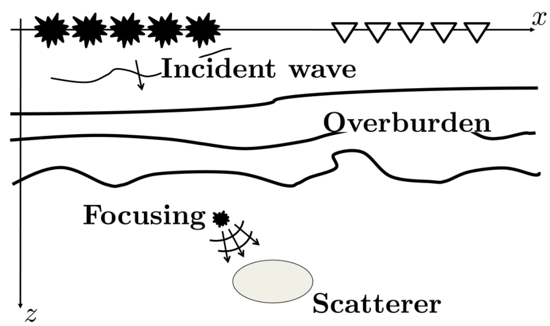

Figure 2: Focusing of the wavefield at depth. A special incident wave, after interacting with the overburden, collapses to a point in the subsurface creating a buried source.

Seismic interferometry (Wapenaar et al., 2005; Curtis et al., 2006; Schuster, 2009) is a technique that allows one to reconstruct the response between two receivers from the cross-correlation of the wavefield measured at these two receivers which are excited by uncorrelated sources surrounding the studied system. In the seismic community, this technique is also known as the *virtual source method* (Bakulin and Calvert, 2006); this refers to the fact that the new response is reconstructed as if one receiver had recorded the response due to a *virtual* source located at the other receiver position.

In this paper, the term *focusing* (Rose, 2001, 2002b) refers to the technique of finding an incident wave that collapses to a spatial delta function $\delta(x-x_0)$ at the location x_0 and at a prescribed time t_0 (i.e., the wavefield is focused at x_0 at t_0). In a one-dimensional medium, we deal with a one-sided problem when observations from only one side of the perturbation are available (e.g., due to the practical consideration that we can only record reflected waves); otherwise, we call it a two-sided problem when we have access to both sides of the medium and account for both reflected and transmitted waves.

We refer to the subjects discussed above as *scattering principles* because they are all related, in different ways, to a *scattering* process. These principles are usually studied as isolated problems but they are related in various ways; hence, understanding their connections offers insight into each of the principles and eventually may lead to new applications. This work is motivated by a simple idea: because the equations that rule these *scattering principles* have a similar functional form, there should be a physical connection that could lead to better comprehension of these principles and to possible applications (e.g., Figure 2).

Visual tour

In this section, we show the relationship between different scattering principles using figures which lead the reader toward a visual understanding of the connections between the principles. Figure 3 illustrates a scattering experiment in a one-dimensional acoustic medium where an impulsive source is located at position $x = 1.44$ km within the scattering region, as shown in Figure 5. The excited wavefield, a band-limited spatial delta function, propagates toward the discontinuities in the model, interacts with them, and generates outgoing scattered waves. We use a time-space finite-difference code to simulate the propagation of the one-dimensional waves and to produce the numerical examples shown in this section. The computed wavefield is shown in Figure 3 and it represents the causal Green's function of the system, G^+. Causality ensures that the wavefield is non zero only in the region delimited by the first arrival (i.e., the direct waves).The slope of the lines, representing the first arrival, depends on the velocity model of Figure 4.

Due to practical limitations, we are usually not able to place a source inside the medium we want to probe, which raises the following question: Is it possible to create the wavefield illustrated in Figure 3 without having a real source at the position $x = 1.44$ km? An initial answer to such question is given by seismic interferometry. This technique allows one to reconstruct the wavefield that propagates between a receiver acting as a virtual source and other receivers located inside the

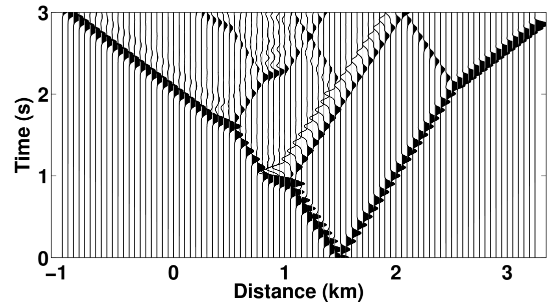

Figure 3: Scattering experiment with a source located at $x = 1.44$ km. The traces are recorded by receivers located along the model shown in Figure 4.

medium (Wapenaar et al., 2005). This technique yields a combination of the causal wavefield and its time-reversed version (i.e., acausal), which is due to the fact that the reconstructed wavefield is propagating between a receiver and a *virtual source*. Without a real (physical) source, one must have non-zero incident waves to create waves that emanate from a receiver. The fundamental steps to reconstruct the Green's function are (Curtis et al., 2006)

1. measure the wavefields $G^+(x,x_{sl})$ and $G^+(x,x_{sr})$ at a receiver located at x (x varies from -1 km to 3 km) excited by impulsive sources located at both sides of the perturbation x_{sl} and x_{sr} (a total of two sources in one dimension) as shown in Figure 5;
2. cross-correlate $G^+(x,x_{sl})$ with $G^+(x_{vs},x_{sl})$, where $x_{vs} = 1.44$ km and vs stands for *virtual source*;
3. cross-correlate $G^+(x,x_{sr})$ with $G^+(x_{vs},x_{sr})$;
4. sum the results computed at the two previous points to obtain $G^+(x,x_{vs})$;
5. repeat this for a receiver located at a different x.

The causal part of the wavefield estimated by seismic interferometry is shown in Figure 6. It is consistent with the result of the scattering experiment of Figure 3 produced with a real source located at $x = 1.44$ km.

We thus have two different ways to reconstruct the same wavefield, but often we are not able to access the part of the medium we want to study and hence we can't place any sources or receivers inside it. In this case we only have access to reflected waves $R(t)$ measured on the left side of the perturbation, i.e. the reflected impulse response measured at $x = 0$ km due to an impulsive source placed at $x = 0$ km. This further limitation raises another question: Can we reconstruct the same wavefield shown in Figure 3 having knowledge only of the scattering data $R(t)$? Since there are neither real sources nor receivers inside the perturbation, we anticipate that the reconstructed wavefield consists of a causal and an acausal part.

For this one-dimensional problem, the answer to this question is given by Rose (2001, 2002a). He shows that we need a particular incident wave in order to collapse the wavefield to a spatial

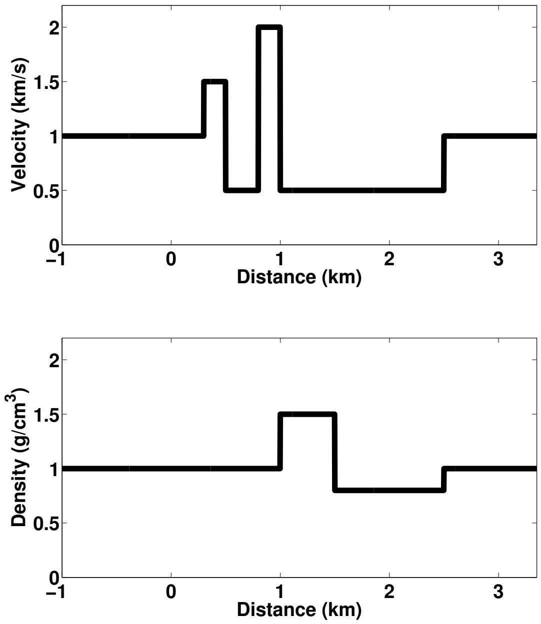

Figure 4: Velocity and density profiles of the one-dimensional model. The perturbation in the velocity is located between $x = 0.3$ km and $x = 2.5$ km and $c_0 = 1$ km/s. The perturbation in the density is located between $x = 1.0$ km and $x = 2.5$ km and $\rho_0 = 1$ g/cm^3.

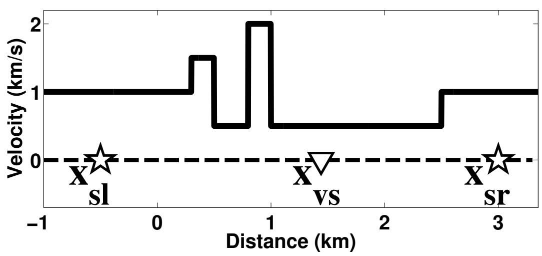

Figure 5: Diagram showing the locations of the real and virtual sources for seismic interferometry. x_{sl} and x_{sr} indicate the two real sources. x_{vs} shows the virtual source location.

delta function after it interacts with the medium, and that this incident wave consists of a spatial delta function followed by the time-reversed solution of the Marchenko equation, as illustrated in Figure 7. Such a solution is obtained by transforming the reflection data.

The Marchenko integral equation (Lamb, 1980; Chadan and Sabatier, 1989) is a fundamental relation of one-dimensional inverse scattering theory. It is an integral equation that relates the reflected scattering amplitude $R(t)$ to the incident wavefield $u(t,t_f)$ which creates a focus in the interior of the medium and ultimately gives the perturbation of the medium. The one-dimensional form of this equation is

$$0 = R(t+t_f) + u(t,t_f) + \int_{-\infty}^{t_f} R(t+t')u(t',t_f)dt', \qquad (1)$$

where t_f is a parameter that controls the focusing location. t_f

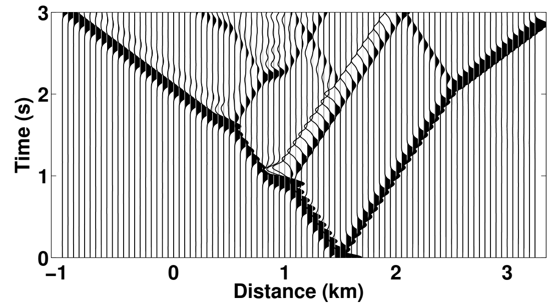

Figure 6: Causal part of the wavefield estimated by seismic interferometry when the receiver located at $x = 1.44$ km acts as a virtual source.

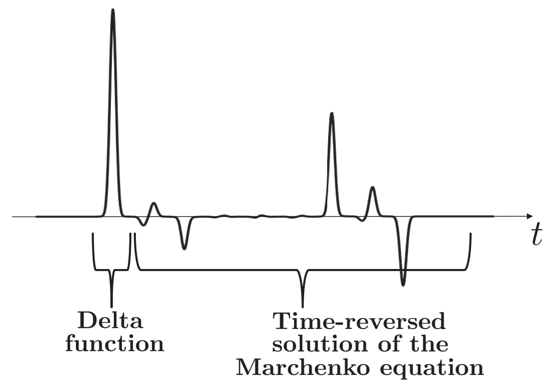

Figure 7: Incident wave that focuses at $t = 0$ s, built using the iterative process discussed by Rose (2002a).

corresponds to the one-way travel time from $x = 0$ km to the location where the wavefield is focused at $t = 0$ s. We solve the Marchenko equation, using the iterative process described in detail in Rose (2002a), and construct the particular incident wave that focuses at location $x = 1.44$ km. After 7 steps of the iterative process, we inject the incident wave in the model and compute the time-space diagram shown in the top panel of Figure 8: it shows the evolution of the wavefield when the incident wave is the particular wave computed with the iterative method. The bottom panel of Figure 8 shows a cross-section of the wavefield at focusing time $t = 0$ s: the wavefield vanishes except at location $x = 1.44$ km; hence the wavefield focuses at this location.

We create a focusing at a location inside the perturbation without having a source or a receiver at such a location and without any knowledge of the medium properties; we only have access to the reflected impulse response measured on the left side of the perturbation. With an appropriate choice of sources and receivers, this experiment can be done in practice (e.g., in a laboratory). Figure 8 however does not resemble the wavefields shown in Figures 3 and 6. But if we denote the wavefield in Figure 8 as $w(x,t)$ and its time-reversed version as $w(x,-t)$, we obtain the wavefield $u(x,t) = w(x,t) + w(x,-t)$ shown in Figure 9. With this process, we effectively go from one-sided to two-sided illumination using reflected waves recorded only

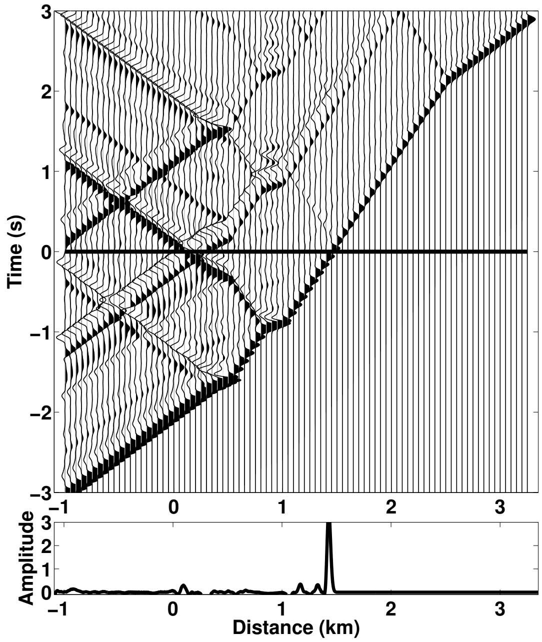

Figure 8: Top: After 7 steps of the iterative process described in Rose (2002a), we inject the particular incident wave in the model and compute the time-space diagram. Bottom: cross-section of the wavefield at $t = 0$ s. The first arrival at $t = 0$ s is non zero at $x = 1.44$ km, and the wavefield for $x < 1.44$ km has a zero crossing at $t = 0$ s.

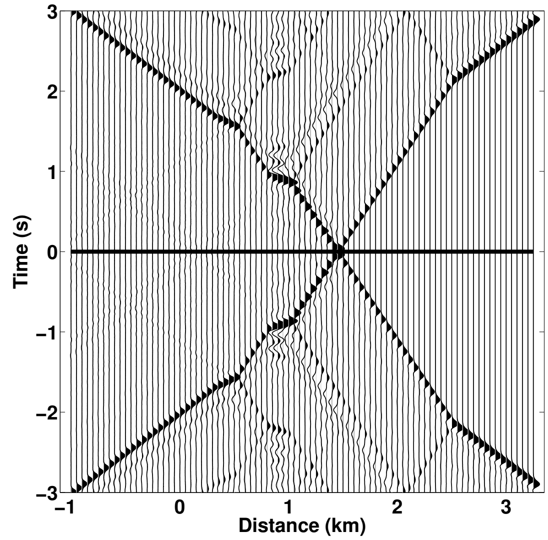

Figure 9: Wavefield that focuses at $x = 1.44$ km at $t = 0$ s without a source or a receiver at this location. This wavefield consists of a causal and an acausal part.

from one side of the perturbation. At early times, waves are propagating only from the left in Figure 8, while in Figure 9 there are waves propagating from both sides. Burridge (1980) shows similar diagrams and explains how to combine such diagrams using causality and symmetry properties. The upper cone in Figure 9 corresponds to the causal Green's function and the lower cone represents the acausal Green's function; we see that the relationship between the two Green's functions is a key element. Note that the wavefield in Figure 9, with a focus in the interior of the medium, is based on reflected data only. We did not use a source or receiver in the medium, and did not know the medium. All necessary information is encoded in the reflected waves.

Conclusions

In the Visual Tour section of this paper, we described the connection between different scattering principles, showing that there are three distinct ways to reconstruct the same wavefield. A physical source, seismic interferometry, and inverse scattering theory allow one to create the same wavestate (see Figure 3) originated by an impulsive source placed at a certain location x_s ($x = 1.44$ km in our examples). Seismic interferometry tells us how to build an estimate of the wavefield without knowing the medium properties, if we have a receiver at the same location x_s of the real source and sources surrounding the medium. Inverse scattering goes beyond and allows us to focus the wavefield inside the medium (at location x_s) without knowing its properties, using only reflected waves recorded at one side of the medium. We show that the interaction between causal and acausal wavefields is a key element to focus the wavefield where there is no real source.

We speculate that many of the insights gained in our one-dimensional framework are still valid in three dimensions. An extension of this work in two or three dimensions would give us the theoretical tools for many useful practical applications. For example, if we knew how to create the three-dimensional version of the incident wavefield shown in Figure 7, we could focus the wavefield to a point in the subsurface to simulate a source at depth and to record data at the surface (Figure 2); these kind of data are of extreme importance for full waveform inversion techniques (Brenders and Pratt, 2007) and subsalt imaging (Sava and Biondi, 2004). Furthermore, we could possibly concentrate the energy of the wavefield inside a hydrocarbon reservoir to fracture the rocks and improve the production of oil and gas (Beresnev and Johnson, 1994).

Acknowledgments

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References

Aki, K., and P. G. Richards, 2002, Quantitative seismology, 2nd ed.: University Science Books, Sausalito.

Bakulin, A., and R. Calvert, 2006, The virtual source method: Theory and case study: Geophysics, **71**, SI139–SI150.

Beresnev, I. A., and P. A. Johnson, 1994, Elastic-wave stimulation of oil production; a review of methods and results: Geophysics, **59**, 1000–1017.

Born, M., and E. Wolf, 1999, Principles of optics: Cambridge University Press, Cambridge.

Brenders, A. J., and R. G. Pratt, 2007, Full waveform tomography for lithospheric imaging: results from a blind test in a realistic crustal model: Geophysical Journal International, **168**, 133–151.

Burridge, R., 1980, The Gelfand-Levitan, the Marchenko, and The Gopinath-Sondhi integral equations of Inverse Scattering theory, regarded in the context of inverse impulse-response problems: Wave Motion, **2**, 305–323.

Chadan, K., and P. C. Sabatier, 1989, Inverse Problems in Quantum Scattering Theory, 2nd edition ed.: Springer, Berlin.

Colton, D., and R. Kress, 1998, Inverse Acoustic and Electromagnetic Scattering Theory: Springer, Berlin.

Curtis, A., P. Gerstoft, H. Sato, R. Snieder, and K. Wapenaar, 2006, Seismic interferometry—turning noise into signal: The Leading Edge, **25**, 1082–1092.

Dorren, H. J. S., E. J. Muyzert, and R. Snieder, 1994, The stability of one-dimensional inverse scattering: Inverse Problems, **10**, 865–880.

Epstein, C. L., 2003, Mathematics of Medical Imaging: Prentice Hall, Upper Saddle River.

Gladwell, G. M. L., 1993, Inverse Problems in Scattering: Kluwer Academic Publishing, Alphen aan den Rijn.

Lamb, G. L., 1980, Elements of soliton theory: Wiley-Interscience, New York.

Rodberg, L. S., and R. M. Thaler, 1967, Introduction to the Quantum Theory of Scattering: Academic Press, New York.

Rose, J. H., 2001, “Single-sided” focusing of the time-dependent Schrödinger equation: Phys. Rev. A, **65**, 012707.

———, 2002a, Single-sided autofocusing of sound in layered materials: Inverse Problems, **18**, 1923–1934.

———, 2002b, Time Reversal, Focusing and Exact Inverse Scattering, *in* Imaging of Complex Media with Acoustic and Seismic Waves: Springer, Berlin, 97–106.

Sava, P., and B. Biondi, 2004, Wave-equation migration velocity analysis - II: Subsalt imaging example: Geophysical Prospecting, **52**, 607–623.

Schuster, G. T., 2009, Seismic Interferometry: Cambridge University Press, Cambridge.

Shull, P. J., 2002, Nondestructive evaluation: theory, techniques, and applications: CRC Press, Boca Raton.

Wapenaar, K., F. Broggini, and R. Snieder, 2011, A proposal for model-independent 3D wave field reconstruction from reflection data: 81st Annual International Meeting, Expanded Abstracts, Society of Exploration Geophysicists, this issue.

Wapenaar, K., J. Fokkema, and R. Snieder, 2005, Retrieving the green’s function in an open system by cross correlation: A comparison of approaches (l): Journal of the Acoustical Society of America, **118**, 2783–2786.