Seismic interferometry for passive and exploration data: reconstruction of internal multiples

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Summary

Exact Green's function representations for seismic interferometry are based on the assumption that the receivers used in the correlation process are surrounded by a closed surface of sources. We investigate two situations for which this condition is not fulfilled: sources only in the subsurface, as in passive seismics, and sources only at the surface, as in exploration seismics. We show that in both cases the full Green's function, including primary and multiply scattered waves, can be reconstructed.

Introduction

It has been shown by many researchers in geophysics, ultrasonics and underwater acoustics that the crosscorrelation of acoustic wavefields recorded by two different receivers yields the response at one of the receiver positions as if there was a source at the other [Claerbout (1968), Duvall et al. (1993), Rickett and Claerbout (1999), Weaver and Lobkis (2001), Campillo and Paul (2003), Roux et al. (2004), Shapiro et al. (2005)]. Various theories have been developed to explain this phenomenon, ranging from diffusion theory for enclosures [Lobkis and Weaver (2001), Weaver and Lobkis (2002)], multiple scattering theory and stationary-phase theory for random media [van Tiggelen (2003), Malcolm et al. (2004), Snieder (2004)] and reciprocity theory for deterministic and random media (non-moving or moving) [Wapenaar et al. (2002, 2004, 2006) Weaver and Lobkis (2004), van Manen et al. (2005)].

The derivations based on reciprocity theory yield exact representations of Green's functions in arbitrary inhomogeneous lossless media. Hence, the reconstructed Green's functions do not only contain the direct wavefield between the two receiver points but also all primary and multiply scattered waves. This requires, however, that the two receivers are surrounded by sources on an arbitrarily shaped closed surface. In reality this condition is seldom fulfilled. In this paper we discuss two distinct situations for which the surface containing the sources is not closed and we discuss the conditions that are needed in order to reconstruct the exact Green's function, including the internal multiples. The first situation we consider corresponds to passive seismic data, for which we usually assume a distribution of natural noise sources along an open surface in the Earth's subsurface. The free surface acts as a mirror, which obviates the need of having sources on a closed surface. This situation has been extensively discussed in the literature, but we include it for completeness. The second situation is that of seismic exploration, with sources at the Earth's surface only. Seismic interferometry for exploration data has been extensively discussed by Schuster et al. (2001, 2004) and Bakulin and Calvert (2004). Impressive results have been obtained for primaries and first order free surface multiples. However, since the surface with sources is not closed, internal multiples are not correctly handled by these methods. Bakulin and Calvert (2006) propose to replace the correlation by a deconvolution process, which acts as a dereverberation filter. This approach partly solves the internal multiple problem but it does not account, for example, for peg-leg multiples. In this paper we reconsider seismic interferometry for exploration data and show how, in theory, all internal multiples can be correctly handled.

Green's function representation

We consider an arbitrary inhomogeneous lossless medium in which we define an arbitrarily shaped closed surface $\partial\mathbb{D}$ with outward pointing normal vector $\mathbf{n} = (n_1, n_2, n_3)$. Inside this surface we define two points $\mathbf{x}_A$ and $\mathbf{x}_B$. In the frequency domain, the Green's function between these two points, $\hat{G}(\mathbf{x}_A, \mathbf{x}_B, \omega)$, can be represented as [Wapenaar et al. (2004, 2006), van Manen et al. (2005)]

$$2\Re\{\hat{G}(\mathbf{x}_A, \mathbf{x}_B, \omega)\} = \oint_{\partial\mathbb{D}} \frac{-1}{j\omega\rho(\mathbf{x})}\Big(\hat{G}^*(\mathbf{x}_A, \mathbf{x}, \omega)\partial_i\hat{G}(\mathbf{x}_B, \mathbf{x}, \omega) - \big(\partial_i\hat{G}^*(\mathbf{x}_A, \mathbf{x}, \omega)\big)\hat{G}(\mathbf{x}_B, \mathbf{x}, \omega)\Big)n_i \mathrm{d}^2\mathbf{x}, \tag{1}$$

where $\Re$ denotes the real part, ω the angular frequency, j the imaginary unit and ρ the mass density. The terms $\hat{G}$ and $\partial_i\hat{G}n_i$ under the integral in the right-hand side of equation 1 represent responses of monopole and dipole sources at $\mathbf{x}$ on $\partial\mathbb{D}$. The products $\hat{G}^*\,\partial_i\hat{G}n_i$ etc. correspond to crosscorrelations in the time domain. Hence, the right-hand side can be interpreted as the integral of the Fourier transform of crosscorrelations of observed wavefields at $\mathbf{x}_A$ and $\mathbf{x}_B$, respectively, due to impulsive sources at $\mathbf{x}$ on $\partial\mathbb{D}$; the integration takes place along the source coordinate $\mathbf{x}$. The left-hand side of equation 1 is the Fourier transform of $G(\mathbf{x}_A, \mathbf{x}_B, t) + G(\mathbf{x}_A, \mathbf{x}_B, -t)$, which is the superposition of the response at $\mathbf{x}_A$ due to an impulsive source at $\mathbf{x}_B$ and its time-reversed version. This reconstructed Green's function is exact and contains, apart from the direct wave between $\mathbf{x}_B$ and $\mathbf{x}_A$, all scattering contributions (primaries and multiples) from inhomogeneities inside as well as outside $\partial\mathbb{D}$. When the medium outside $\partial\mathbb{D}$ is homogeneous, equation 1 can be approximated by

$$2\Re\{\hat{G}(\mathbf{x}_A, \mathbf{x}_B, \omega)\} \approx \frac{2}{\rho c}\oint_{\partial\mathbb{D}} \hat{G}^*(\mathbf{x}_A, \mathbf{x}, \omega)\hat{G}(\mathbf{x}_B, \mathbf{x}, \omega)\mathrm{d}^2\mathbf{x}, \tag{2}$$

where c is the propagation velocity. Since the right-hand side contains one crosscorrelation product of monopole responses only, this representation is better suited for seismic interferometry than equation 1. For a detailed analysis of the approximations in equation 2, see Wapenaar and Fokkema (2006). Evaluation of either equation 1 or 2 requires that sources are available on a closed surface $\partial\mathbb{D}$ around the observation points $\mathbf{x}_A$ and $\mathbf{x}_B$. In the following sections we discuss two situations for which $\partial\mathbb{D}$ is not closed.

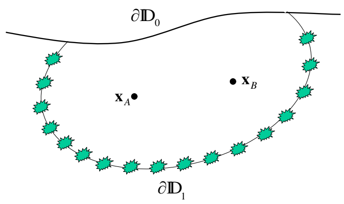

Fig. 1: *Configuration for passive data.*

Configuration for passive data

For the situation of passive data we assume that natural sources are available in the subsurface and that the responses of these sources are measured by receivers at or below the free surface. We divide the closed surface $\partial\mathbb{D}$ into a part $\partial\mathbb{D}_0$ coinciding with the free surface and a part $\partial\mathbb{D}_1$ containing the sources in the subsurface, see Figure 1. For this situation equation 1 needs to be evaluated over $\partial\mathbb{D}_1$ only. This is exact as long as $\partial\mathbb{D}_0$ and $\partial\mathbb{D}_1$ together form a closed surface. Hence, the direct wave as well as the primaries and multiples in $\hat{G}(\mathbf{x}_A, \mathbf{x}_B, \omega)$ are correctly reconstructed by the integral along the sources on $\partial\mathbb{D}_1$. A more intuitive explanation is that the free surface $\partial\mathbb{D}_0$ acts as a mirror which obviates the need of having sources on a closed surface. Note that when the sources at $\partial\mathbb{D}_1$ are uncorrelated noise sources, the right-hand side of equation 2 reduces to a direct crosscorrelation of the observed wavefields at $\mathbf{x}_A$ and $\mathbf{x}_B$, see Draganov et al. (2006) for a real data example.

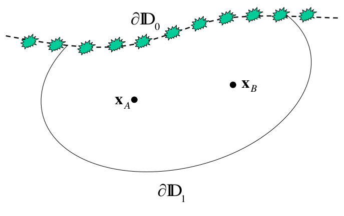

Fig. 2: *Configuration for exploration data.*

Configuration for exploration data

For the situation of exploration data, sources are only available at the acquisition surface. Again we divide the closed surface $\partial\mathbb{D}$ into two parts, this time a part $\partial\mathbb{D}_0$ coinciding with the acquisition surface and an arbitrarily chosen source-free part $\partial\mathbb{D}_1$ in the subsurface, see Figure 2. In the following we assume that surface related multiples have been eliminated, hence the acquisition

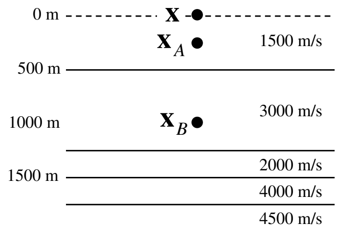

Fig. 3: *Horizontally layered medium.*

surface $\partial\mathbb{D}_0$ is assumed non-reflecting (if we would assume a free surface there would be no integral left to be evaluated). Assuming the responses of the sources at $\partial\mathbb{D}_0$ are measured by receivers at $\mathbf{x}_A$ and $\mathbf{x}_B$ in the subsurface (for example in a VSP, a vertical array, a horizontal well, or at the ocean bottom), crosscorrelation and integration along the sources on $\partial\mathbb{D}_0$ yields an approximation of the Green's function $\hat{G}(\mathbf{x}_A, \mathbf{x}_B, \omega)$. The fact that the integral over $\partial\mathbb{D}_1$ cannot be evaluated due to the absence of sources in the subsurface means that not only the amplitudes of the direct wave and primary reflections may be erroneously reconstructed, but also that the internal multiples are incorrectly handled and that spurious multiples may occur. The occurrence of spurious multiples is extensively discussed by Snieder et al. (2006). Here we illustrate it with a simple plane-wave experiment. Consider the horizontally layered medium of Figure 3. A downgoing plane wave is incident to this configuration at $z = 0$ (not shown); the ray parameter is $p = 0.2$ s/km, which corresponds to an incidence angle of 17.5 degrees in the first layer. The responses at $\mathbf{x}_A$ and $\mathbf{x}_B$ are shown in Figure 4. For this plane-wave experiment we omit the integral over $\partial\mathbb{D}_0$ in equation 2 and replace it by a direct crosscorrelation. The result is shown in Figure 5, where it is compared with the exact plane-wave response between $\mathbf{x}_A$ and $\mathbf{x}_B$. Note the asymmetry and the occurrence of spurious multiples in the correlation result. For this simple example a better result could easily be obtained by selecting the first arrival in the response at $\mathbf{x}_A$ by means of a time window and correlating this with the full response at $\mathbf{x}_B$ (comparable to what Bakulin and Calvert (2004) do in their virtual source method). However, in more

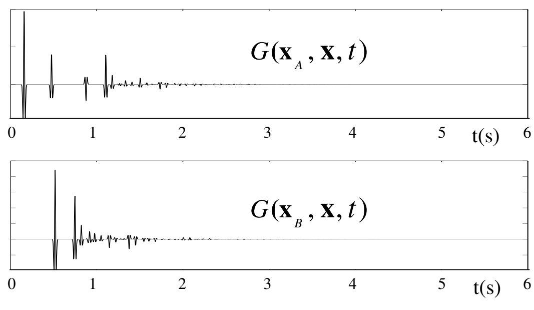

Fig. 4: *Responses at $\mathbf{x}_A$ and $\mathbf{x}_B$.*

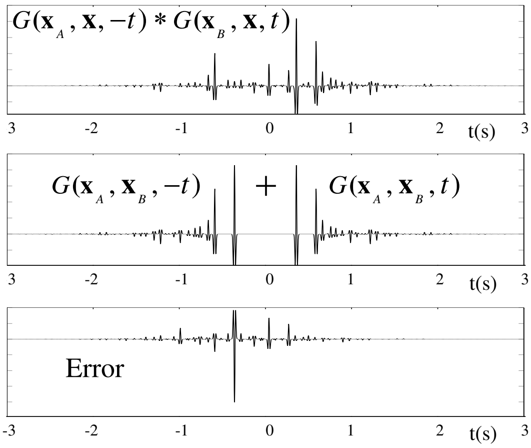

Fig. 5: *Crosscorrelation result, exact response and error. The vertical scale is the same in all panels.*

complicated situations this is not trivial. Moreover, it introduces other approximations that will not be further discussed here, since the aim of this paper is to derive an exact interferometric relation for exploration data.

Analysis of the neglected boundary integral

Consider again the configuration of Figure 2. The fact that the closed boundary integral of equation 1 or 2 needs to be replaced by an open boundary integral over $\partial\mathbb{D}_0$ implies a number of approximations, as we have seen above. Let us have a closer look at the neglected part of the integral over $\partial\mathbb{D}_1$ in the exact representation of equation 1. Let $\partial\mathbb{D}_1$ be a half-sphere with radius $r_\mathbb{D}$. If we take $r_\mathbb{D} \to \infty$ and assume that the medium is homogeneous outside some finite domain $\mathbb{D}_\mathrm{f}$, then the Green's functions under the integral are $O(1/r_\mathbb{D})$ and each of the correlation products is $O(1/r_\mathbb{D}^2)$. In the corresponding convolution-type theorem the two terms of $O(1/r_\mathbb{D}^2)$ cancel each other, making the integrand $O(1/r_\mathbb{D}^3)$. However, in the correlation-type theorem of equation 1 this cancellation does not take place, which means that the integrand is $O(1/r_\mathbb{D}^2)$. Since the surface area of the integration boundary $\partial\mathbb{D}_1$ increases with $r_\mathbb{D}^2$, the integral over $\partial\mathbb{D}_1$ in equation 1 (and also in equation 2) is $O(1)$. In other words, the boundary integral over $\partial\mathbb{D}_1$ does not vanish when $r_\mathbb{D} \to \infty$. For a plane-wave experiment, as in the example above, we arrive at a similar conclusion. For $z_\mathbb{D} \to \infty$ the Green's functions are $O(1)$ and so are the crosscorrelations. Again no cancellation takes place and, since the integral is omitted, the end result is also $O(1)$.

Modified extinction condition

The integral over $\partial\mathbb{D}_1$ does not vanish when $r_\mathbb{D} \to \infty$, assuming a homogeneous medium outside some finite domain, but what happens when the medium is inhomogeneous throughout the lower half-space? Due to internal multiple scattering, the integrand will be $O(f(r_\mathbb{D})/r_\mathbb{D}^2)$, where $f(r_\mathbb{D})$ is a decaying function accounting for trans-

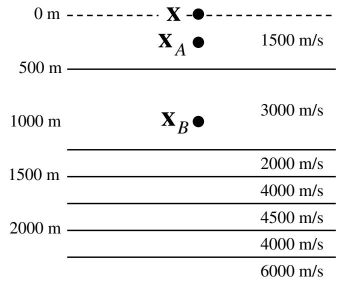

Fig. 6: *Horizontally layered medium, with two layers added.*

mission loss. The precise behavior of $f(r_\mathbb{D})$ depends on the type and distribution of the inhomogeneities, but what matters is that it will vanish for $r_\mathbb{D} \to \infty$. The integration surface area increases again with $r_\mathbb{D}^2$, hence, the integral is now $O(f(r_\mathbb{D}))$, which means that it vanishes for $r_\mathbb{D} \to \infty$. For a plane-wave experiment in a horizontally layered medium we arrive at a similar conclusion, following the same reasoning as in the previous section.

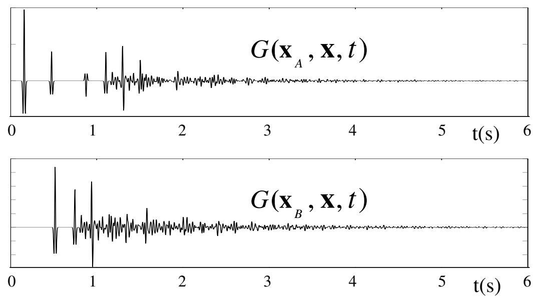

Fig. 7: *Responses at* $\mathbf{x}_A$ *and* $\mathbf{x}_B$*.*

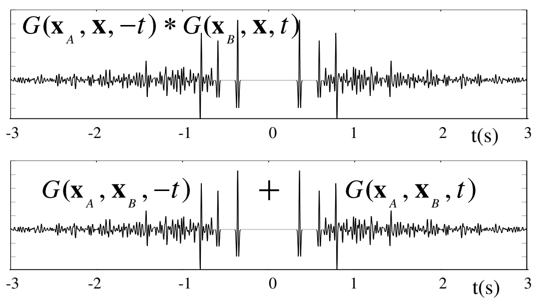

Fig. 8: *Crosscorrelation result and exact response. The vertical scale is the same in both panels.*

Hence, equation 1, with closed surface $\partial\mathbb{D}$ replaced by the acquisition surface $\partial\mathbb{D}_0$ (see Figure 2), is exact when the medium is inhomogeneous throughout the lower half-space (and equation 2 is a good approximation for the same situation).

This is a quite remarkable result. Usually one starts with deriving a modelling or processing scheme for a simplified situation; complications only arise when it is applied to a more realistic situation. Here we see the opposite happening. For the situation of a simple subsurface configuration embedded in a homogeneous medium, equation 1 or 2 (with $\partial\mathbb{D}$ replaced by $\partial\mathbb{D}_0$) involves erroneous amplitudes and spurious multiples. For the more realistic situation of an inhomogeneous subsurface, equation 1 (with $\partial\mathbb{D}$ replaced by $\partial\mathbb{D}_0$) becomes exact.

We illustrate this again for a plane-wave experiment in a horizontally layered medium. Figure 6 shows the same configuration as in Figure 3, but with two layers added. For the same incident plane wave with ray parameter $p = 0.2$ s/km, critical reflection occurs at the lower interface, hence $f(z_{\mathbb{D}})$ vanishes directly below this interface, which is why adding more layers does not make any difference. The responses at $\mathbf{x}_A$ and $\mathbf{x}_B$ are shown in Figure 7. The main difference with the responses in Figure 4 is the longer coda. The crosscorrelation result is shown in Figure 8, where it is compared with the exact plane-wave response between $\mathbf{x}_A$ and $\mathbf{x}_B$. Of course the response is more complicated than in Figure 5, but the match is perfect (the error for this idealized situation is computational noise only, in the order of 10^{-13}). Note that the crosscorrelation of the long codas have contributed to the suppression of spurious multiples at early times.

Concluding remarks

From the theory discussed in this paper as well as from the numerical example it follows that seismic interferometry applied to exploration data (Figure 2) benefits from the fact that the Earth's subsurface is inhomogeneous. Errors that would occur in the reconstructed Green's function when the response of only a few layers would be available are suppressed by crosscorrelating the full response of the inhomogeneous subsurface. This leads to the recommendation that much longer traces should be recorded than the usual four seconds in seismic exploration. The reconstruction of the Green's function is the result of a complex interference of crosscorrelated primaries and multiply scattered events, present in the coda of the response. It has been observed before that coda waves are surprisingly stable [Fink (1997), Snieder and Scales (1998)], hence, this should not be a limiting factor for practical applications. Note that, despite the complexity of the coda, this reconstruction process is fully deterministic and thus does not rely on diffusivity and equipartitioning assumptions, as in some of the references mentioned in the introduction.

Throughout the paper we have assumed that the medium is lossless. Investigations by Slob et al. (2006) for electromagnetic passive data indicate that when the losses are small, interferometry yields Green's functions with correct traveltimes and approximate amplitudes. It remains to be investigated how anelastic losses will degrade the Green's function reconstruction for exploration data, as discussed in this paper.

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