Reconstruction of the elastodynamic reflection response in an inhomogeneous medium using crosscorrelation

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Summary

In recent years, several authors have shown with modeling results how one can reconstruct the acoustic reflection response using crosscorrelation. This process was named Seismic Interferometry. Here, we show the relationship that can be used to reconstruct the reflection response Green's function in the elastodynamic case. This relation was derived using the Rayleigh-Betti reciprocity theorem. Modeling results using this relation show its validity.

Introduction

Seismic interferometry is the process of obtaining a reflection response at point A as if from a source at point B resulting from the crosscorrelation process of observations at A and B. These observations can be real reflection measurements at the points A and B at the surface from sources elsewhere along the surface (Schuster, 2001). The reflection response can also be reconstructed from passive noise measurements at the surface at the points A and B resulting from subsurface sources. This was first proved by Claerbout (1968) for a 1D acoustic medium. Later, Wapenaar proved it for a 3D inhomogeneous acoustic medium using one-way wave equation reciprocity theorem of the correlation type (Wapenaar et al., 2002). In Wapenaar et al. (2004), this relationship was generalized also for elastic medium using Rayleigh-Betti reciprocity theorem. In this paper, we show the relationship for reconstructing the elastodynamic reflection response from passive noise measurements and sustain it with modeling results.

Relation between the reflection and the transmission Green's functions.

Let us have an elastic inhomogeneous anisotropic lossless medium bounded by a free surface. In this medium we choose a volume $\mathcal{D}$ such that part of its bounding surface $\partial\mathcal{D}$ is a piece of the free surface ($\partial\mathcal{D}_0$) and the other part ($\partial\mathcal{D}_1$) is an arbitrary shaped surface inside the medium. For this configuration, in the frequency domain we can write the following relation between the reflection and the transmission Green's functions (Wapenaar et al., 2004):

$$2\Re\left\{\hat{G}^{v,t}_{p,q}\left(\boldsymbol{x}_A,\boldsymbol{x}_B,\omega\right)\right\} = -\int_{\partial\mathcal{D}_1}\Big[\left\{\hat{G}^{v,f}_{p,i}\left(\boldsymbol{x}_A,\boldsymbol{x},\omega\right)\right\}^*\hat{G}^{v,h}_{q,i}\left(\boldsymbol{x}_B,\boldsymbol{x},\omega\right) + \left\{\hat{G}^{v,h}_{p,i}\left(\boldsymbol{x}_A,\boldsymbol{x},\omega\right)\right\}^*\hat{G}^{v,f}_{q,i}\left(\boldsymbol{x}_B,\boldsymbol{x},\omega\right)\Big]\,d^2\boldsymbol{x}. \tag{1}$$

On the left-hand side of equation 1 is the real part of the measured particle velocity (^{v}) in the p-direction at point A due to a traction source (^{t}) in the q-direction at point B. Both A and B are at the free surface. On the right-hand side, the symbol * stands for complex-conjugation. This means that on the right-hand side we have the crosscorrelation of the observed particle velocities at the points A and B in the directions p and q due to subsurface sources of force type (^{f}) and deformation rate (^{h}) along the boundary $\partial\mathcal{D}_1$. Equation 1 is exact and it allows us to reconstruct the elastodynamic Green's function at the surface from passive measurements due to subsurface sources.

We can look at the Green's functions from the subsurface source as a sum of a part propagating initially from the sources into the volume $\mathcal{D}$ and a part propagating initially outward from the volume, i.e.,

$$\hat{G}^{v,f}_{p,i}\left(\boldsymbol{x}_A,\boldsymbol{x},\omega\right) = \hat{G}^{(in)v,f}_{p,i}\left(\boldsymbol{x}_A,\boldsymbol{x},\omega\right) + \hat{G}^{(out)v,f}_{p,i}\left(\boldsymbol{x}_A,\boldsymbol{x},\omega\right). \tag{2}$$

The outward propagating part $\hat{G}^{(out)v,f}_{p,i}$ is registered at the surface after it is back-scattered from inhomogeneities outside the volume $\mathcal{D}$. If the medium outside the volume $\mathcal{D}$ is assumed to be homogeneous, then $\hat{G}^{(out)v,f}_{p,i}$ will not be back-scattered and thus will be zero at the registration points A and B. In this case, we can approximate equation 3 with

$$2\Re\left\{\hat{G}^{v,t}_{p,q}\left(\boldsymbol{x}_A,\boldsymbol{x}_B,\omega\right)\right\} \approx -\int_{\partial\mathcal{D}_1}\left\{\hat{G}^{v,\phi}_{p,k}\left(\boldsymbol{x}_A,\boldsymbol{x},\omega\right)\right\}^*\hat{G}^{v,\phi}_{q,k}\left(\boldsymbol{x}_B,\boldsymbol{x},\omega\right)d^2\boldsymbol{x}, \tag{3}$$

which is more suitable for use for exploration purposes. In the above relation, the superscript ϕ stands for quasi P-wave sources (when $k = 1$) and quasi S-wave sources (when $k = 2,\ 3$). These quasi P- and S-sources are obtained by applying flux-normalized decomposition at the source level. The accuracy of relation 3 depends on the curvature of the boundary $\partial\mathcal{D}_1$. When $\partial\mathcal{D}_1$ is planar the only approximation is that the evanescent fields are not taken into account.

When inhomogeneities are present outside the volume $\mathcal{D}$, then $\hat{G}^{(out)}$ will be back-scattered and thus non-zero at the observation points A and B. After crosscorrelation of these back-scattered arrivals we will receive ghost events in the reconstructed reflection response. We write relation 3 as

$$2\Re\left\{\hat{G}^{v,t}_{p,q}\left(\boldsymbol{x}_A,\boldsymbol{x}_B,\omega\right)\right\} + \text{Error} \approx \\ -\int_{\partial\mathcal{D}_1}\left\{\hat{G}^{v,\phi}_{p,k}\left(\boldsymbol{x}_A,\boldsymbol{x},\omega\right)\right\}^*\hat{G}^{v,\phi}_{q,k}\left(\boldsymbol{x}_B,\boldsymbol{x},\omega\right)d^2\boldsymbol{x}, \quad (4)$$

where the Error part on the left-hand side accounts for these ghost events. However, if the sources in the subsurface are randomly distributed in depth, then during the integration process over the source surface $\partial\lceil_1$ the ghost events do not interfere constructively and are strongly attenuated. We have showed this with modeling results for the case of acoustic medium (see (Draganov et al., 2004)).

Numerical modeling results

In Figure 1 we show the model used to verify equation 4. The subsurface consists of three layers separated by two lightly dipping boundaries. There are 307 receivers at the free surface starting at $x_1 = 2100$ m every 15 m. The receivers record the vertical and the horizontal components of the particle velocity. The sources are randomly distributed in depth between levels $x_3 = 650$ m and $x_3 = 800$ m. In the horizontal direction x_1 the sources are still regularly spaced starting from 2100 m till 5700 m with a horizontal distance between neighboring sources of 21 m. At each source position two shots were modeled - one with a P-source and the second with a S-source. Thus, for each subsurface source position we record four traces at each receiver position at the surface.

Following relation 4, we first reconstruct the vertical particle velocity that can be observed at the surface from a vertical traction source at $(x_1, x_3) = (3900, 0)$ m. This is done in the following way. For each subsurface shot position, we take the observed vertical particle velocity shot gather resulting from a P-source. Then we extract a "master" trace at position $x_1 = 3900$ m and correlate it with the whole shot gather. This process is repeated also for the measured vertical particle velocity shot gather from a S-source. Then, we sum both correlated results. This represents the integrand on the right-hand side of equation 4. This procedure is repeated for all subsurface source positions and at the end the results from all the steps are summed together to give the integration result in equation 4. The final result after muting the negative times is given in Figure 2. In Figure 3 we show the directly modeled vertical particle velocity observed at the surface from a vertical traction source at $x_1 = 3900$ m.

Looking at Figures 2 and 3 we see that it is difficult to compare them as the directly modeled reflection response

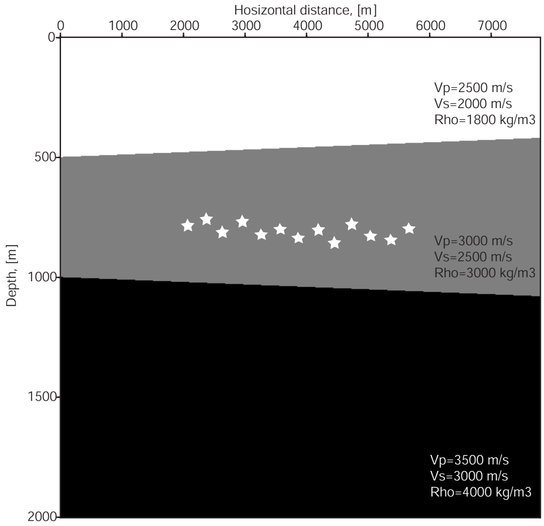

Fig. 1: *Subsurface model of three layers separated by two dipping boundaries. The receivers at the surface are distributed between* $x_1 = 2100$ *and* $x_1 = 5700$ *m every 15 m. The sources are randomly distributed in the vertical direction between depth levels* $x_3 = 650$ *and* $x_3 = 800$ *m. In the horizontal direction the sources are kept regularly spaced between* $x_1 = 2100$ *and* $x_1 = 5700$ *m every 21 m.*

in obscured by the presence of surface waves, while the reconstructed reflection response does not have surface waves (it is shown in Figure 2 as it was obtained from the correlations). This is explained with the fact that in the integration over the source surface $\partial\mathcal{D}_1$ the different sources contribute differently to the reconstructed response (Wapenaar et al., 2004). When we do not have sources close to the surface only the reflection response is reconstructed without the surface waves. To be able to compare the results accurately we muted the delta function from the reconstructed reflection response and filtered out the surface waves from the directly modeled reflection response. The results are shown in Figures 4 and 5. We see that the reconstruction according equation 4 is very accurate. The presence of a reflector below the sources have not caused ghost events to appear . The randomness of the subsurface sources in the vertical direction have "healed" the picture.

In Figures 6 and 7 we also show the reconstructed and the directly modeled horizontal particle velocity component from a traction source in the horizontal direction at the surface at $x_1 = 3900$ m. Again, the match is very good.

Conclusions

We showed that in a 3D inhomogeneous solid medium the elastodynamic reflection response can be reconstructed from transmission responses observed at the surface from separate P- and S- sources in the subsurface. Inhomogeneities below the sources can cause ghost events to appear in the reconstructed reflection response. When the subsurface sources, though, are randomly distributed in

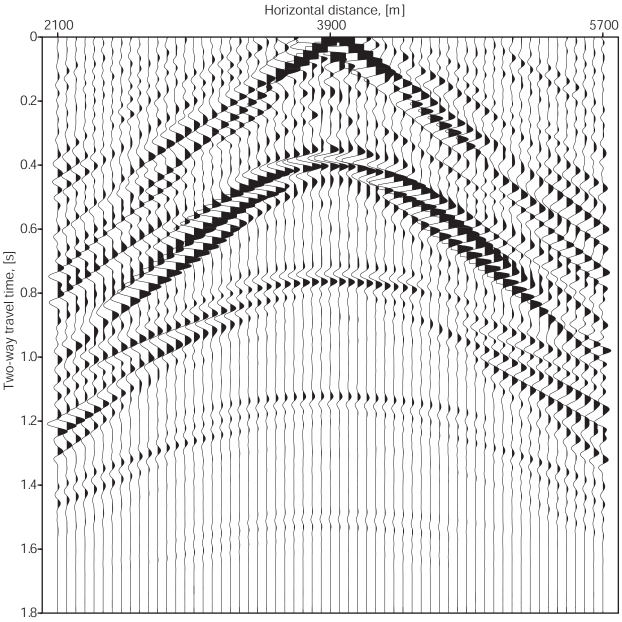

Fig. 2: *Reconstructed vertical particle velocity reflection response* $\hat{G}^{v,t}_{3,3}$ *as if from a vertical traction source at* $x_1 = 3900$ *m.*

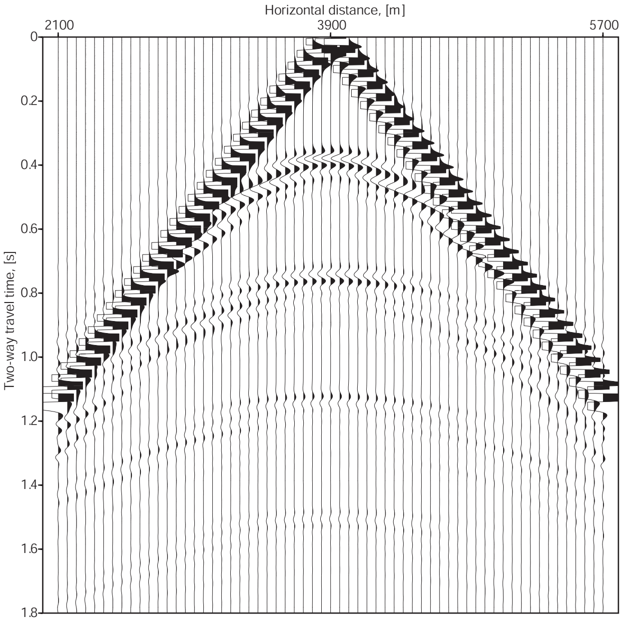

Fig. 3: *Directly modeled vertical particle velocity reflection response* $\hat{G}^{v,t}_{3,3}$ *from a vertical traction source at* $x_1 = 3900$ *m.*

depth, the ghost events in the reconstructed reflection response are strongly suppressed due to incoherent summation during the reconstruction procedure. When there are no subsurface sources close to the surface, no surface waves are reconstructed.

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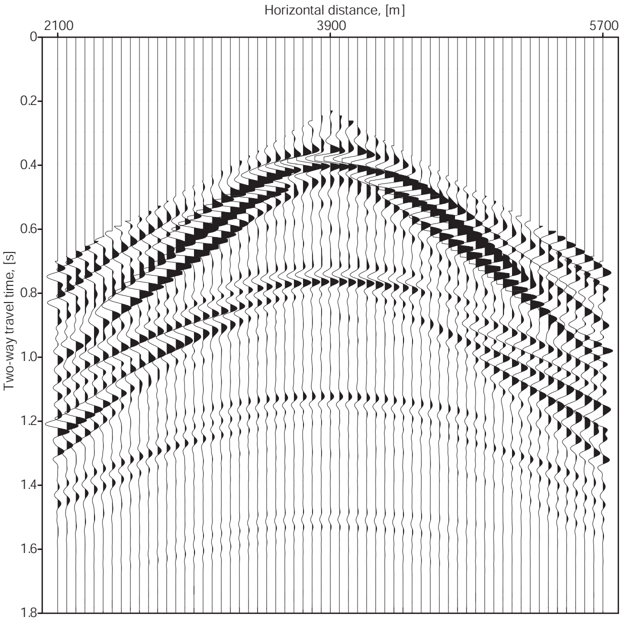

Fig. 4: *Reconstructed vertical particle velocity reflection response* $\hat{G}^{v,t}_{3,3}$ *as if from a vertical traction source at* $x_1 = 3900$ *m after muting the delta function.*

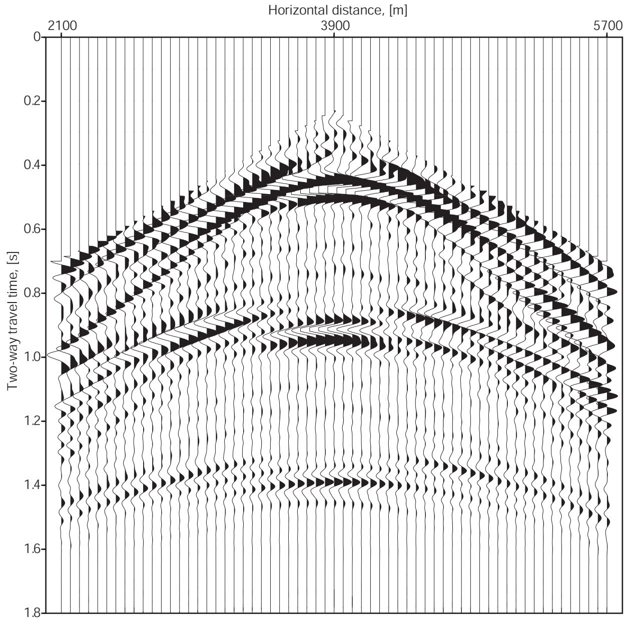

Fig. 6: *Reconstructed horizontal particle velocity reflection response* $\hat{G}^{v,t}_{1,1}$ *as if from a horizontal traction source at* $x_1 = 3900$ *m after muting the delta function.*

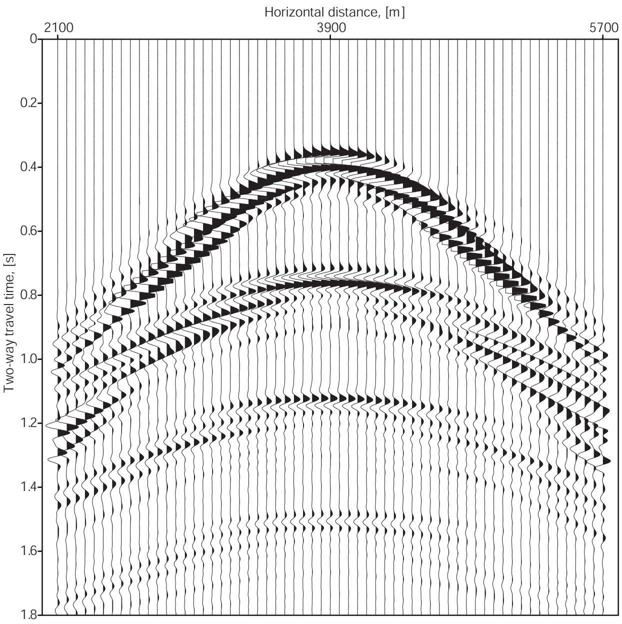

Fig. 5: *Directly modeled vertical particle velocity reflection response* $\hat{G}^{v,t}_{3,3}$ *from a vertical traction source at* $x_1 = 3900$ *m. The surface waves have been filtered out.*

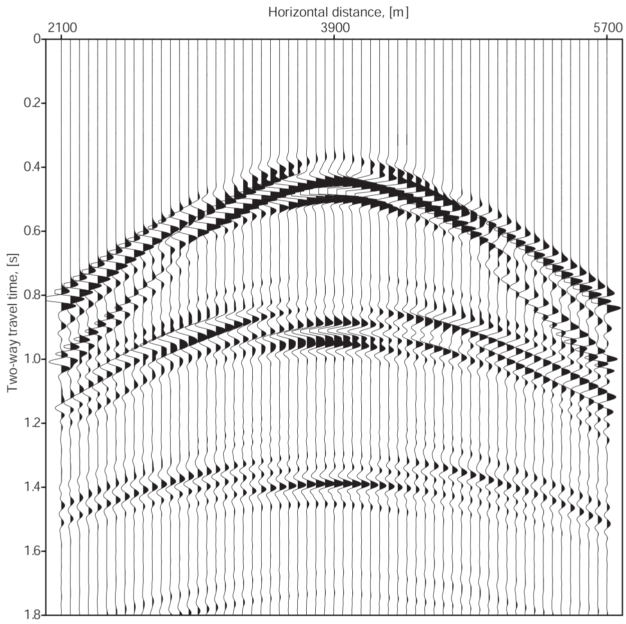

Fig. 7: *Directly modeled horizontal particle velocity reflection response* $\hat{G}^{v,t}_{1,1}$ *from a horizontal traction source at* $x_1 = 3900$ *m. The surface waves have been filtered out.*