Migration with correction for transmission losses in arbitrary inhomogeneous media

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Summary

We show how a correction for transmission loss can be derived from energy-conservation principles in horizontally layered media. Next we derive a similar correction for laterally varying media from a correlation type reciprocity theorem, a.k.a. power reciprocity theorem. We conclude this paper by demonstrating on some synthethic migration examples, the improved recovery of the reflection-coefficient as a function of angle.

Introduction

Imaging below complex and high contrast structures, e.g. a salt body, is a difficult task. Previously we discussed the effects of neglecting multi-pathing and/or transmission-losses in wavefield extrapolation [3]. Multi-pathing needs to be accounted for to produce kinematically correct results, see [5], but transmission-losses need to be considered to obtain results that are also *dynamically* correct, see [2, 6].
The subject of multi-pathing in ray-tracing has received a lot of attention over the years; as a result multi-valued arrivals can be handled quite accurately now, [8], but calculating amplitudes in caustic points is still a problem. Gaussian beams, [7], do not have this problem, but their use is not yet an established method.
Although likely to accompany multi-pathing, transmission loss has received far less attention in the seismic community. Schleicher et al [9] proposed a method based on local plane wave transmission- and reflection coefficients, and not allowing for caustics. Wapenaar and Herrmann [10] proposed a method to correct for transmission-losses in arbitrary media; they demonstrated it on wavefield extrapolation in acoustic, laterally homogeneous, fine-layered media. In [3] we generalized the implementation to laterally varying media and demonstrated it on downward extrapolation of a plane wave through a syncline interface. In this paper we apply the transmission loss correction to redatuming and show that the resulting estimate of the reflection coefficient as a function of angle, improves.

Transmission loss illustrated

In migration undoing propagation effects is usually performed by complex conjugating the forward propagator, i.e. reversing time and interchanging sources and receivers. This way one can properly undo the kinematics, but energy lost due to reflection cannot be recovered; one cannot fully undo dynamic propagation effects this way.

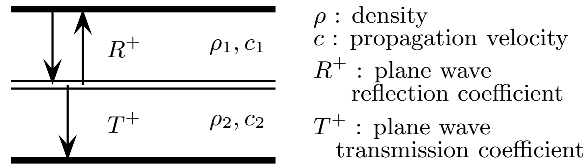


Fig. 1: Plane wave at normal incidence with flat interface

We illustrate this by considering a medium consisting of two homogeneous half-spaces separated by a flat interface and a plane wave at normal incidence with the interface, fig 1. The energy-balance for fig 1 is the familiar relation

$$1 - R^{+,*}R^+ = \frac{\rho_1 c_1}{\rho_2 c_2} T^{+,*}T^+. \tag{1}$$

The $*$-superscript denotes complex conjugation. By substituting the flux-normalized counterpart of T^+,

$$\mathcal{T}^+ = \sqrt{\frac{\rho_1 c_1}{\rho_2 c_2}} T^+, \tag{2}$$

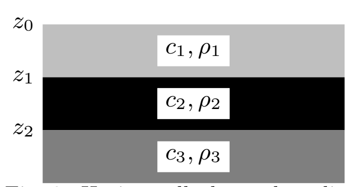
we simplify (1) to

$$1 - R^{+,*}R^+ = \mathcal{T}^{+,*}\mathcal{T}^+. \tag{3}$$

See Ursin [13], for more on flux-normalization in laterally homogeneous media. Flux-normalization also makes up- and downgoing transmission-coefficients equal; $\mathcal{T}^+ = \mathcal{T}^-$ as opposed to $T^+ = \frac{\rho_2 c_2}{\rho_1 c_1} T^-$. For plane waves at an angle this symmetry and equation (1) are also valid. Analoguous relations hold for flux-normalized wavefields in laterally inhomogeneous media, see the next section.
From (3) we can express $\{\mathcal{T}^+\}^{-1}$ as

$$\{\mathcal{T}^+\}^{-1} = \left(1 - R^{+,*}R^+\right)^{-1} \mathcal{T}^{+,*}. \tag{4}$$

Using the complex conjugate forward propagator in migration implies that $\{\mathcal{T}^+\}^{-1}$ is approximated by $\mathcal{T}^{+,*}$. But for strong reflectivity $\{\mathcal{T}^+\}^{-1} \approx \mathcal{T}^{+,*}$ gives a severe underestimation of the emitted energy.



	$c[\frac{m}{s}]$	$\rho[\frac{kg}{m^3}]$
1	1400	1400
2	2250	2250
3	1500	1500

Fig. 2: Horizontally layered medium for illustrating effects of neglecting transmission loss

To illustrate the consequences of neglecting transmission loss in migration, we add a third horizontal layer, and

take the medium parameters shown in fig 2.
In the (p_x, ω)-domain (p_x the horizontal slowness and ω angular frequency) the primary reflection response at z_0 from the interface at z_2 is

$$X(p_x, z_0) = W^-(p_x, z_0; z_2) R_2^+(p_x) W^+(p_x, z_2; z_0); \quad (5)$$

$W^\pm$ describes propagation between z_0 and z_2. We have omitted the ω-dependency for notational convenience and will keep doing so in the rest of this paper. We redatum $X(p_x, z_0)$ to z_2 by multiplying equation (5) from the left with $\{W^-\}^*(p_x, z_0; z_2)$ and from the right with $\{W^+\}^*(p_x, z_2; z_0)$. The phase will be handled correctly, but let us now analyze the amplitudes. Using

$$|W^-(p_x, z_0; z_2)| = |\mathcal{T}_1^-|$$
$$|W^+(p_x, z_2; z_0)| = |\mathcal{T}_1^+| = |\mathcal{T}_1^-| \triangleq |\mathcal{T}_1|,$$

it is clear that

$$|W^{-,*} X W^{+,*}| = |\mathcal{T}_1|^2 |R_2| |\mathcal{T}_1|^2. \quad (6)$$

We plotted equation (6) as a function of p_x, see fig 3. The errors are clear; the absolute value of the redatumed estimate of R_2 is not just smaller over the whole p_x-range, the p_x-dependence is not even recovered correct in a qualitative sence. Instead of rising from a minimum at $p_x = 0$ to a maximum at the critical angle, it has an approximately constant value for sub-critical values of p_x and goes to zero at the critical angle.

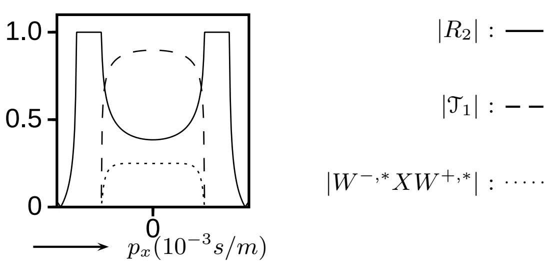


Fig. 3: Result of redatuming in the (p_x, ω)-domain without transmission loss correction

Transmission loss correction

Flux-normalizing forward operators in laterally homogeneous media works the same as equation (2), flux-normalizing the transmission coefficient of plane waves. For laterally variant media it is less trivial.
To derive a relation similar to equation (4) for one-way Green's functions in laterally varying media, Wapenaar introduced one-way reciprocity-theorems for flux-normalized wavefields [11, 12]. After discretizing the reciprocity theorem analogue of equation (3), deriving the transmission-loss correction is straightforward matrix-algebra.

For the seismic situation we choose the configuration illustrated in fig 4, two infinite parallel planes enclosing

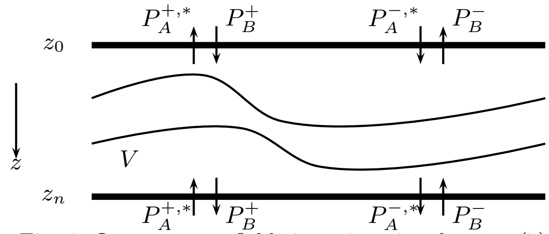


Fig. 4: One way wavefields in reciprocity theorem (7)

a volume V; z_0 is the surface where the data are measured, z_n the subsurface-level where we want to redatum the data to. In the region $z_0 < z < z_n$ the medium corresponds to the actual medium, but above and below we choose non-scattering media.
The up- and downgoing wavefields of states A and B are related by a reciprocity theorem of the correlation-type, or power reciprocity theorem,

$$\int_{z=z_0} \left[P_A^{+,*} P_B^+ - P_A^{-,*} P_B^- \right] d^2\mathbf{x}$$
$$\approx \int_{z=z_n} \left[P_A^{+,*} P_B^+ - P_A^{-,*} P_B^- \right] d^2\mathbf{x}, \quad (7)$$

in the (x, ω)-domain. The two sides are only approximately equal because evanescent wavefields are neglected. In the remainder of this paper we will assume the half-space below z_n to be source-free, i.e. $P_A^- = P_B^- = 0$ at z_n. If both states are also identical, $P_B^\pm = P_A^\pm = P^\pm$, equation (7) is the continuous, laterally varying analogue of the flux-normalized energy-balance equation, (3).
We discretize the wavefields $P_{A,B}^\pm$ at z_0 and z_n by N receivers at equi-distant positions and collect the corresponding values in column vectors $\mathbf{P}_{A,B}^\pm(z_0)$ and $\mathbf{P}_{A,B}^+(z_n)$. The discrete version of equation (7) is

$$\{\mathbf{P}_A^+(z_0)\}^H \mathbf{P}_B^+(z_0) - \{\mathbf{P}_A^-(z_0)\}^H \mathbf{P}_B^-(z_0)$$
$$= \{\mathbf{P}_A^+(z_n)\}^H \mathbf{P}_B^+(z_n), \quad (8)$$

where the superscript in $\{\mathbf{P}\}^H$ means transposition and complex conjugation.
Given a downgoing unit point source at z_0, position j, we denote the downgoing transmission response at z_n by the column-vector $\mathbf{W}_{g,j}^+(z_n, z_0)$. The subscript g indicates that $\mathbf{W}_{g,j}^+(z_n, z_0)$ is a generalized primary; besides the primary it also contains the internal multiples due to the inhomogeneous medium between z_0 and z_n.
The upgoing reflection response at z_0 of the same downgoing point source is given by the column-vector $\mathbf{X}_j(z_0, z_0|z_n)$. This is the part of the deconvolved reflection-data, stripped of free-surface multiples from z_0 and reflections from below $z \geq z_n$ (remember we choose the medium above z_0 and below z_n non-scattering).
We collect the columns $\mathbf{W}_{g,j}^+(z_n, z_0)$ and $\mathbf{X}_j(z_0, z_0|z_n)$ in the square matrices

$$\mathbf{W}_g^+ = \mathbf{W}_g^+(z_n, z_0), \quad \text{and} \quad \mathbf{X} = \mathbf{X}(z_0, z_0|z_n),$$

respectively. The matrix $\mathbf{W}_g^+$ represents the forward extrapolation operator from z_0 to z_n.
Using these matrices we express $\mathbf{P}_{A,B}^-(z_0)$ and $\mathbf{P}_{A,B}^+(z_n)$ in terms of $\mathbf{P}_{A,B}^+(z_0)$ by

$$\mathbf{P}_{A,B}^+(z_n) = \mathbf{W}_g^+(z_n, z_0)\mathbf{P}_{A,B}^+(z_0), \quad (9)$$

$$\mathbf{P}_{A,B}^-(z_0) = \mathbf{X}(z_0, z_0|z_n)\mathbf{P}_{A,B}^+(z_0). \quad (10)$$

Substituting equations (9) and (10) into (8), gives

$$\mathbf{I} - \mathbf{X}^H\mathbf{X} = \{\mathbf{W}_g^+\}^H\mathbf{W}_g^+. \quad (11)$$

Note the similarity between (11) and (3), with $\mathbf{W}_g^+$ corresponding to $\mathcal{T}^+$ and $\mathbf{X}$ to R^+. The $\mathcal{T}^+ = \mathcal{T}^-$ symmetry for laterally homogeneous media translates into

$$\mathbf{W}_g^-(z_0, z_n) = \{\mathbf{W}_g^+(z_n, z_0)\}^T \quad (12)$$

for laterally variant media. Similarly their inverses obey

$$\mathbf{F}_g^-(z_n, z_0) = \{\mathbf{F}_g^+(z_0, z_n)\}^T.$$

Analoguous to (4) the inverse extrapolation operator for downgoing wavefields can be expressed as

$$\mathbf{F}_g^+ = [\mathbf{I} - \mathbf{X}^H\mathbf{X}]^{-1}\{\mathbf{W}_g^+\}^H. \quad (13)$$

For practical computations we expand the correction-factor and terminate the series after K terms.

$$\mathbf{F}_g^{+,(K)} = \sum_{k=0}^{K}[\mathbf{X}^H\mathbf{X}]^k\{\mathbf{W}_g^+\}^H. \quad (14)$$

For weak reflections we can use the zero order approximation of $\mathbf{F}_g^+$, i.e. take the conventional approach and simply use the conjugate of $\mathbf{W}_g^+$. For stronger reflections we can evaluate (14), which involves K matrix-multiplications. There is a considerable amount of redundancy in (14) :

1. If $K = L^2$ and $L \in \mathbb{N}$, raising any square matrix to the power K takes only $2L$ matrix-multiplies; $\mathbf{A}^K = (\mathbf{A}^L)^L$. Exploiting this fact is called Paterson's rule by Golub and Van Loan, [4].

2. The cross correlation $\mathbf{X}^H\mathbf{X}$ is hermitian; and so is any summation of integer powers of $\mathbf{X}^H\mathbf{X}$.

We can exploit these two features to evaluate equation (14) with effectively $\sqrt{K}$ matrix multiplies in stead of K, without any approximation.
We extrapolate the full reflection data $\mathbf{X}(z_0, z_0|\infty)$, containing the reflections from $0 < z < \infty$, downward from z_0 to z_m by

$$\mathbf{X}(z_m, z_m|\infty) = \mathbf{F}_g^-(z_m, z_0)\mathbf{X}(z_0, z_0|\infty)\mathbf{F}_g^+(z_0, z_m). \quad (15)$$

When obtained with the K-th order approximation $\mathbf{F}_g^{+,(K)}$, we label redatuming result $\mathbf{X}^{(K)}(z_m, z_m|\infty)$ similarly. Finally we estimate the reflection-coefficient at (x_j, z_m). Transforming each column of the matrix $\mathbf{X}(z_m, z_m|\infty)$ to the (p_x, ω)-domain and summing all frequency-components for constant p_x for each column j, yields (de Bruin et al. [1]),

$$\langle\tilde{\mathbf{R}}_j(z_m)\rangle = \frac{1}{N_\omega}\sum_\omega \tilde{\mathbf{X}}_j(z_m, z_m|\infty). \quad (16)$$

Again, we add a superscript (K) when we used $\mathbf{F}_g^{+,(K)}$ to undo propagation.

Synthethic data examples

We demonstrate the effect of equations (14), (15), and (16) on a synthethic data-set, obtained from the syncline model in fig 5. For the medium-parameters we used the ones from fig 2. Instead of working in the (p_x, ω)-domain, we model the wavefields with finite-difference and choose the configuration shown in fig. 6.

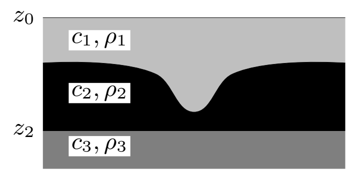


Fig. 5: syncline medium

We put the layer boundaries at $z_0 = 0m$, and $z_2 = 600m$. We use $N = 195$ sources and receivers, sampled at $\Delta x = 16m$. We sample the finite-difference grid at $\delta x = \delta z = 4m$ and $\delta t = 0.8ms$. To model forward propagation between z_0 and z_2, and the corresponding reflections, the medium needs to be non-scattering for $z \geq z_2$. We achieve this by setting $c_3 = c_2$ and $\rho_3 = \rho_2$ for modelling $\mathbf{W}_g^+(z_0, z_2)$ and $\mathbf{X}(z_0, z_0|z_2)$; in practice we will derive $\mathbf{X}(z_0, z_0|z_2)$ from the full reflection data $\mathbf{X}(z_0, z_0|\infty)$ by muting the reflections from $z \geq z_2$, instead of modeling it separately.

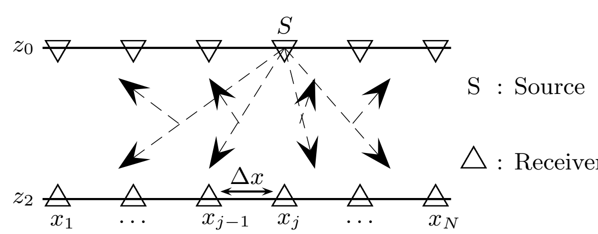


Fig. 6: Source/receiver configuration

Fig 7(a) shows the time-domain representation of the middle column of $\mathbf{X}(z_0, z_0|\infty)$, i.e. the reflection-response of the syncline medium for a source at (z_0, x_{98}), right above the syncline. Fig 7(b) shows $\mathbf{X}_{98}(z_0, z_0|z_2)$, the reflection response for the same source position, but then from the medium without the bottom-layer.

Fig 7(c) shows $\{\mathbf{W}^{+}_{g,98}(z_0, z_2)\}^H$, i.e. the zero order approximation for downward extrapolation to (z_2, x_{98}), right below the syncline.

The result of applying this downward extrapolation to (z_2, x_{98}) is shown in fig 7(d). Clearly the kinematics are correct, but if we analyze the amplitudes by looking at the estimated reflection coefficient $\langle\tilde{\mathbf{R}}_{98}(z_2)\rangle^{(0)}$, see fig 8, we see the same kind of error as demonstrated in fig 3. By using instead $\mathbf{F}^{+,(1)}$ or even $\mathbf{F}^{+,(4)}$ for downward extrapolation, the estimated reflection coefficient closes in on the theoretical result.

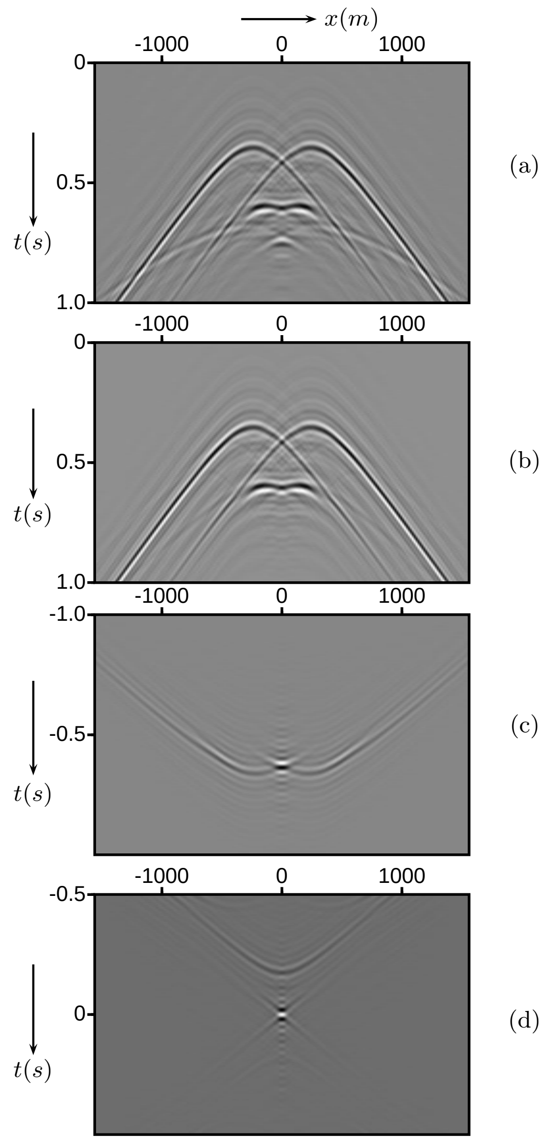


Fig. 7: Modeling and redatuming results from syncline medium, fig 5 : (a) (x,t)-representation of $\mathbf{X}_{98}(z_0, z_0|\infty)$; (b) (x,t)-representation of $\mathbf{X}_{98}(z_0, z_0|z_2)$; (c) (x,t)-representation of $\mathbf{F}^{+,(0)}_{98}(z_2, z_0)$; (d) (x,t)-representation of $\mathbf{X}^{(0)}_{98}(z_2, z_2|\infty)$

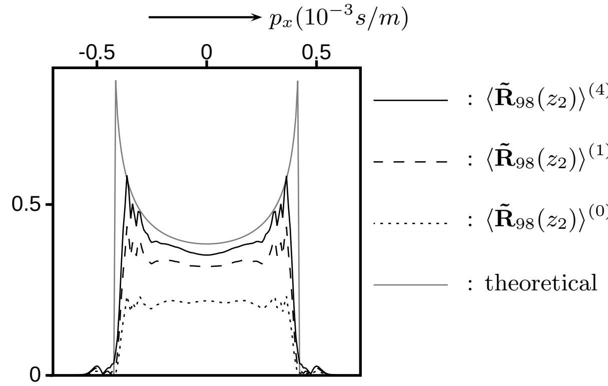


Fig. 8: Reflection coefficients

Conclusions

We have proposed a method for correcting transmission loss due to wave propagation through arbitrary, acoustic media. We have demonstrated its effects on migration in two high contrast media. Without the transmission loss correction it is not even possible to estimate just the qualitative p_x-dependence of the reflection coefficient. With the transmission loss correction the correspondence between the estimated reflection coefficient and the theoretical result clearly improves, both qualitatively and quantitatively.

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