Relations between reflection and transmission responses of 3-D inhomogeneous media

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Summary

Relations between reflection and transmission responses of horizontally layered media have been formulated by many authors. We derive similar relations for 3-D inhomogeneous media and we indicate some applications.

Introduction

In this paper we present a unified approach for deriving relationships between seismic reflection and transmission responses in 3-D inhomogeneous media. In all relations, the codas due to internal multiple scattering are included. We consider situations with and without a free surface on top of the configuration; below a specific depth level we assume that the medium is homogeneous. Hence, the responses of interest are (1) reflection responses at the upper boundary, with and without free surface multiples at the upper boundary, (2) transmission responses between the upper and lower boundary, with and without free surface multiples at the upper boundary, (3) reflection responses at the lower boundary, with and without free surface multiples at the upper boundary.

One-way reciprocity theorems

We use acoustic reciprocity as a starting point for deriving the relations between the various reflection and transmission responses. In general, acoustic reciprocity formulates a relation between two acoustic states in one and the same domain [de Hoop (1988), Fokkema and van den Berg (1993)]. We distinguish between two-way and one-way reciprocity theorems. In this paper we choose to work with the one-way reciprocity theorem because the concepts of reflection and transmission apply to downgoing and upgoing wave fields (rather than to full wave fields). Consider a domain $\mathcal{D}$ embedded between horizontal boundaries $\partial\mathcal{D}_0$ and $\partial\mathcal{D}_m$ (with $\partial\mathcal{D}_m$ below $\partial\mathcal{D}_0$). For this configuration the one-way reciprocity theorem of the convolution type for states A and B reads in the space-frequency domain

$$\int_{\partial\mathcal{D}_0}\{P_A^+P_B^- - P_A^-P_B^+\}\mathrm{d}^2\mathbf{x} = \int_{\partial\mathcal{D}_m}\{P_A^+P_B^- - P_A^-P_B^+\}\mathrm{d}^2\mathbf{x}, \quad (1)$$

where $P^+ = P^+(\mathbf{x},\omega)$ and $P^- = P^-(\mathbf{x},\omega)$ are flux-normalized downgoing and upgoing wave fields, respectively. Furthermore, ω denotes the angular frequency and $\mathbf{x}$ the Cartesian coordinate vector, defined as $\mathbf{x} = (x_1, x_2, x_3)$; throughout this paper the x_3-axis is pointing downward. We speak of a 'convolution type' reciprocity theorem because the products in the frequency domain ($P_A^+P_B^-$ etc.) correspond to convolutions in the time domain. This one-way reciprocity theorem is based on a coupled system of one-way wave equations for flux-normalized downgoing and upgoing waves. It holds for primary and multiply reflected waves with any propagation angle (including evanescent wave modes) in 3-D inhomogeneous media (Wapenaar and Grimbergen, 1996). In equation (1) it is assumed that the medium parameters in both states are identical and that the domain $\mathcal{D}$ is source-free. A more general expression can be found in the reference mentioned above.

The one-way reciprocity theorem of the correlation type reads

$$\int_{\partial\mathcal{D}_0}\{P_A^{+*}P_B^+ - P_A^{-*}P_B^-\}\mathrm{d}^2\mathbf{x} = \int_{\partial\mathcal{D}_m}\{P_A^{+*}P_B^+ - P_A^{-*}P_B^-\}\mathrm{d}^2\mathbf{x}, \quad (2)$$

where * denotes complex conjugation. We speak of 'correlation type' because the products in the frequency domain ($P_A^{+*}P_B^+$ etc.) correspond to correlations in the time domain. In addition to the assumptions for the convolution type theorem, here it is assumed that the medium is lossless and that evanescent wave modes can be neglected. As a consequence, any result obtained from equation (2) will be spatially band-limited.

Specification of the states A and B

In this section we define the states A and B that will be used in the one-way reciprocity theorems for deriving the relations between the reflection and transmission responses. Figure 1 shows four possible situations for state A. The boundaries $\partial\mathcal{D}_0$ and $\partial\mathcal{D}_m$ are chosen at depth levels $x_{3,0}+\epsilon$ and $x_{3,m}-\epsilon$, respectively (with $x_{3,m} > x_{3,0}$ and ϵ a vanishing positive constant). As mentioned in the introduction, we consider responses without (states A-1 and A-2 in Figure 1) and with free surface multiples (states A-3 and A-4 in Figure 1). In states A-1 and A-3 we choose a source for downgoing waves at $\mathbf{x}_A = (\mathbf{x}_{H,A}, x_{3,0})$, just above the upper boundary $\partial\mathcal{D}_0$. Here we used the subscript $_H$ to denote the horizontal coordinates, i.e., $\mathbf{x}_H = (x_1, x_2)$. Hence, $\mathbf{x}_{H,A}$ denotes the horizontal coordinates of $\mathbf{x}_A$, i.e., $\mathbf{x}_{H,A} = (x_{1,A}, x_{2,A})$. In states A-2 and A-4 we consider a source for upgoing waves at $\mathbf{x}'_A = (\mathbf{x}'_{H,A}, x_{3,m})$, just below the lower boundary $\partial\mathcal{D}_m$. The downgoing and upgoing wave fields at $\partial\mathcal{D}_0$ and $\partial\mathcal{D}_m$ are given in Table 1 for the different situations. The notation is as follows. Reflection responses are denoted by R^+ and R^-, transmission responses by T^+ and T^-. For example, $R_0^+(\mathbf{x}, \mathbf{x}_A, \omega)$ is the reflection response of the inhomogeneous medium in $\mathcal{D}$, including all internal multiples, for a source at $\mathbf{x}_A$ and a receiver at $\mathbf{x}$. The subscript $_0$ denotes that no free surface multiples are included; when this subscript is absent it means that free

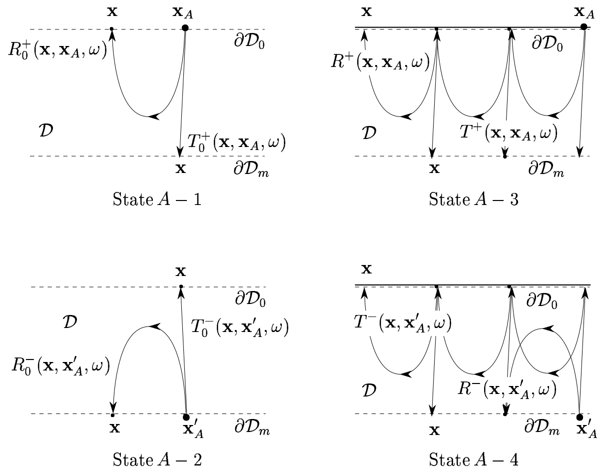

Fig. 1: States for the acoustic one-way reciprocity theorems. State A-1: At $\mathbf{x}_A$ just above $\partial\mathcal{D}_0$ there is a source for downgoing waves. The half-spaces above $\partial\mathcal{D}_0$ and below $\partial\mathcal{D}_m$ are homogeneous. The half-space below $\partial\mathcal{D}_m$ is source-free. State A-2: As in state A-1, but with a source for upgoing waves at $\mathbf{x}'_A$ just below $\partial\mathcal{D}_m$. State A-3: As in state A-1, but with a free surface just above $\partial\mathcal{D}_0$. State A-4: As in state A-3, but with a source for upgoing waves at $\mathbf{x}'_A$ just below $\partial\mathcal{D}_m$.

surface multiples are included. The superscript $^+$ denotes that this is a response of a downgoing source wave field. Finally, $s_A(\omega)$ is the source spectrum for state A and r^- is the reflection coefficient of the free surface for upgoing waves; it is defined as $r^- = -1$.

States B-1 to B-4 are defined analogous to states A-1 to A-4, except that the source points $\mathbf{x}_A$ and $\mathbf{x}'_A$ are replaced by $\mathbf{x}_B$ and $\mathbf{x}'_B$, respectively. The reciprocity theorems (1) and (2) can be applied to find relations between any of the states A-1 to A-4 on the one hand and any of the states B-1 to B-4 on the other hand. In the following sections we make a selection of combinations that lead to relevant relations between reflection and/or transmission responses.

Source-receiver reciprocity

In this section we derive source-receiver reciprocity relations. Although these relations are well-known for full wave fields (i.e., solutions of the two-way wave equation), they are less obvious for one-way wave fields (i.e., solutions of the coupled one-way wave equations for downgoing and upgoing waves). We substitute the expressions for the one-way wave fields in states A-1 and B-1 (Table 1) into the one-way reciprocity theorem of the convolution type (equation 1). Dividing the result by $s_A(\omega)s_B(\omega)$ we thus obtain

$$\int_{\partial\mathcal{D}_0}\Big[\delta(\mathbf{x}_H-\mathbf{x}_{H,A})R_0^+(\mathbf{x}_H,x_{3,0},\mathbf{x}_B,\omega)- \quad (3)$$
$$R_0^+(\mathbf{x}_H,x_{3,0},\mathbf{x}_A,\omega)\delta(\mathbf{x}_H-\mathbf{x}_{H,B})\Big]\mathrm{d}^2\mathbf{x}=0,$$

or

$$R_0^+(\mathbf{x}_A,\mathbf{x}_B,\omega)=R_0^+(\mathbf{x}_B,\mathbf{x}_A,\omega), \qquad (4)$$

for $\mathbf{x}_A$ and $\mathbf{x}_B$ just above $\partial\mathcal{D}_0$. This equation describes source-receiver reciprocity for the reflection response without the free surface multiples, observed just above the boundary $\partial\mathcal{D}_0$.

A similar result for the reflection response observed just below the lower boundary $\partial\mathcal{D}_m$ is obtained by substitution of states A-2 and B-2, yielding

$$R_0^-(\mathbf{x}'_A,\mathbf{x}'_B,\omega)=R_0^-(\mathbf{x}'_B,\mathbf{x}'_A,\omega), \qquad (5)$$

for $\mathbf{x}'_A$ and $\mathbf{x}'_B$ just below $\partial\mathcal{D}_m$. Substitution of states A-3 and B-3 into equation (1) yields for the reflection response including the free surface multiples, observed at the free surface

$$R^+(\mathbf{x}_A,\mathbf{x}_B,\omega)=R^+(\mathbf{x}_B,\mathbf{x}_A,\omega). \qquad (6)$$

Similarly, combining states A-4 and B-4 yields

$$R^-(\mathbf{x}'_A,\mathbf{x}'_B,\omega)=R^-(\mathbf{x}'_B,\mathbf{x}'_A,\omega). \qquad (7)$$

Finally, source-receiver reciprocity for the transmission responses is obtained by substituting either states A-2 and B-1 or states A-4 and B-3 into equation (1), yielding

$$T_0^+(\mathbf{x}'_A,\mathbf{x}_B,\omega)=T_0^-(\mathbf{x}_B,\mathbf{x}'_A,\omega) \qquad (8)$$

and

$$T^+(\mathbf{x}'_A,\mathbf{x}_B,\omega)=T^-(\mathbf{x}_B,\mathbf{x}'_A,\omega). \qquad (9)$$

Equations (4), (5) and (8) were previously derived in the wavenumber domain by Haines (1988).

From reflection to transmission

In this section we derive the first relation between reflection and transmission responses, for the situation without free surface multiples. We substitute the expressions for the one-way wave fields in states A-1 and B-1 (Table 1) into the one-way reciprocity theorem of the correlation type (equation 2). Dividing the result by $s_A^*(\omega)s_B(\omega)$ we thus obtain

$$\int_{\partial\mathcal{D}_m}\{T_0^+(\mathbf{x},\mathbf{x}_A,\omega)\}^*T_0^+(\mathbf{x},\mathbf{x}_B,\omega)\mathrm{d}^2\mathbf{x}=\delta(\mathbf{x}_{H,B}-\mathbf{x}_{H,A})$$
$$-\int_{\partial\mathcal{D}_0}\{R_0^+(\mathbf{x},\mathbf{x}_A,\omega)\}^*R_0^+(\mathbf{x},\mathbf{x}_B,\omega)\mathrm{d}^2\mathbf{x}, \qquad (10)$$

for $\mathbf{x}_A$ and $\mathbf{x}_B$ just above $\partial\mathcal{D}_0$. Note that this equation is not exact, since evanescent wave modes have been neglected in the derivation of the one-way reciprocity theorem of the correlation type. Similar expressions have been derived before by Herman (1992) using a two-way reciprocity theorem and by Wapenaar and Herrmann (1993) using the one-way reciprocity theorem (equation

State A-1:	source point $\mathbf{x}_A$ just above $\partial\mathcal{D}_0$; homogeneous half-spaces above $\partial\mathcal{D}_0$ and below $\partial\mathcal{D}_m$; no sources below $\partial\mathcal{D}_m$
$\mathbf{x} \in \partial\mathcal{D}_0$	$P_A^+(\mathbf{x}, \mathbf{x}_A, \omega) = \delta(\mathbf{x}_H - \mathbf{x}_{H,A}) s_A(\omega)$ $P_A^-(\mathbf{x}, \mathbf{x}_A, \omega) = R_0^+(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega)$
$\mathbf{x} \in \partial\mathcal{D}_m$	$P_A^+(\mathbf{x}, \mathbf{x}_A, \omega) = T_0^+(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega)$ $P_A^-(\mathbf{x}, \mathbf{x}_A, \omega) = 0$

State A-2:	source point $\mathbf{x}'_A$ just below $\partial\mathcal{D}_m$; homogeneous half-spaces above $\partial\mathcal{D}_0$ and below $\partial\mathcal{D}_m$; no sources above $\partial\mathcal{D}_0$
$\mathbf{x} \in \partial\mathcal{D}_0$	$P_A^+(\mathbf{x}, \mathbf{x}'_A, \omega) = 0$ $P_A^-(\mathbf{x}, \mathbf{x}'_A, \omega) = T_0^-(\mathbf{x}, \mathbf{x}'_A, \omega) s_A(\omega)$
$\mathbf{x} \in \partial\mathcal{D}_m$	$P_A^+(\mathbf{x}, \mathbf{x}'_A, \omega) = R_0^-(\mathbf{x}, \mathbf{x}'_A, \omega) s_A(\omega)$ $P_A^-(\mathbf{x}, \mathbf{x}'_A, \omega) = \delta(\mathbf{x}_H - \mathbf{x}'_{H,A}) s_A(\omega)$

State A-3:	source point $\mathbf{x}_A$ just above $\partial\mathcal{D}_0$; free surface just above $\partial\mathcal{D}_0$; homogeneous source-free half-space below $\partial\mathcal{D}_m$
$\mathbf{x} \in \partial\mathcal{D}_0$	$P_A^+(\mathbf{x}, \mathbf{x}_A, \omega) = \delta(\mathbf{x}_H - \mathbf{x}_{H,A}) s_A(\omega) + r^- P_A^-(\mathbf{x}, \mathbf{x}_A, \omega)$ $P_A^-(\mathbf{x}, \mathbf{x}_A, \omega) = R^+(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega)$
$\mathbf{x} \in \partial\mathcal{D}_m$	$P_A^+(\mathbf{x}, \mathbf{x}_A, \omega) = T^+(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega)$ $P_A^-(\mathbf{x}, \mathbf{x}_A, \omega) = 0$

State A-4:	source point $\mathbf{x}'_A$ just below $\partial\mathcal{D}_m$; free surface, no sources, just above $\partial\mathcal{D}_0$; homogeneous half-space below $\partial\mathcal{D}_m$
$\mathbf{x} \in \partial\mathcal{D}_0$	$P_A^+(\mathbf{x}, \mathbf{x}'_A, \omega) = r^- P_A^-(\mathbf{x}, \mathbf{x}'_A, \omega)$ $P_A^-(\mathbf{x}, \mathbf{x}'_A, \omega) = T^-(\mathbf{x}, \mathbf{x}'_A, \omega) s_A(\omega)$
$\mathbf{x} \in \partial\mathcal{D}_m$	$P_A^+(\mathbf{x}, \mathbf{x}'_A, \omega) = R^-(\mathbf{x}, \mathbf{x}'_A, \omega) s_A(\omega)$ $P_A^-(\mathbf{x}, \mathbf{x}'_A, \omega) = \delta(\mathbf{x}_H - \mathbf{x}'_{H,A}) s_A(\omega)$

Table 1. States for the acoustic one-way reciprocity theorems

2). Equation (10) is the basis for deriving the coda of the transmission response from the cross-correlation of the reflection response. Note that the inverse of the transmission coda, in combination with an inverse primary propagator, may be used in seismic reflection imaging to obtain an image in which the internal multiple scattering effects are suppressed.

From transmission to reflection

In this section we derive the second relation between reflection and transmission responses, this time for the situation with free surface multiples. We substitute the expressions for the one-way wave fields in states A-3 and B-3 (Table 1) into the one-way reciprocity theorem of the correlation type (equation 2) and subsequently use the source-receiver reciprocity relations (6) and (9). Dividing the result by $s_A^*(\omega) s_B(\omega)$ we thus obtain

$$2\Re\big[R^+(\mathbf{x}_A, \mathbf{x}_B, \omega)\big] = \delta(\mathbf{x}_{H,B} - \mathbf{x}_{H,A}) - \int_{\partial\mathcal{D}_m} \{T^-(\mathbf{x}_A, \mathbf{x}, \omega)\}^* T^-(\mathbf{x}_B, \mathbf{x}, \omega) \mathrm{d}^2\mathbf{x}, \tag{11}$$

for $\mathbf{x}_A$ and $\mathbf{x}_B$ at the free surface, just above $\partial\mathcal{D}_0$ and where $\Re\{\cdot\}$ denotes the real part. When the transmission responses $T^-(\mathbf{x}_A, \mathbf{x}, \omega)$ and $T^-(\mathbf{x}_B, \mathbf{x}, \omega)$ with free surface multiples are known for a sufficient range of $\mathbf{x}$-values, equation (11) is an explicit expression for the real part of the reflection response $R^+(\mathbf{x}_A, \mathbf{x}_B, \omega)$. Since the reflection response in the time domain is causal, the imaginary part of $R^+(\mathbf{x}_A, \mathbf{x}_B, \omega)$ is obtained via the Hilbert transform. Equation (11) is the theoretical basis for acoustic daylight imaging (Wapenaar et al., 2002).

Free surface multiple elimination

In order to find a relation between the reflection responses with and without free surface multiples, we substitute the expressions for the one-way wave fields in states A-1 and B-3 (Table 1) into the one-way reciprocity theorem of the convolution type (equation 1). Applying the source-receiver reciprocity relation (4) to the result we obtain

$$R_0^+(\mathbf{x}_A, \mathbf{x}_B, \omega) - R^+(\mathbf{x}_A, \mathbf{x}_B, \omega) = \int_{\partial\mathcal{D}_0} R_0^+(\mathbf{x}_A, \mathbf{x}, \omega) R^+(\mathbf{x}, \mathbf{x}_B, \omega) \mathrm{d}^2\mathbf{x}, \tag{12}$$

for $\mathbf{x}_A$ and $\mathbf{x}_B$ just above $\partial\mathcal{D}_0$. When the response $R^+(\mathbf{x}_A, \mathbf{x}_B, \omega)$ with free surface multiples is known (from decomposed seismic measurements), equation (12) is an integral equation of the second kind for the response $R_0^+(\mathbf{x}_A, \mathbf{x}_B, \omega)$ without free surface multiples. A method for free surface multiple elimination, based on a similar type of equation but derived in a different way, has previously been proposed by Kennett (1979) and Berkhout (1982) and implemented by Verschuur et al. (1992). Fokkema and van den Berg (1993) used a two-way reciprocity theorem to derive a similar multiple elimination scheme. Equation (12) may be seen as the 'one-way counterpart' of the result of Fokkema and van den Berg (1993).

Next we substitute states A-4 and B-1 into equation (1). Applying the source-receiver reciprocity relation (9) to the result we obtain

$$T_0^+(\mathbf{x}'_A, \mathbf{x}_B, \omega) - T^+(\mathbf{x}'_A, \mathbf{x}_B, \omega) = \int_{\partial\mathcal{D}_0} T^+(\mathbf{x}'_A, \mathbf{x}, \omega) R_0^+(\mathbf{x}, \mathbf{x}_B, \omega) \mathrm{d}^2\mathbf{x}. \quad (13)$$

for $\mathbf{x}_B$ just above $\partial\mathcal{D}_0$ and $\mathbf{x}'_A$ just below $\partial\mathcal{D}_m$. This is an explicit expression for the transmission response $T_0^+(\mathbf{x}'_A, \mathbf{x}_B, \omega)$ without free surface multiples in terms of the transmission response $T^+(\mathbf{x}'_A, \mathbf{x}, \omega)$ with free surface multiples and $R_0^+(\mathbf{x}, \mathbf{x}_B, \omega)$.

Seismic interferometry

Schuster (2001) proposed to evaluate an integral over the product of $\{R_0^+(\mathbf{x}_A, \mathbf{x}, \omega)\}^*$ and $R^+(\mathbf{x}_B, \mathbf{x}, \omega)$, which corresponds to a cross-correlation in the time domain of two reflection responses recorded at $\mathbf{x}_A$ and $\mathbf{x}_B$, respectively. He showed that this leads to a new reflection response observed at $\mathbf{x}_A$, as if there was a source at $\mathbf{x}_B$. We investigate the integral over the product of $\{R_0^+(\mathbf{x}_A, \mathbf{x}, \omega)\}^*$ and $R^+(\mathbf{x}_B, \mathbf{x}, \omega)$ by substituting states A-1 and B-3 into the one-way reciprocity theorem of the correlation type (equation 2) and employing the appropriate source-receiver reciprocity relations. We thus obtain

$$R^+(\mathbf{x}_A, \mathbf{x}_B, \omega) = \delta(\mathbf{x}_{H,B} - \mathbf{x}_{H,A}) - \int_{\partial\mathcal{D}_0} \{R_0^+(\mathbf{x}_A, \mathbf{x}, \omega)\}^* R^+(\mathbf{x}_B, \mathbf{x}, \omega) \mathrm{d}^2\mathbf{x} - \int_{\partial\mathcal{D}_m} \{T_0^-(\mathbf{x}_A, \mathbf{x}, \omega)\}^* T^-(\mathbf{x}_B, \mathbf{x}, \omega) \mathrm{d}^2\mathbf{x}, \quad (14)$$

for $\mathbf{x}_A$ and $\mathbf{x}_B$ just above $\partial\mathcal{D}_0$. This equation shows that an integral over the correlation of reflection responses recorded at $\mathbf{x}_A$ and $\mathbf{x}_B$ (the first integral on the right-hand side along the common source coordinate $\mathbf{x}$ at $\partial\mathcal{D}_0$ of both responses) indeed contributes to the reflection response $R^+(\mathbf{x}_A, \mathbf{x}_B, \omega)$. Following Schuster (2001), we call this seismic reflection interferometry.

Expressions for reflectivity 'from below'

In this section we derive expressions for the reflection response for upgoing waves at the bottom of the configuration. We substitute the expressions for the one-way wave fields in states A-1 and B-2 (Table 1) into the one-way reciprocity theorem of the correlation type (equation 2). Dividing the result by $s_A^*(\omega)s_B(\omega)$ we thus obtain

$$\int_{\partial\mathcal{D}_m} \{T_0^+(\mathbf{x}, \mathbf{x}_A, \omega)\}^* R_0^-(\mathbf{x}, \mathbf{x}'_B, \omega) \mathrm{d}^2\mathbf{x} = -\int_{\partial\mathcal{D}_0} \{R_0^+(\mathbf{x}, \mathbf{x}_A, \omega)\}^* T_0^+(\mathbf{x}'_B, \mathbf{x}, \omega) \mathrm{d}^2\mathbf{x}, \quad (15)$$

for $\mathbf{x}_A$ just above $\partial\mathcal{D}_0$ and $\mathbf{x}'_B$ just below $\partial\mathcal{D}_m$. When the reflection response $R_0^+(\mathbf{x}, \mathbf{x}_A, \omega)$ and the transmission response $T_0^+(\mathbf{x}, \mathbf{x}_A, \omega)$, both without free surface multiples, are known, equation (15) is an integral equation of the first kind for the reflection response $R_0^-(\mathbf{x}, \mathbf{x}'_B, \omega)$ for upgoing waves at the bottom of the configuration, just below $\partial\mathcal{D}_m$, without free surface multiples. $R_0^-(\mathbf{x}, \mathbf{x}'_B, \omega)$ may be used for imaging the configuration between the boundaries $\partial\mathcal{D}_0$ and $\partial\mathcal{D}_m$ 'from below', in addition to conventional imaging 'from above'. Other relevant expressions for the reflectivity from below are obtained by substituting states A-3 and B-4 or states A-4 and B-4 into equation (2).

Conclusions

We have presented a unified derivation for relations between reflection and transmission responses of 3-D inhomogeneous media, using one-way reciprocity theorems of the convolution type and of the correlation type (equations 1 and 2). Note that because all relations apply to reflection and/or transmission responses for downgoing and/or upgoing wave fields, in practice a decomposition of measurements into downgoing and upgoing wave fields is required as a preprocessing step.

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