AVA Correction for Migration in Highly Discontinuous Media

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Summary

Current macro-model–based one-way wavefield extrapolation ignores the practical highly discontinuous character of rock formations. The main effects of this highly discontinuous behaviour on transmitted wavefields are angle-dependent dispersion and amplitude losses. In this paper the angle-dependent dispersion and amplitude losses are quantified by a proposed extended macro model, which is much related to the current macro model except that two stochastic parameters on the rock formations are added. The stochastic parameters are derived based on the observation of power-law behaviour of subsurface heterogeneity. Explicit extrapolation operators are used to perform forward and inverse wavefield extrapolation. A true-amplitude prestack migration scheme is proposed, that can correctly eliminate the dispersion effects of the fine-layering together with the geometrical spreading effects in imaging the subsurface based on the extended macro model. A synthetic seismic experiment is designed to test the migration scheme.

Introduction

The main process in seismic migration is the elimination of propagation effects from the seismic measurements. Usually these propagation effects are quantified by a macro model, which contains the main geological boundaries in the subsurface and the average velocities between these boundaries. However, the rock formations as revealed by well logs and core samples are highly discontinuous at all length scales down to a few centimeters and smaller. Extensive studies on wave propagation through (1-D) finely layered media (e.g., O'Doherty and Anstey, 1971) have shown that short period internal multiples may seriously affect the apparent propagation properties of the seismic wavefield. The main effects are angle-dependent dispersion and amplitude losses of the transmitted wavefield. It is obvious that the macro model does not include the details of rock formations at such small scales. Hence, the seismic processing based on the macro model will not compensate for these angle-dependent dispersions and amplitude losses. This may result in dispersed images and erroneous amplitude-variation-with-angle (AVA) effects. It is not realistic to attempt to construct a velocity model that includes the correct details of the rock formations. Wapenaar et al. (1994) introduced an extended macro model to account for the AVA correction. The extended macro model aims to replace a finely layered medium by a homogeneous anisotropic medium with anelastic losses in such a way that the transmission response of this replacement medium is effectively the same as the transmission response of the original finely layered medium.

In this paper, we further develop the idea of the extended macro model. Instead of describing the model with frequency-dependent vertical and horizontal complex phase velocities as introduced in Wapenaar et al. (1994), we directly derive the dispersion relation for the replacement medium based on the apparent transmission response of the original finely layered medium, and then define the average velocities and stochastic parameters as those used by Wapenaar et al. (1994) as a new parameterized extended macro model. This makes that the proposed replacement model has a less severe angle limitation for mimicking the wave propagation in the true finely layered medium and it is able to handle fine-layering structures that are either horizontal or tilted. By developing phase-shift operators that govern primary wave propagation in the anisotropic replacement medium with anelastic losses, based on the derived dispersion relation, and then designing short explicit extrapolation operators in the space-frequency domain based on these phase-shift operators, we can perform wavefield depth extrapolation in a true subsurface that is moderately inhomogeneous at the macro scale and highly discontinuous at the sub-wavelength scale. Thus, a true-amplitude prestack migration scheme is proposed, that can correctly eliminate the dispersion effects of the fine-layering together with the geometrical spreading effects in imaging the subsurface based on the extended macro model. Several authors (Wapenaar and Herrmann, 1993; Widmaier et al., 1996) have proposed to include fine-layering effects into migration. The extended macro model we use is much related to the conventional macro model used for current migration schemes, except that two stochastic parameters on the rock formations need to be added. The stochastic parameters are derived based on the observation of power-law behaviour of subsurface heterogeneity (Walden and Hosken, 1985), which can be estimated from well logs, from VSP data, or directly from the reflection measurements based on the proposed imaging scheme.

Transmission response of a layered medium

The transmission response of a plane wave with incidence angle θ propagating through a finely layered medium can be described in terms of wavefield extrapolation operators (Burridge and Chang, 1989; Shapiro and Zien, 1993; Wapenaar et al., 1994) as

$$\tilde{W}_g^+(z_m, z_0, \theta, \omega) = \tilde{W}_p^+(z_m, z_0, \theta, \omega)\tilde{C}(z_m, z_0, \theta, \omega), \quad (1)$$

where $\tilde{W}_p^+(z_m, z_0, \theta, \omega)$ is the extrapolation operator for the primary wave, and $\tilde{C}(z_m, z_0, \theta, \omega)$ is a correction operator that accounts for the fine-layering scattering. We call $\widetilde{W}_g^+(z_m, z_0, \theta, \omega)$ the generalized primary extrapolation operator. The primary extrapolation operator is given by

$$\widetilde{W}_p^+(z_m, z_0, \theta, \omega) = \exp(-j\omega \cos\phi\, \Delta z / c_0)\,, \qquad (2)$$

where $\Delta z = z_m - z_0$, c_0 denotes the average velocity of the finely layered medium, and ϕ is defined as $\cos\phi = \sqrt{1-(c_s^2/c_1^2)(\sin\theta)^2}$. Here c_s and c_1 denotes the root-mean-square velocity and the velocity of the first layer medium, respectively. Based on the observation of power-law behaviour of subsurface heterogeneity (Walden and Hosken, 1985), the correction operator may be expressed as (Wapenaar et al., 1994)

$$\widetilde{C}(z_m, z_0, \theta, \omega) = \exp\{-A(\omega)(\cos\phi)^{\alpha-n}\, \Delta z / c_0\}\,, \quad (3)$$

where α is the value of the power, $n = 4$ when the layered medium contains velocity contrasts only, and $n = 0$ for density contrasts only. With the power-spectrum of the reflection coefficients, $R(\omega)$, being defined as $R(\omega) = \upsilon|\omega|^\alpha$, and its Hilbert transform being given by $I(\omega) = \upsilon \tan(\alpha\pi/2) sign(\omega)|\omega|^\alpha$, $A(\omega)$ reads

$$A(\omega) = 0.5R(\omega) + j0.5I(\omega)\,. \qquad (4)$$

Extended macro model

The extended macro model aims to replace a finely layered medium by a (piecewise) homogeneous anisotropic medium with anelastic losses in such a way that the primary transmission response of this replacement medium is effectively the same as that of the original finely layered medium. Wapenaar et al. (1994) proposed an extended macro model that is described using frequency-dependent vertical and horizontal complex phase velocities. Due to only matching the coefficients of zeroth-order and second-order rayparameters in the Taylor series expansion, the proposed model has a high angle limitation for the transmission response of the replacement medium to mimic the generalized primary. As an alternative, we directly derive the dispersion relation for the replacement medium based on the generalized primary, and then define the average velocities and stochastic parameters, that determine the dispersion relation, as a parameterized extended macro model. Thus the proposed method also satisfies the Kramers-Kronig relations for all propogation angles.

For a plane wave with an incident angle of θ, the corresponding horizontal wavenumber reads $k_x = (\omega/c_1)\sin\theta$. From its transmission response as expressed with equations (2) and (3), we can derive a related complex vertical wavenumber as

$$k_z = (\omega\cos\phi - jA(\omega)(\cos\phi)^{\alpha-n})/c_0\,, \qquad (5)$$

where $\cos\phi = \sqrt{1-c_s^2 k_x^2/\omega^2}$. For a given k_x value we can solve k_z from equation (5). This defines a dispersion relation for the replacement medium. For a finely layered structure that makes an angle of β with the horizontal direction, the corresponding dispersion relation can be obtained as

$$k_z = \cos\theta(\omega\cos\phi - jA(\omega)(\cos\phi)^{\alpha-n})/c_0 \cos(\theta-\beta)\,, \quad (6)$$

where

$$\cos\phi = \sqrt{1-\frac{c_s^2}{\omega^2}\left[k_x^2\cos 2\beta + \frac{\omega^2}{c_1^2}\sin^2\beta - k_x(\omega^2/c_1^2 - k_x^2)^{1/2}\sin 2\beta\right]}$$

Here c_s and c_1 can be approximately replaced using the average velocity of c_0. Thus the three parameters (together with the dip-angle of the fine-layering structure): the average velocity c_0 and the stochastic parameters α and υ, provide a parameterized extended macro model, which is much related to the conventional macro model used for current migration schemes, except that two stochastic parameters on the rock formations have been added.

Phase-shift operator

From the dispersion relation we can obtain a phase-shift extrapolation operator that can efficiently perform wavefield depth extrapolation and imaging in a laterally invariant medium. The forward phase-shift extrapolation operator for downgoing waves for $\omega > 0$ reads

$$\widetilde{W}_g^+(z_{i+1}, z_i, k_x, \omega, c_0, \upsilon, \alpha, \beta) = \exp(-jk_z\Delta z)\,, \quad (7)$$

and the inverse phase-shift extrapolation (downward continuation) operator for upgoing waves is expressed as

$$\widetilde{F}_g^-(z_{i+1}, z_i, k_x, \omega, c_0, \upsilon, \alpha, \beta) = \{\exp[-j(k_z^-)^*\Delta z]\}^*\,, \quad (8)$$

where the superscript * denotes the complex conjugate, and we have (Zhang et al., 2001)

$$k_z^- = k_z(-k_x, \omega, c_0, \upsilon, \alpha, \beta). \qquad (9)$$

The complex conjugate for k_z^- in equation (9) makes the inverse phase-shift operator able to compensate for the attenuation and dispersion due to the fine-layering scattering. However, the stability due to boosting amplitudes appears to be a crucial problem for the inverse phase-shift operator. Hence, even for laterally invariant media, we can not directly apply the phase-shift operator of equation (9) to perform inverse wavefield extrapolation.

The stability can be achieved by modifying the part of k_z^- associated to high angles, that is, we set the imaginary part of k_z^- positive for incident angles exceeding a given value. In this way, we can get a conditional stable operator for a finite number of depth extrapolation steps.

Explicit extrapolation operator

The explicit extrapolation operator scheme provides an approximation to determine the spatial forward and inverse extrapolation operators in a laterally varying medium. That is to find a short operator in the space-frequency domain such that its spatial Fourier transform matches the phase-shift operator in the required range of angles of propagation as accurate as possible and its amplitudes decay outside this propagation region. For the explicit extrapolation operator scheme, the task to stabilize the inverse extrapolation operators by suppressing the wavefield with a high angle of propagation can be easily accomplished through taking the requirement of stability into account when determining the maximum angle of propagation.

There exists a number of available methods for designing symmetric short operators. However, an asymmetric short operator needs to be designed here. The weighted least-squares method (Thorbecke and Rietveld, 1994) can derive a stable, highly accurate short operator with a very low computational cost in an isotropic lossless medium. The method has been extended to design asymmetric short operators by Zhang et al. (2001). In this paper we further adapt the weighted least-squares method to design short operators for an anisotropic lossy medium.

Examples

Based on the proposed explicit extrapolation operator scheme, we can develop a table-driven AVA prestack migration scheme. A synthetic seismic experiment is designed to test the migration scheme. The seismic data set is generated through numerical modeling of the acoustic wave equation in a true finely layered medium using the reflectivity method. The synthetic velocity well-log of the layered medium is illustrated in the left of Figure 1, and the corresponding macro model is shown in the right of Figure 1. One shot gather is shown in Figure 2. The migrated results for this shot gather with the extended macro model as well as with the standard macro model are illustrated in Figures 3(a) and 3(b). The migration with the standard macro model means that the highly discontinuous properties of the velocity distribution are ignored. From Figure 3 we see that the main reflectors are imaged clearly. Figure 4 shows the comparisons of the reflectivity of the two migrated results with the exact reflection coefficients at the depth level of 1100m. The exact reflection coefficients are obtained based on the velocity and density contrasts at the second boundary of the macro model. From Figure 4 we can see that the migration result, that takes fine-layering scattering into account, better approaches the exact reflection coefficients. The small discrepancy between the exact reflection coefficients and this migrated reflectivity is mainly due to the fact that the migrated reflectivity is related to the real highly discontinuous velocity model, as shown in the left of Figure 1.

The reason that we use a laterally invariant medium is that the only available modeling tool for the true velocity model, consisting of 21000 layers as illustrated in Figure 1, is the reflectivity method. However, the migration algorithm we use is based on the explicit extrapolation operators, that has been proven to be effective for both isotropic and anisotropic laterally varying media. Hence, it is reasonable to state that the proposed scheme can efficiently be applied for a practical subsurface that is moderately inhomogeneous at the macro scale and highly discontinuous at the sub-wavelength scale.

Conclusions

A method for wavefield depth extrapolation in a subsurface that is moderately inhomogeneous at the macro scale and highly discontinuous at the sub-wavelength scale has been presented. Moreover, a prestack migration scheme, that can image the true reflectivity of the subsurface by eliminating the geometrical spreading effects as well as the dispersion effects of the finely-layering, has been proposed. The statistical behaviour exhibited by the subsurface heterogeneity (fractal Brownian motion) is used to set up a parameterized extended macro model, that can quantify the effects on the transmission response of the short period internal multiples due to the highly discontinuous properties of the rock formations. The proposed AVA migration scheme is implemented based on the extended macro model by pre-calculating an operator table and then selecting the space-variant operator during the depth extrapolation process.

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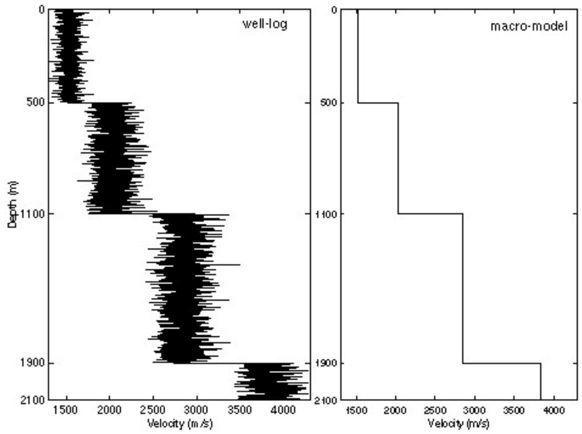

Figure 1. A synthetic velocity well log for a layered medium. The right is the corresponding macro model.

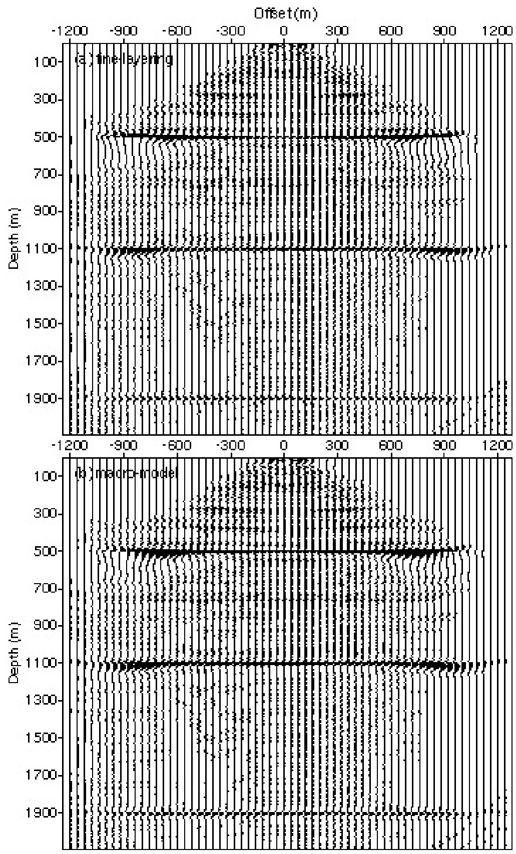

Figure 3. Shot-record migration results. The result in (a) is obtained using an extended macro model that takes the fine-layering scattering into account. The result in (b) is obtained using the macro model that ignores the highly discontinuous properties of the velocity distribution.

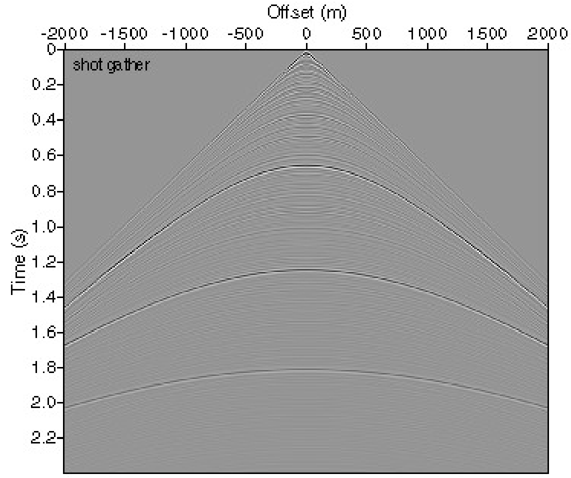

Figure 2. Shot gather for a zero-phase Ricker wavelet with a peak frequency of 40Hz. The shot gather is obtained using the reflectivity method in the true finely layered medium.

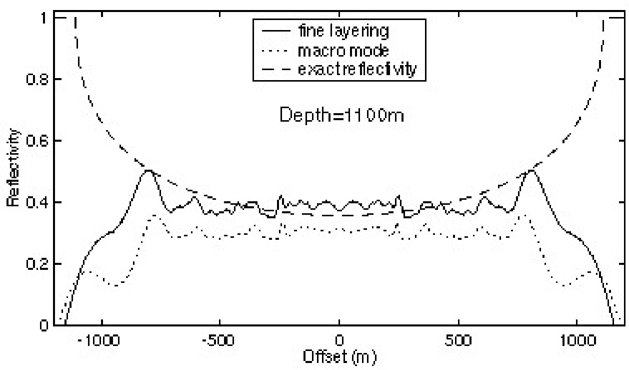

Figure 4. Comparison of the reflectivity of the two migration results with the exact reflection coefficients at the depth level of the second main reflector. The solid line denotes the migration result at the depth level of 1100m obtained using an extended macro model that takes the fine-layering scattering into account, the dotted line the migration result obtained using the macro model that ignores the highly discontinuous properties of the velocity distribution, and the dashed line the exact reflection coefficients based on the velocity and density contrasts at the second boundary of the macro model.