Pressure-dependent scaling behaviour of the transmission response of reservoir rocks

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Summary

Ultrasonic experiments carried out on several reservoir sandstone samples have shown a specific pressure-dependent behaviour of the transmission response. Apart from the well-known velocity increase as ambient pressure increases, the amplitude and the time axis are scaled as a function of the pressure. In order to quantify the scaling phenomenon two approaches are used. First, a heuristically derived model from the experimental data is tested on numerically simulated data. On the other hand, an analytically derived model from a modified version of the O'Doherty-Anstey expression for the transmission response through finely layered media is also analyzed and tested on numerically simulated data. Both scaling models present two scalar parameters that relate a wavelet recorded at a higher ambient pressure with another recorded at a relatively lower one. Estimating these parameters from measurements for a range of different ambient pressures gives valuable information about the pressure-dependent behaviour of the reservoir rock.

Introduction

A couple of years ago, ultrasonic transmission measurements have been carried out on Rotliegend reservoir sandstone samples (den Boer et al., 1996; Swinnen, 1997). The experiments were carried out for a range of different ambient pressures. Figure 1 shows the transmission responses for pressures ranging from 2 MPa (the latest arrival) to 20 MPa (the first arrival).

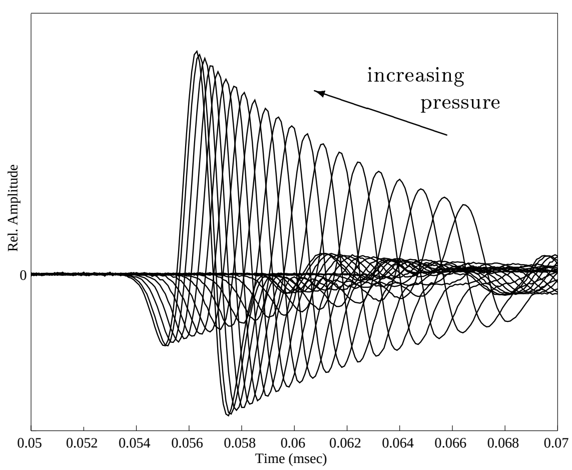

Fig. 1: *Transmission responses of Rotliegend reservoir sandstone for varying ambient pressure.*

It appeared that not only the arrival time reduces when the ambient pressure increases, but that the width of the wavelet reduces by approximately the same relative amount. In other words, the time-axis seems to be scaled by a single factor when the ambient pressure is changed from one value to another. Also the amplitude changes with changing pressure. It has been carefully checked that these time and amplitude changes are not source effects, but that it is the propagation through the sandstone that changes with changing pressure.

Heuristically derived scaling model

A simple scaling relation was derived from the experimental data by den Boer et al. (1996) and further studied by Swinnen (1997). A pair of recorded traces are related by

$$w_B(t) = \beta\, w_A(\frac{t}{\alpha}), \tag{1}$$

where $w_B(t)$ and $w_A(t)$ denote the transmission responses at two different ambient pressures and α and β are the scaling parameters. The first one stretches the time axis with respect to $t = 0$, while the second one affects the wavelet amplitude. In the frequency domain, the scaling relation becomes

$$W_B(\omega) = \beta\, \alpha\, W_A(\alpha\, \omega). \tag{2}$$

In order to calculate α and β, two traces are compared in the time domain, and using a normalized least-squares criterion, both scaling parameters are found by optimizing the match between the scaled trace with the recorded trace at a lower pressure. For example, Figure 2 shows the

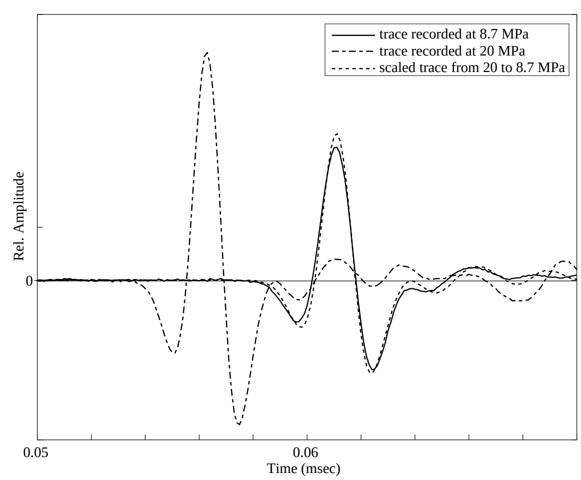

Fig. 2: *Illustration of the scaling behaviour* $w_B(t) = \beta\, w_A(\frac{t}{\alpha})$.

transmission responses recorded at 8.7 MPa (solid) and at 20 MPa (dashed-dotted) and a scaled version of the latter trace (dotted) that approximately coincides with the former.

The binary layered medium approach

We start by making a strong simplification, that is, we assume that the sandstone is horizontally layered. Although this is not very realistic, it is a suitable starting point for studying the scaling behaviour analytically. The second assumption is that the layered medium consists of only two types of material (hence the name binary layered medium); the layer thicknesses are in some way randomly distributed. The third assumption is that changes in the ambient pressure do not affect the layer thicknesses, but only the material parameters. With these assumptions the depth-dependent normal-incidence plane-wave reflectivity $r(z)$ obeys the following scaling relation

$$r_B(z) = \beta\, r_A(z), \tag{3}$$

where the subscripts A and B refer to two different ambient pressure states. When the material parameters of both layer types react similarly (in a relative sense) to changes in the ambient pressure then $\beta = 1$; when they react differently, then $\beta \neq 1$. The average slowness $\bar{s}$ of the material obeys the following relation

$$\bar{s}_B = \alpha\, \bar{s}_A. \tag{4}$$

It is beyond the scope of this paper to specify the scaling factors α and β and their mutual relation. For our analysis it is sufficient to assume that relations (3) and (4) hold for some value of α and β.

In the following section we will evaluate the scaling behaviour of the transmission response of binary layered media analytically.

Scaling behaviour of the transmission response

The normal-incidence plane-wave transmission response of a layered medium can be expressed in the frequency domain in terms of a 'generalized primary' propagator $\mathcal{W}(z_1, z_0, \omega)$, according to

$$\begin{aligned} \mathcal{W}(z_1, z_0, \omega) &= \mathcal{P}(z_1, z_0, \omega)\mathcal{M}(z_1, z_0, \omega) \\ &= \exp\{-j\omega\bar{s}\Delta z\}\exp\{-\mathcal{A}(2\omega\bar{s})\Delta z\}, \end{aligned} \tag{5}$$

where $\Delta z = z_1 - z_0$. The first exponential describes the (flux-normalized) primary propagation from depth level z_0 to z_1 and the second exponential accounts for the internal multiples generated at all interfaces between those two depth levels. The function $\mathcal{A}$ is the Fourier transform of the 'causal part' of $\mathcal{S}(z)$, according to

$$\mathcal{A}(k) = \int_0^\infty \exp\{-jkz\}\mathcal{S}(z)\mathrm{d}z, \tag{6}$$

where $\mathcal{S}(z)$ is the autocorrelation of the reflection function $r(z)$, expressed by,

$$\mathcal{S}(z) = \frac{1}{\Delta z - z}\int_{z_0}^{z_1 - z} r(\zeta)r(\zeta + z)\mathrm{d}\zeta. \tag{7}$$

Note that equation (5) is the well-known O'Doherty-Anstey relation (O'Doherty and Anstey, 1971), except that $\mathcal{A}(k)$ in equation (6) is expressed in terms of a spatial rather than a temporal autocorrelation function. The depth-time conversion takes place in equation (5), where $\mathcal{A}(k)$ is evaluated at $k = 2\omega\bar{s}$. Assuming $r(z)$ obeys equation (3), $\mathcal{A}(k)$ has the following scaling behaviour

$$\mathcal{A}_B(k) = \beta^2\mathcal{A}_A(k), \tag{8}$$

where the subscripts A and B refer again to two different ambient pressure states. For these two pressure states the generalized primary propagators read

$$\begin{aligned} \mathcal{W}_A(z_1, z_0, \omega) &= \mathcal{P}_A(z_1, z_0, \omega)\mathcal{M}_A(z_1, z_0, \omega) \\ &= \exp\{-j\omega\bar{s}_A\Delta z\}\exp\{-\mathcal{A}_A(2\omega\bar{s}_A)\Delta z\} \end{aligned} \tag{9}$$

$$\begin{aligned} \mathcal{W}_B(z_1, z_0, \omega) &= \mathcal{P}_B(z_1, z_0, \omega)\mathcal{M}_B(z_1, z_0, \omega) \\ &= \exp\{-j\omega\bar{s}_B\Delta z\}\exp\{-\mathcal{A}_B(2\omega\bar{s}_B)\Delta z\} \end{aligned} \tag{10}$$

or, using equations (4) and (8),

$$\begin{aligned} &\mathcal{W}_B(z_1, z_0, \omega) \\ &\quad= \exp\{-j\alpha\omega\bar{s}_A\Delta z\}\exp\{-\beta^2\mathcal{A}_A(2\alpha\omega\bar{s}_A)\Delta z\} \\ &\quad= \mathcal{P}_A(z_1, z_0, \alpha\omega)[\mathcal{M}_A(z_1, z_0, \alpha\omega)]^{\beta^2}. \end{aligned} \tag{11}$$

Numerical experiments

We are faced with two different scaling models for the transmission response. The former one (equation (1)) was heuristically derived analyzing the experimental data. The second one (equation (11)) was analytically derived from a modified O'Doherty-Anstey relation and making some assumptions on the pressure-dependent behaviour of the sandstone. We have performed numerical simulations for the transmission response through a binary layered medium, in order to check whether it is a convenient starting model for analyzing the pressure-dependent rock behaviour.

The proposed sandstone model consists of a stack of horizontal acoustic layers with just two alternating velocities, and the thickness of each bed is a random variable following an exponential distribution with mean d. The density is considered constant for both material types. The assumption is that as the ambient pressure increases one of the material changes its velocity while the other remains fixed. Thus the reflectivity and the average slowness change according to equations (3) and (4).

The total transmission response is calculated by means of forward modelling of the acoustic wave equation in the frequency domain, considering a plane-wave incident from the top and calculating the plane-wave transmitted to the bottom of the stack of horizontal parallel layers. After calculating the transmission impulse response, convolution with a Ricker wavelet is carried out.

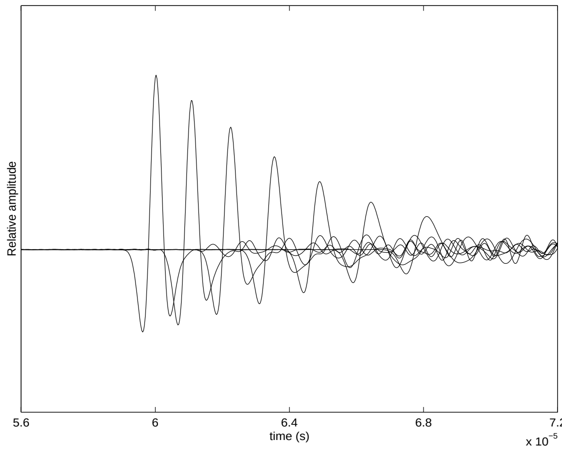

Fig. 3: *Transmission responses (convolved with Ricker wavelet) for varying impedance contrast.*

In Figure 3, transmitted zero-phase Ricker wavelets through binary layered media are shown. The trace with the fastest arrival corresponds to the smallest impedance contrast and average slowness. The later arrivals are calculated by fixing one velocity and decreasing the other, that is increasing the impedance contrast and the average slowness of the models. Note the similar scaling behaviour observed in the experimental data for different ambient pressures (Figure 1).

The first and third arrivals in Figure 3 are used to check both equation (1) and equation (11). The corresponding rock model parameters for these two cases are shown in Table 1.

	Total path	Mean (d) thickness	Impedance contrast
Model A	20 cm	0.3 mm	3200 m/s - 3500 m/s
Model B			3000 m/s - 3500 m/s

Table 1: *Binary layered model parameters*

Even though the former scaling model was derived from real data, it does not give any explicit relation between the scaling parameters (α and β in equation (1)) and some physical characteristic of the reservoir rock. In spite of the latter, it is worthwhile to scale one trace from a low impedance contrast to a higher impedance contrast, applying the same least-squares technique previously used with the experimental data. The results can be seen in Figure 4. Although the match is good, some differences in the amplitude of the main lobes and in the coda can be seen.

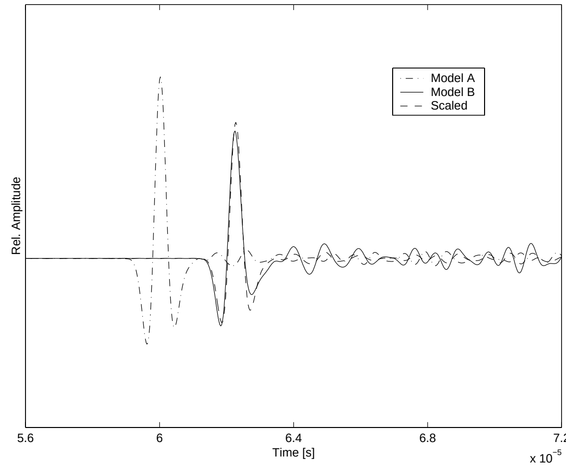

Fig. 4: *First arrival (Model A) and third arrival (Model B) of Figure 3, and scaled version from A to B using* $w_B(t) = \beta\, w_A(\frac{t}{\alpha})$.

On the other hand, the analytically derived scaling model of equation (11) presents scaling parameters somehow related to the acoustic characteristics of the layered model via equations (3) and (4). From the model velocities and using those equations, it is possible to calculate the scaling parameters $\alpha = 1.035$ and $\beta = 1.718$. They are used to scale the transmission response from model A to model B. The direct path delay is subtracted from both responses according to the corresponding time delay for each model. Thus equation (11) simplifies to

$$\mathcal{M}_B(z_1, z_0, \omega) = [\mathcal{M}_A(z_1, z_0, \alpha\omega)]^{\beta^2}. \qquad (12)$$

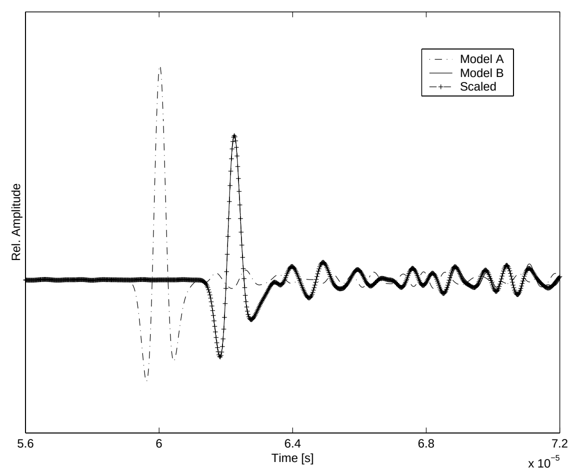

Fig. 5: *First arrival (Model A) and third arrival (Model B) of Figure 3, and scaled version from A to B using* $\mathcal{M}_B(z_1, z_0, \omega) = [\mathcal{M}_A(z_1, z_0, \alpha\omega)]^{\beta^2}$.

As follows from the latter formulation, the scaling model must be applied to the transmission impulse response, that is, with no source function included. As a first step in our forward modelling of the wave equation, the plane wave transmission response through the stack of layers is calculated. Then the scaling relation (equation (12)) is applied. Finally, the transmission impulse responses are convolved with a Ricker wavelet and shifted back to the correct arrival time. In Figure 5 the results are displayed. The match between the scaled version of the transmission response of model A ('+' marked line) and the transmission response of model B (solid line) is almost exact. This is another numerical confirmation of the O'Doherty-Anstey relation, although we know it is only an approximation to the total transmission response through a layered sequence.

Discussion and conclusions

The attenuation and dispersion of seismic energy due to superposition of internal multiples may well be the cause of the scaling of the transmitted wavelets through a rock sample under varying ambient pressure. The wavelet scaling produced by changes in the impedance contrast between layers corresponds to the pressure-dependent scaling behaviour observed in the experimental data. This correspondance suggests that as the ambient pressure increases, the average slowness and the impedance contrast within the rock decrease.

Two different scaling models were analyzed. The heuristically derived scaling model of equation (1) has been tested with numerical simulated data giving good results. However, the relation between the scaling parameters and the acoustic characteristics of the rock is not explicit.

The analytically derived scaling model of equation (11) has been tested with numerical data giving even better results (compare Figures 4 and 5), despite it has not been applied to the real data yet. This result was expected as long as a binary layered medium approach was used and the scaling relation was derived from a modified version of the well-known O'Doherty-Anstey relation. Our hypothesis is that as pressure changes, some variation in the velocity of one constituent material occurs. The α and β parameters are directly related with acoustic characteristics of the layered model (i.e. reflectivity and average slowness).

Equation (11) quantifies the scaling behaviour of the transmission response. Note that in both terms at the right-hand side the frequency is scaled with the same factor α. This agrees with our earlier observation that the arrival time and the width of the wavelet scale by approximately the same amount when the ambient pressure is changed. The exponent β^2 in the second term accounts for the amplitude change, but has an effect on the phase as well. Since this exponent is applied to a frequency-dependent term, there is not a simple scaling relation in the time-domain.

Although we have made a number of simplifying assumptions, it is worthwhile to use equation (11) as a first approximate model for observations like those in Figure 1. Estimating the parameters α and β from that type of measurements for a range of different ambient pressures gives valuable information about the pressure-dependent behaviour of the reservoir rock. However, more realistic 3-D scattering models within the rock matrix may also present the scaling behaviour of the transmission response. Present research is focused on that direction.

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