3-D Depth Migration in VTI Media with Explicit Extrapolation Operators

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Summary

We present a space-frequency domain 3-D depth migration scheme for imaging transversely isotropic media with a vertical axis of symmetry (VTI). The anisotropy is described in terms of Thomsen parameters; however, it can accommodate a wide range of anisotropy rather than the weak anisotropy. In the proposed scheme the 3-D wavefield depth extrapolation is implemented by incorporating optimum 1-D spatial convolution operators with McClellan transformations. A high-accuracy, 1-D short convolution operator for VTI media is designed by combining the weighted least squares and weighted quadratic programming methods; and McClellan transformation operators are extended to handle unequal inline and crossline spacings. The proposed scheme can account for anisotropy without additional computational cost in comparison with the isotropic case. The performance of the algorithm is demonstrated by 3-D migration impulse responses.

Introduction

Many sedimentary rocks exhibit anisotropy. Furthermore, these sedimentary rocks can be described, to a good approximation, as being transversely isotropic (TI) with a symmetry axis perpendicular to the bedding plane. Failure to account for anisotropy in migration algorithms may lead to large position errors or a complete loss of steeply dipping structures. Authors such as Ristow (1999), Uzcategui (1995), and Zhang et al. (2001) have proposed algorithms by which 2-D TI media may be imaged. Several authors also have developed 3-D migration algorithms for anisotropic media. Ristow (1998) suggested a 3-D implicit finite-difference depth migration scheme based on multi-way splitting algorithms. Ball (1995) and Nolte et al. (2000) implemented anisotropic 3-D depth migration on field data.

Explicit depth extrapolation methods can correctly image arbitrarily dipping reflectors even in the presence of severe lateral velocity variations for 3-D isotropic media (Hale, 1991a). McClellan transformations offer a computationally efficient scheme for 3-D explicit depth extrapolation by implementing 2-D circularly symmetric spatial convolutions with 1-D convolution operators. Since the wavefield extrapolation operators keep the circular symmetry in VTI media, we can still use McClellan transformations to reduce the computational cost of 3-D explicit depth extrapolations in VTI media. Thus the anisotropic effects are only related to the design of the 1-D convolution operator. Besides McClellan transformation operators (which are entirely the same as those in the isotropic case), the accuracy and computational cost of the 3-D explicit depth extrapolation depend on how well the 1-D operators approximate the circularly symmetric anisotropic phase-shift operators in the radial direction over the propagation region, and how many points we use to design the desired 1-D operators. In this paper, we design the 1-D operators by combining the weighted least squares and weighted quadratic programming methods following Zhang et al. (2001). More attention is paid to reduce the length of the 1-D operators or accommodate the sparse spatial sampling by controlling the maximum angle of propagation in the design of 1-D operators. On the McClellan transformation operators, we follow Hale's (1991a) work and further make a modification to account for unequal inline and crossline spacings.

Phase-shift extrapolation operator

For VTI media the phase-velocity of *qP* wave can be expressed exactly in Thomsen notation (Tsvankin,1996) as

$$V^2(\theta)/V_{P0}^2 = 1+\varepsilon\sin^2\theta - f/2 + f/2\sqrt{\left(1+2\varepsilon\sin^2\theta/f\right)^2 - 2(\varepsilon-\delta)\sin^2 2\theta/f}, \quad (1)$$

where $f = 1 - V_{S0}^2/V_{P0}^2$, V_{P0} and V_{S0} are, respectively, the *qP* and *qSV* wave velocities in the vertical direction, and ε and δ are the Thomsen parameters. Note that eq.(1) is valid for both weak and strong anisotropy. The phase angle θ is given by

$$\sin\theta = V(\theta)k_r/\omega, \qquad \cos\theta = V(\theta)k_z/\omega, \quad (2)$$

where $k_r = \sqrt{k_x^2+k_y^2}$. Substituting eq.(2) into eq.(1) yields

$$(1-f)k_z^4 + bk_z^2 + c = 0, \quad (3)$$

where

$$b = 2(1-f+\varepsilon-f\delta)k_r^2 - (2-f)\,\omega^2/V_{P0}^2,$$

$$c = \omega^4/V_{P0}^4 + (1-f)(1+2\varepsilon)k_r^4 - (2-f+2\varepsilon)k_r^2\,\omega^2/V_{P0}^2.$$

The roots of eq.(3), which are related to the downward and upward going *qP* waves, are given by

$$k_z^2 = \left(-b-\sqrt{b^2-4(1-f)c}\right)\Big/(2-2f). \quad (4)$$

The solution of k_z with a positive real part (or negative imaginary part) will provide an exact circularly symmetric forward phase-shift extrapolation operator for downgoing waves as

$$\tilde{W}^+\left(k_r,\omega/V_{P0},\varepsilon,\delta,V_{S0}/V_{P0}\right) = \exp\left(-jk_z\Delta z\right). \quad (5)$$

Thus the inverse phase-shift extrapolation (downward continuation) operator for upgoing waves is obtained as

$$\tilde{F}^-(k_r,\cdots) \approx \left[\tilde{W}^+\left(k_r,\omega/V_{P0},\varepsilon,\delta,V_{S0}/V_{P0}\right)\right]^*, \quad (6)$$

where the superscript * denotes the complex conjugate, and the superscripts + and – refer to downgoing and upgoing, respectively.

Design of 1-D convolution operators

McClellan transformations offer a computationally efficient scheme to implement 2-D circularly symmetric convolutions with 1-D convolution operators. Hence, it becomes the key of the 3-D explicit depth extrapolation in VTI media to find a 1-D short convolution operator such that its spatial Fourier transform matches the radial slice of 2-D circularly symmetric phase-shift operators, such as the operators in eq.(6) or eq.(5), in the required range of angles of propagation as accurate as possible and its amplitudes outside this propagation region are less than unity. The nonlinear least-squares (Holberg, 1988) and Taylor series expansion (Hale, 1991b) methods have been proposed to design the 1-D convolution operators. However, Holberg's method has a high computational cost, and Hale's is more efficient but less accurate (whose errors are unnecessarily small for small wavenumbers and grow rapidly with increasing wavenumbers). The weighted least-squares method (Thorbecke and Rietveld, 1994), although without constraints on the amplitudes, can derive a stable, high accurate 1-D convolution operator with a very low computational cost. Furthermore, this method is advanced into the weighted quadratic programming method (Zhang et al., 2001) by introducing linear constraints on the amplitudes both in and outside the propagation region to guarantee the amplitudes less than a given maximum value, such as 1.001. In this paper, we design the 1-D operators by combining the weighted least squares and weighted quadratic programming methods, that is to find a short operator to match the phase-shift operator in eq.(6) in the required range of angles of propagation by first the weighted least-squares method and then the weighted quadratic programming method if the former fails to make the amplitudes less than a given value.

One of the attractive qualities of the explicit depth extrapolation with the convolution operators is that the maximum angle of propagation can be controlled in the design of the operators. This provides a good tool to suppress the aliasing due to sparse spatial sampling. For a fixed spatial sampling interval Δx, the maximum angle of propagation can be determined from the following,

$$\pi^2 V_{P0}^2/(\Delta x^2 \omega_{\max}^2) \geq \sin^2\theta/[1+\varepsilon\sin^2\theta - f/2 + f/2\sqrt{(1+2\varepsilon\sin^2\theta/f)^2 - 2(\varepsilon-\delta)\sin^2 2\theta/f}], \quad (7)$$

where $\omega_{\max}$ is the maximum frequency of the seismic data. To design the convolution operators with the required range of angles of propagation satisfying eq.(7), we can suppress the aliasing effectively.

The design method of the operators that we use can derive a very short, highly accurate operator at the cost of decreasing the maximum angle of propagation. This also provides a good way to reduce the computational cost of the 3-D depth extrapolation. Fig.1 illustrates the behavior of the proposed design method.

3-D explicit depth extrapolation

The 1-D convolution operator can be expressed in the wavenumber domain as

$$\tilde{H}(k_r) = h_0 + 2\sum_{i=1}^{n} h_i \cos(ik_r\Delta x), \quad (8)$$

where $h_i\,(i=0,n)$ are the coefficients of the short operator, and n is the number of points that we use to design this short operator. Taking the 2-D inverse Fourier transform of eq.(8) with respect to k_x and k_y yields

$$H(x,y) = h_0\delta(x,y) + 2\sum_{i=1}^{n} h_i \Re^{-1}(\cos(i\sqrt{k_x^2+k_y^2}\Delta x)). \quad (9)$$

Thus the wavefield explicit depth extrapolation in the space-frequency domain can be expressed as

$$P(x,y,z+\Delta z,\omega) = h_0 P(x,y,z,\omega) + 2\sum_{i=1}^{n} h_i P(x,y,z,\omega) * \Re^{-1}(\cos(i\sqrt{k_x^2+k_y^2}\Delta x)), \quad (10)$$

where the operator $\Re^{-1}$ denotes the inverse Fourier transform, and the sign * denotes the convolution. Define

$$P(x,y,z,\omega) * \Re^{-1}(\cos(i\sqrt{k_x^2+k_y^2}\Delta x)) = g_i$$

and

$$\Re^{-1}(\cos(\sqrt{k_x^2+k_y^2}\Delta x)) = G.$$

Based on the following trigonometric identity

$$\cos(ik_r\Delta x) = 2\cos((i-1)k_r\Delta x)\cos(k_r\Delta x) - \cos((i-2)k_r\Delta x), \quad (11)$$

we have recursive relations as follows:

$$g_i = 2g_{i-1} * G - g_{i-2}, \quad i = 2,n, \qquad g_1 = P(x,y,z,\omega) * G, \quad g_0 = P(x,y,z,\omega). \quad (12)$$

Thus the wavefield depth extrapolation can be rewritten as

$$P(x,y,z+\Delta z,\omega) = h_0 g_0 + 2\sum_{i=1}^{n} h_i g_i. \quad (13)$$

Eqs (12) and (13) propose an algorithm to carry out 3-D depth extrapolation with 1-D convolution operators. Now the key is how to implement the function convolving with the operator G.

McClellan (Hale, 1991a) suggested an approximation

$$\cos(\sqrt{k_x^2+k_y^2}\Delta x) \approx -1 + 0.5(1+\cos(k_x\Delta x))(1+\cos(k_y\Delta x)). \quad (14)$$

From eq.(14) we have

$$G(x,y) = 1/8(\delta_{-1,-1} + \delta_{1,-1} + \delta_{1,1} + \delta_{-1,1}) + 1/4(\delta_{-1,0} + \delta_{1,0} + \delta_{0,1} + \delta_{0,-1}) - 1/2\delta_{0,0}, \quad (15)$$

where $\delta_{i,j} = \delta(x+i\Delta x, y+j\Delta x)$. Thus the function convolving with G can be achieved with the differences of this function. Hale (1991a) proposed another approximation of $\cos(k_r\Delta x)$ to reduce the errors in eq.(14), which leads to a high-order difference in the convolution with G. This accuracy increase is got at an increase in the computational cost. Blacquiere (1991) uses an optimization procedure to further improve this approximation.

Foregoing discussion is based on equal inline and crossline spacings, i.e. $\Delta x = \Delta y$. Substituting the approximation

$$P * (\delta_{i,1} + \delta_{i,-1} - 2\delta_{i,0}) \approx \Delta x^2/\Delta y^2\, P * (\bar{\delta}_{i,1} + \bar{\delta}_{i,-1} - 2\bar{\delta}_{i,0}),$$

where $\bar{\delta}_{i,j} = \delta(x + i\Delta x, y + j\Delta y)$, into eq.(15) we have

$$G(x,y) = r/8(\bar{\delta}_{-1,-1} + \bar{\delta}_{1,-1} + \bar{\delta}_{1,1} + \bar{\delta}_{-1,1}) + (2-r)/4 \quad (16)$$
$$(\bar{\delta}_{-1,0} + \bar{\delta}_{1,0}) + r/4(\bar{\delta}_{0,1} + \bar{\delta}_{0,-1}) - r/2\bar{\delta}_{0,0},$$

where $r = \Delta x^2 / \Delta y^2$. Thus the algorithm can directly apply to unequal inline and crossline spacings.

Examples

The algorithm is developed based on pre-calculating an anisotropic 1-D operator table following Zhang et al. (2001) and then picking the space-variant operators during the depth extrapolation process. Two impulse responses for weak and strong anisotropy are illustrated in Figs 2 and 3, where we use the original McClellan transformations and the 12-point 1-D operators designed for a maximum propagation angle of 60^0 (as the thin lines shown in Fig.1) with $\Delta x = \Delta y = 30m$ and $\Delta z = 15m$. Fig.4 shows the impulse response of the same case as Fig.2 except that the 9-point operators designed for a maximum propagation angle of 44^0 (as the dashed lines shown in Fig.1) are used to handle the sparse spatial sampling of $\Delta x = \Delta y = 40m$ with $\Delta z = 15m$. The comparison of the impulse responses for the equal and unequal inline and crossline spacings is shown in Fig.5, where the 3:2 spacing ratio is used for the unequal spacing case and 12-point 1-D operators are employed in both impulse responses.

Conclusions

We have presented a table-driven anisotropic depth migration scheme for imaging 3-D heterogeneous media. Similar to the isotropic explicit finite-difference methods, the proposed scheme can correctly image steeply dipping structures. A high-accuracy 1-D short convolution operator is introduced by using new design methods of operators. This results in good imaging quality, as shown in the impulse responses. Besides using McClellan transformations, the computational cost is further reduced by designing a short 1-D operator with smaller maximum angle of propagation. Also we can suppress the aliasing due to sparse spatial sampling by properly designing the 1-D operators. Furthermore, we propose a low-cost modification to the McClellan transformation operators that can be applied directly for unequal inline and crossline spacings while retaining much of the speed and quality, as demonstrated by the comparison of the impulse responses.

Acknowledgements

The first author J. Zhang thanks the Netherlands Research Centre for Integrated Solid Earth Science for supporting his research fellowship at Delft University of Technology.

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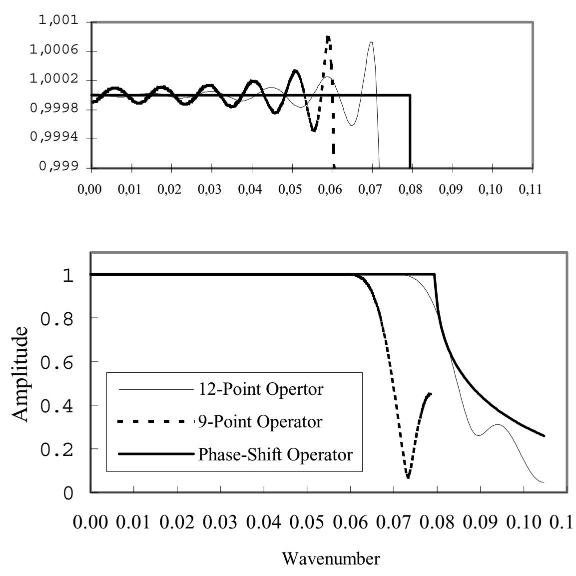

Figure 1. Comparison of the wavenumber spectra between convolution operators and exact phase-shift operator. The thin lines denote the 12-point operator with a maximum propagation angle of 60^0 and $\Delta x = 30m$, the dashed lines the 9-point operator with a maximum propagation angle of 44^0 and $\Delta x = 40m$. The top shows a detailed view around 1.0.

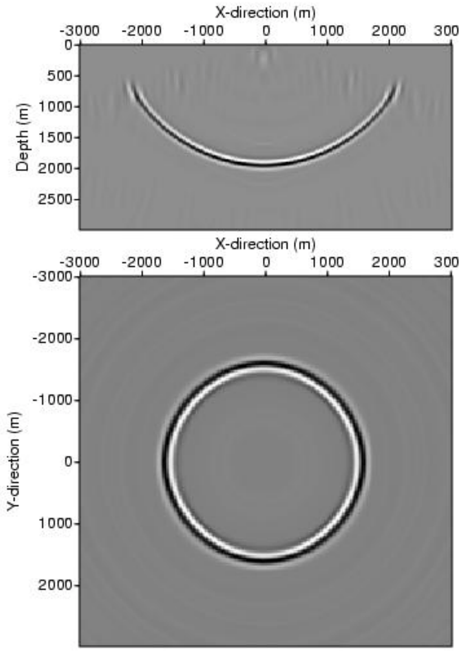

Figure 2. 3-D migration impulse response of weak anisotropy. The Thomsen parameters are $\varepsilon = 0.2$ and $\delta = 0.1$ with $V_{P0} = 2000m/s$ and $V_{S0} = 1000m/s$. The top is for crossline coordinate $y = 0$, and the bottom is a depth slice for the depth $z = 1350m$ with $\Delta z = 15m$.

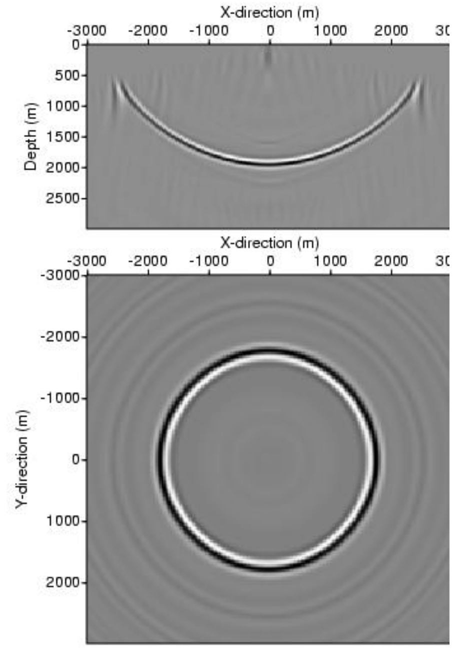

Figure 3. Result of the strong anisotropy. The impulse response is for the same case as that in Fig.2, except that the Thomsen parameters are changed as $\varepsilon = 0.4$ and $\delta = 0.2$. The top is for crossline coordinate $y = 0$, and the bottom is a depth slice for the depth $z = 1350m$.

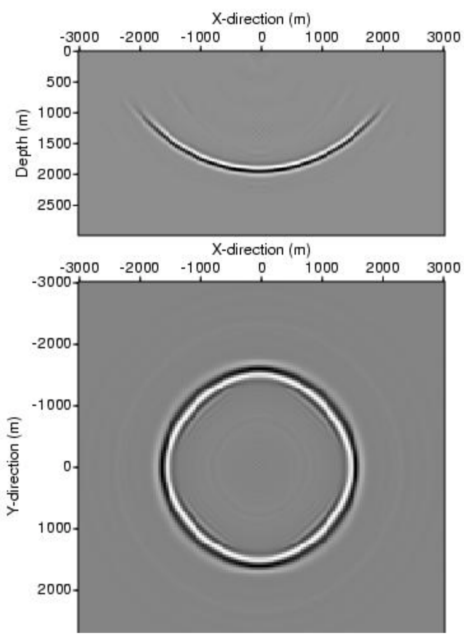

Figure 4. Result of short operators. The impulse response is same as that in Fig.2, except that 9-point operators designed for a maximum propagation angle of 44^0 are used

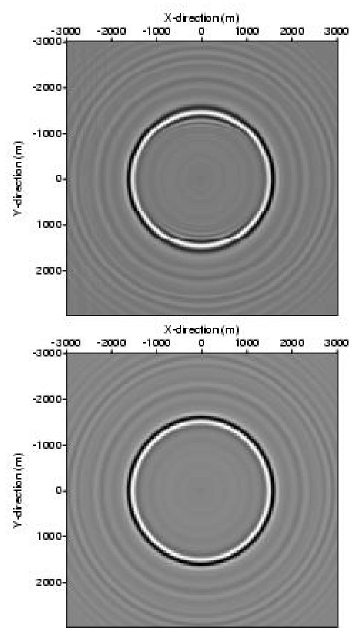

Figure 5. Comparison of depth slices at $z = 1350m$ with $\Delta z = 15m$ for the equal and unequal grid spacings. The top is for $\Delta x \times \Delta y = 20 \times 30$, and the bottom is for $\Delta x = \Delta y = 20m$. The physical parameters are same as those for Fig.2