Convolution type interaction of time-lapse acoustic wave fields

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Summary

By calculating a convolution type integral involving two time-lapse wave fields, at a certain depth, one obtains a representation, which is equivalent to a difference wave field at the recording level. This difference wave field shows no difference reflections from temporal contrasts above the interaction depth. If the wave fields, appearing in the interaction integral, are calculated correctly, the resulting integral then only generates difference reflections proportional to temporal contrasts below the interaction depth. A progressive elimination procedure of difference reflections with depth is shown by finite difference examples.

Wave field equations

We consider two time-lapse acoustic wave fields. The onsets of the reference and monitor wave fields are at the time instant $t^{(1)}$ and $t^{(2)}$, respectively. Between these time instants changes in the medium parameters may have occured. We assume that the time-lapse interval $t^{(2)} - t^{(1)}$ is much larger than the time increments at which discernable dynamic changes occur in the wave field quantities $\{p^{(1)}, v_k^{(1)}\}$ and $\{p^{(2)}, v_k^{(2)}\}$. We also assume that during these latter time increments the reference and monitor acoustic medium parameters density and compressibility, $\{\rho^{(1)}, \kappa^{(1)}\}$ and $\{\rho^{(2)}, \kappa^{(2)}\}$, may be approximated by constant functions of time. Applying a Fourier transformation with respect to time, we consider the acoustic reference wave field equations,

$$\partial_k p^{(1)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}},\omega\right) + j\omega\rho^{(1)}\left(\mathbf{x}\right) v_k^{(1)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}},\omega\right) = f_k^{(1)}\left(\omega\right)\delta\left(\mathbf{x}-\mathbf{x}^{\mathrm{S}}\right),$$

$$\partial_k v_k^{(1)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}},\omega\right) + j\omega\kappa^{(1)}\left(\mathbf{x}\right) p^{(1)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}},\omega\right) = q^{(1)}\left(\omega\right)\delta\left(\mathbf{x}-\mathbf{x}^{\mathrm{S}}\right), \quad (1)$$

and the acoustic monitor wave field equations,

$$\partial_k p^{(2)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}},\omega\right) + j\omega\rho^{(2)}\left(\mathbf{x}\right) v_k^{(2)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}},\omega\right) = f_k^{(2)}\left(\omega\right)\delta\left(\mathbf{x}-\mathbf{x}^{\mathrm{S}}\right),$$

$$\partial_k v_k^{(2)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}},\omega\right) + j\omega\kappa^{(2)}\left(\mathbf{x}\right) p^{(2)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}},\omega\right) = q^{(2)}\left(\omega\right)\delta\left(\mathbf{x}-\mathbf{x}^{\mathrm{S}}\right), \quad (2)$$

in which $\mathbf{x} \in \mathcal{R}^3$ and ω is the angular frequency. $\mathcal{R}^3$ is divided by the planar surface, $x_3 = x_3^1$, into an upper half-space $\mathcal{D}^{\mathrm{t}}$, for which $x_3 < x_3^1$, and a lower half-space $\mathcal{D}^{\mathrm{b}}$, for which $x_3 > x_3^1$. The source distributions with source functions, $\{q^{(1)}, f_k^{(1)}\}$ and $\{q^{(2)}, f_k^{(2)}\}$, in Eqs. (1) and (2), are located at $\mathbf{x}^{\mathrm{S}} \in \mathcal{D}^{\mathrm{t}}$. Rewriting Eqs. (2) yields

$$\partial_k p^{(2)} + j\omega\rho^{(1)} v_k^{(2)} = -j\omega\Delta^{\mathrm{tl}}\rho v_k^{(2)} + f_k^{(2)}\delta\left(\mathbf{x}-\mathbf{x}^{\mathrm{S}}\right),$$
$$\partial_k v_k^{(2)} + j\omega\kappa^{(1)} p^{(2)} = -j\omega\Delta^{\mathrm{tl}}\kappa p^{(2)} + q^{(2)}\delta\left(\mathbf{x}-\mathbf{x}^{\mathrm{S}}\right), \quad (3)$$

with temporal contrast functions given by

$$\Delta^{\mathrm{tl}}\rho = \rho^{(2)} - \rho^{(1)} \quad \text{and} \quad \Delta^{\mathrm{tl}}\kappa = \kappa^{(2)} - \kappa^{(1)}. \quad (4)$$

By subtracting Eqs. (1) from Eqs. (3) we construct the following difference wave field equations

$$\partial_k p^{\mathrm{dif}} + j\omega\rho^{(1)} v_k^{\mathrm{dif}} = f_k^{\mathrm{dif}}, \quad \text{in} \quad \mathcal{R}^3,$$
$$\partial_k v_k^{\mathrm{dif}} + j\omega\kappa^{(1)} p^{\mathrm{dif}} = q^{\mathrm{dif}}, \quad \text{in} \quad \mathcal{R}^3, \quad (5)$$

in which the difference wave field quantities are given by

$$p^{\mathrm{dif}} = p^{(2)} - p^{(1)} \quad \text{and} \quad v_k^{\mathrm{dif}} = v_k^{(2)} - v_k^{(1)}. \quad (6)$$

The difference source quantities in Eqs. (5) are obtained as

$$f_k^{\mathrm{dif}} = -j\omega\Delta^{\mathrm{tl}}\rho v_k^{(2)} + \Delta^{\mathrm{tl}} f_k,$$
$$q^{\mathrm{dif}} = -j\omega\Delta^{\mathrm{tl}}\kappa p^{(2)} + \Delta^{\mathrm{tl}} q, \quad (7)$$

in which

$$\Delta^{\mathrm{tl}} f_k = \left(f_k^{(2)} - f_k^{(1)}\right)\delta\left(\mathbf{x}-\mathbf{x}^{\mathrm{S}}\right),$$
$$\Delta^{\mathrm{tl}} q = \left(q^{(2)} - q^{(1)}\right)\delta\left(\mathbf{x}-\mathbf{x}^{\mathrm{S}}\right). \quad (8)$$

Next, we assume the following decomposition of the difference wave field quantities

$$p^{\mathrm{dif}} = p^{\mathrm{dif,d}} + p^{\mathrm{dif,u}}, \quad \text{in} \quad \mathcal{R}^3$$
$$v_k^{\mathrm{dif}} = v_k^{\mathrm{dif,d}} + v_k^{\mathrm{dif,u}}, \quad \text{in} \quad \mathcal{R}^3. \quad (9)$$

The wave field components $\{p^{\mathrm{dif,d}}, v_k^{\mathrm{dif,d}}\}$ are governed by the wave field equations,

$$\partial_k p^{\mathrm{dif,d}} + j\omega\rho^{(1)} v_k^{\mathrm{dif,d}} = f_k^{\mathrm{dif,d}}, \quad \text{in} \quad \mathcal{R}^3,$$
$$\partial_k v_k^{\mathrm{dif,d}} + j\omega\kappa^{(1)} p^{\mathrm{dif,d}} = q^{\mathrm{dif,d}}, \quad \text{in} \quad \mathcal{R}^3, \qquad (10)$$

with source quantities

$$\{f_k^{\mathrm{dif,d}}, q^{\mathrm{dif,d}}\} = \{f_k^{\mathrm{dif}}, q^{\mathrm{dif}}\}, \qquad \text{in} \quad \mathcal{D}^{\mathrm{t}},$$
$$\{f_k^{\mathrm{dif,d}}, q^{\mathrm{dif,d}}\} = \{0, 0\}, \qquad \text{in} \quad \mathcal{D}^{\mathrm{b}}. \qquad (11)$$

The wave field equations of the wave field components $\{p^{\mathrm{dif,u}}, v_k^{\mathrm{dif,u}}\}$ are given by

$$\partial_k p^{\mathrm{dif,u}} + j\omega\rho^{(1)} v_k^{\mathrm{dif,u}} = f_k^{\mathrm{dif,u}}, \quad \text{in} \quad \mathcal{R}^3,$$
$$\partial_k v_k^{\mathrm{dif,u}} + j\omega\kappa^{(1)} p^{\mathrm{dif,u}} = q^{\mathrm{dif,u}}, \quad \text{in} \quad \mathcal{R}^3, \qquad (12)$$

with source quantities

$$\{f_k^{\mathrm{dif,u}}, q^{\mathrm{dif,u}}\} = \{0, 0\}, \qquad \text{in} \quad \mathcal{D}^{\mathrm{t}},$$
$$\{f_k^{\mathrm{dif,u}}, q^{\mathrm{dif,u}}\} = \{f_k^{\mathrm{dif}}, q^{\mathrm{dif}}\}, \qquad \text{in} \quad \mathcal{D}^{\mathrm{b}}. \qquad (13)$$

Observe that the difference wave fields defined by Eqs. (5), (10) and (12) propagate in the reference medium. The decomposition of Eqs. (9) is invoked by choosing different supports for the temporal contrast functions of Eqs. (7).

Reciprocity theorem

In this section the acoustic reciprocity theorem of the convolution type is used. This theorem has its roots in Green's theorem for Laplace's equation and Helmholtz's extension to the wave equation. A reciprocity theorem interrelates the wave field quantities of two admissible states, A and B, that occur in one and the same time-invariant domain $\mathcal{D} \subset \mathcal{R}^3$ [1]. In forward/inverse source and scattering problems one state is identified with a physical wave field while the other state is identified with a computational or so called Green's wave field. In time-lapse problems both states are identified with physical wave fields. The respective wave field equations for State A and State B are given by

$$\partial_k p^{A,B}(\mathbf{x},\omega) + j\omega\rho^{A,B}(\mathbf{x})\, v_k^{A,B}(\mathbf{x},\omega) = f_k^{A,B}(\mathbf{x},\omega),$$
$$\partial_k v_k^{A,B}(\mathbf{x},\omega) + j\omega\kappa^{A,B}(\mathbf{x})\, p^{A,B}(\mathbf{x},\omega) = q^{A,B}(\mathbf{x},\omega). \qquad (14)$$

The reciprocity theorem of the convolution type (in the time-domain the multiplications represent convolutions) is obtained as [2],

$$\int_{\mathbf{x}\,\in\,\partial\mathcal{D}} \left(v_k^A p^B - p^A v_k^B\right) \nu_k \,\mathrm{d}A$$
$$+\int_{\mathbf{x}\,\in\,\mathcal{D}} j\omega \left[\left(\rho^B - \rho^A\right) v_k^A v_k^B - \left(\kappa^B - \kappa^A\right) p^A p^B \right] \mathrm{d}V$$
$$= \int_{\mathbf{x}\,\in\,\mathcal{D}} \left(f_k^B v_k^A - f_k^A v_k^B + q^A p^B - q^B p^A\right) \mathrm{d}V, \qquad (15)$$

in which the normal ν_k is pointing outward $\mathcal{D}$. Using the reference and monitor wave fields of Eqs. (1) and (2), taking the source position of the reference wave field to be $\mathbf{x}^{\mathrm{R}}$ instead of $\mathbf{x}^{\mathrm{S}}$, and using only q sources, application of the reciprocity theorem yields, omitting ω,

$$\int_{\mathbf{x}\,\in\,\partial\mathcal{D}} \Big[v_k^{(1)}\left(\mathbf{x};\mathbf{x}^{\mathrm{R}}\right) p^{(2)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}}\right) - p^{(1)}\left(\mathbf{x};\mathbf{x}^{\mathrm{R}}\right) v_k^{(2)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}}\right) \Big] \mathrm{d}A$$
$$+ \int_{\mathbf{x}\,\in\,\mathcal{D}} j\omega \Big[\Delta^{\mathrm{tl}}\rho v_k^{(1)}\left(\mathbf{x};\mathbf{x}^{\mathrm{R}}\right) v_k^{(2)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}}\right) - \Delta^{\mathrm{tl}}\kappa p^{(1)}\left(\mathbf{x};\mathbf{x}^{\mathrm{R}}\right) p^{(2)}\left(\mathbf{x};\mathbf{x}^{\mathrm{S}}\right) \Big] \mathrm{d}V$$
$$= q^{(1)} p^{(2)}\left(\mathbf{x}^{\mathrm{R}};\mathbf{x}^{\mathrm{S}}\right) - q^{(2)} p^{(1)}\left(\mathbf{x}^{\mathrm{R}};\mathbf{x}^{\mathrm{S}}\right). \qquad (16)$$

Hence, the sum of a boundary integral and a volume integral of temporal contrast sources is equivalent to a difference measurement.

Interaction integral

Taking the boundary integral of Eq. (16) we define the following convolution type interaction integral,

$$\hat{I}^{\mathrm{conv}}\left(x_3^1;\mathbf{x}^{\mathrm{R}},\mathbf{x}^{\mathrm{S}}\right) \stackrel{\mathrm{def}}{=} \int_{\mathbf{x}_T\,\in\,\mathcal{R}^2} \Big[v_3^{(1)}\left(\mathbf{x}_T, x_3^1;\mathbf{x}^{\mathrm{R}}\right) p^{(2)}\left(\mathbf{x}_T, x_3^1;\mathbf{x}^{\mathrm{S}}\right) - p^{(1)}\left(\mathbf{x}_T, x_3^1;\mathbf{x}^{\mathrm{R}}\right) v_3^{(2)}\left(\mathbf{x}_T, x_3^1;\mathbf{x}^{\mathrm{S}}\right) \Big] \mathrm{d}\mathbf{x}_T. \qquad (17)$$

The integration is with respect to the transverse coordinate $\mathbf{x}_T = (x_1, x_2)$, at a depth x_3^1. Consider the wave fields, $\{p^{(1)}, v_k^{(1)}\}(\mathbf{x};\mathbf{x}^{\mathrm{R}})$ and $\{p^{(1)}, v_k^{(1)}\}(\mathbf{x};\mathbf{x}^{\mathrm{S}})$, which differ with respect to their source positions. Applications of the reciprocity theorem of Eq. (15) to these wave fields, with respect to the domain $\mathcal{D}^{\mathrm{b}}$, leads to

$$0 = \int_{\mathbf{x}_T\,\in\,\mathcal{R}^2} \Big[v_3^{(1)}\left(\mathbf{x}_T, x_3^1;\mathbf{x}^{\mathrm{R}}\right) p^{(1)}\left(\mathbf{x}_T, x_3^1;\mathbf{x}^{\mathrm{S}}\right) - p^{(1)}\left(\mathbf{x}_T, x_3^1;\mathbf{x}^{\mathrm{R}}\right) v_3^{(1)}\left(\mathbf{x}_T, x_3^1;\mathbf{x}^{\mathrm{S}}\right) \Big] \mathrm{d}\mathbf{x}_T. \qquad (18)$$

In the derivation we have taken into account that contribution at $(x_1^2 + x_2^2) \to \infty$ and $x_3 \to \infty$ vanish. Because there is no contrast in the medium parameters between the two states, and inside the domain of application, $\mathcal{D}^{\mathrm{b}}$, both states have no sources, the two volume intgrals of Eq. (15) also vanish in Eq. (18). Subtracting Eq. (18) from Eq. (17), using Eq. (6), yields

$$I^{\mathrm{conv}}\left(x_3^1; \mathbf{x}^{\mathrm{R}}, \mathbf{x}^{\mathrm{S}}\right) = \int_{\mathbf{x}_T \in \mathcal{R}^2} \Big[v_3^{(1)}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{R}}\right) p^{\mathrm{dif}}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{S}}\right) - p^{(1)}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{R}}\right) v_3^{\mathrm{dif}}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{S}}\right) \Big] \mathrm{d}\mathbf{x}_T. \quad (19)$$

Next, consider the wave fields, $\{p^{(1)}, v_k^{(1)}\}(\mathbf{x}; \mathbf{x}^{\mathrm{R}})$ and $\{p^{\mathrm{dif,d}}, v_k^{\mathrm{dif,d}}\}(\mathbf{x}; \mathbf{x}^{\mathrm{S}})$, the latter wave field being governed by Eqs. (10). Applications of the reciprocity theorem of Eq. (15) to these wave fields, with respect to the domain $\mathcal{D}^{\mathrm{b}}$, leads to

$$0 = \int_{\mathbf{x}_T \in \mathcal{R}^2} \Big[v_3^{(1)}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{R}}\right) p^{\mathrm{dif,d}}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{S}}\right) - p^{(1)}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{R}}\right) v_3^{\mathrm{dif,d}}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{S}}\right) \Big] \mathrm{d}\mathbf{x}_T. \quad (20)$$

Subtracting Eq. (20) from Eq. (19), using Eq. (9), yields

$$I^{\mathrm{conv}}\left(x_3^1; \mathbf{x}^{\mathrm{R}}, \mathbf{x}^{\mathrm{S}}\right) = \int_{\mathbf{x}_T \in \mathcal{R}^2} \Big[v_3^{(1)}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{R}}\right) p^{\mathrm{dif,u}}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{S}}\right) - p^{(1)}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{R}}\right) v_3^{\mathrm{dif,u}}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{S}}\right) \Big] \mathrm{d}\mathbf{x}_T. \quad (21)$$

Taking the wave fields, $\{p^{(1)}, v_k^{(1)}\}(\mathbf{x}; \mathbf{x}^{\mathrm{R}})$ and $\{p^{\mathrm{dif,u}}, v_k^{\mathrm{dif,u}}\}(\mathbf{x}; \mathbf{x}^{\mathrm{S}})$, the latter wave field being governed by Eqs. (12), and applying Eq. (15) to these wave fields, with respect to the domain $\mathcal{D}^{\mathrm{t}}$, gives

$$q^{(1)} p^{\mathrm{dif,u}}\left(\mathbf{x}^{\mathrm{R}}, \mathbf{x}^{\mathrm{S}}\right) = \int_{\mathbf{x}_T \in \mathcal{R}^2} \Big[v_k^{(1)}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{R}}\right) p^{\mathrm{dif,u}}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{S}}\right) - p^{(1)}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{R}}\right) v_k^{\mathrm{dif,u}}\left(\mathbf{x}_T, x_3^1; \mathbf{x}^{\mathrm{S}}\right) \Big] \mathrm{d}\mathbf{x}_T. \quad (22)$$

Using Eq. (21) and (22) the interaction quantity is expressed as

$$I^{\mathrm{conv}}\left(x_3^1; \mathbf{x}^{\mathrm{R}}, \mathbf{x}^{\mathrm{S}}\right) = q^{(1)} p^{\mathrm{dif,u}}\left(\mathbf{x}^{\mathrm{R}}, \mathbf{x}^{\mathrm{S}}\right). \quad (23)$$

We arrive at a representation of I^{conv} in terms of the component difference wave field $p^{\mathrm{dif,u}}$. This wave field has contrast sources depending on the temporal contrasts inside $\mathcal{D}^{\mathrm{b}}$. Temporal contrasts inside $\mathcal{D}^{\mathrm{t}}$ do not generate difference reflections in the difference gather $p^{\mathrm{dif,u}}$, and hence in I^{conv}. If I^{conv} is calculated incorrectly residual difference reflection energy from temporal contrasts inside $\mathcal{D}^{\mathrm{t}}$ will appear. Hence, minimizing this energy should give the correct reference and monitor medium parameters inside $\mathcal{D}^{\mathrm{t}}$. The reference and monitor two-way wave fields appearing in I^{conv} can be calculated from data by applying a wave field decomposition, into down-going and up-going one-way wave fields, at the recording level [3].

Numerical example

We consider the model in Figure (1)*t* with the reference and monitor velocities and densities given in Table (1). With a finite difference code two acoustic time-lapse wave fields are simulated. The source is placed in the middle at the top of the model at (0,0) m. The receivers are placed at 0 m depth. Figure (1)*b* shows a difference gather p^{dif}, obtained by subtracting a single monitor shot-gather from a single reference shot-gather. Next, a number of reference shot-gathers, with different lateral source positions $\mathbf{x}_T^{\mathrm{R}}$, at $\mathbf{x}_3^{\mathrm{R}} = 0\,\mathrm{m}$, with receivers at $x_3 = x_3^1$, are modeled. Also, a single monitor shot-gather with source position, $(\mathbf{x}_T^{\mathrm{S}}, \mathbf{x}_3^{\mathrm{S}}) = (0, 0)$, and also with receivers at $x_3 = x_3^1$, is modeled. Using these shot-gathers we calculate the interaction integral of Equation (17) at $x_3^1 = 400\,\mathrm{m}$ and $x_3^1 = 650\,\mathrm{m}$. The resulting time-domain integrals $I^{\mathrm{conv}}(x_3; \mathbf{x}_T^{\mathrm{R}}, x_3^{\mathrm{R}}, \mathbf{x}_T^{\mathrm{S}}, x_3^{\mathrm{S}})$ are shown in Figure (2)*l* and (2)*r*, respectively. To test if I^{conv} represents a difference measurement $p^{\mathrm{dif,u}}$, the monitor model is changed such that it equals the reference model for $x_3 < 400\,\mathrm{m}$ (halfway the diamond-shaped object). The resulting dif-

	$c^{(1)}$ [m/s]	$\rho^{(1)}$ [kg/m^3]
background	1800	1500
diamond-shaped object	2500	2000
lower layer	2700	2300
	$c^{(2)}$ [m/s]	$\rho^{(2)}$ [kg/m^3]
background	1800	1500
diamond-shaped object	2700	2200
lower layer	2900	2400

Table 1: *Reference and monitor velocities and densities,* $c^{(1)}$,$\rho^{(1)}$ *and* $c^{(2)}$ *and* $\rho^{(2)}$.

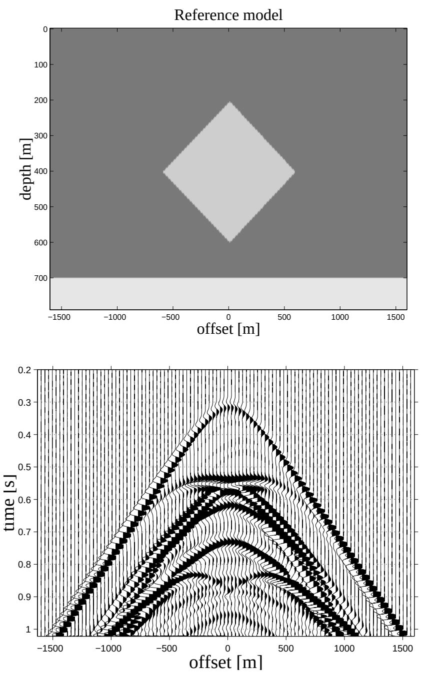

Fig. 1: *top (t): model; bottom (b): Difference wave field at* $x_3 = 0\,\mathrm{m}$

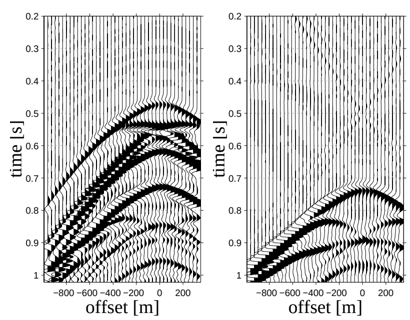

Fig. 2: *left (l): interaction integral at* $x_3 = 400\,\mathrm{m}$*; right (r): interaction integral at* $x_3 = 650\,\mathrm{m}$.

ference gather is shown in Figure (3)*t*. One observes that Fig. (3)*t* is very similar to Fig (2)*l*. We proceed by further altering the monitor model such that their is no temporal contrast in the diamond shaped-object. The only temporal contrast is in the lower layer. In Figure (3)*b* the difference gather is shown. The difference reflections associated with the temporal contrast in the diamond-shaped object, visible in Figure (1)*b* have disappeared. Note the similarity of Figure (3)*b* with Figure (2)*r*.

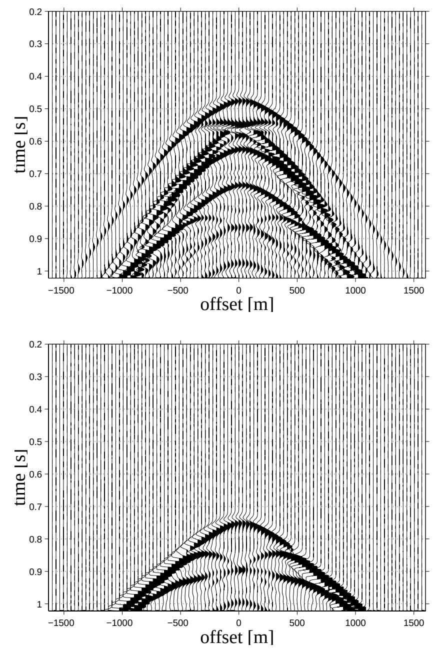

Fig. 3: *top (t): difference wave field at* $x_3 = 0\,\mathrm{m}$*, no temporal contrasts for* $x_3 < 400\,\mathrm{m}$*; bottom (b): difference wave field at* $x_3 = 0\,\mathrm{m}$*, no temporal contrasts for* $x_3 < 650\,\mathrm{m}$.

Conclusions

The interaction integral of Eq. (17) simulates an hypothetical difference measurement through media which show no temporal contrast above the interaction depth. The integral can be used as a forward model in time-lapse inverse problems by minimizing difference reflection energy. It is ideally suited to solve nonrepeatability problems by calculating it below the temporal contrasts at shallow depths.

References

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