Velocity replacement techniques in inhomogeneous media

Renée Smit, Jacob Fokkema, Evert Slob and Kees Wapenaar, Centre for Technical Geoscience, Delft Univ. of Techn.

Summary

In this paper we present a theory how, using Rayleigh's reciprocity theorem, an acoustic medium with velocity contrasts in the horizontal and vertical direction can successively be homogenized to the constant background velocity. The resulting expressions are formulated in terms of pseudo-differential operators that act on the original wavefield. The actual replacement is achieved by solving an integral equation of the second kind. The solution of the latter is obtained by means of a Neumann solution. In this way primaries as well as internal multiples are dealt with. The proposed replacement technique can be used in imaging procedures to obtain on each level a genuine decomposition into up- and downgoing wavefields. Furthermore on each level the reflection function can then be evaluated with respect to the same background medium.

Introduction

In the following analysis we consider the action of a point source of the injection type in an acoustic medium. The seismic medium response is recorded with a set of point receivers. We assume that the seismic preprocessing has resulted in a configuration model whereby the semi-infinite upper half-space is homogeneous in nature characterized by the acoustic wavespeed c_0. In this halfspace the point sources and receivers are positioned. The lower semi-infinite halfspace is our domain of interest and is occupied with an inhomogeneous medium characterized by the wavespeed $c(\mathbf{x})$. We further assume that there is no contrast in mass density and hence is constant throughout our whole configuration and consequently immaterial in our further analysis. In this paper it is our aim starting from the original configuration to successively replace the inhomogeneous lower halfspace in small steps with the homogeneous background material (velocity replacement) and subsequently determine the resulting wavefield in the modified configuration.

Contrast formulation

Our starting point is Rayleigh's reciprocity theorem formulated in the space-frequency domain with frequency parameter $s = j\omega$ (Fokkema and van den Berg [1]). The spatial domain of consideration is depicted in Figure 1. The left-hand side shows the starting configuration, which is denoted by state 0. The inhomogeneous medium is divided in the x_3-direction in thin slabs of width Δx_3 in which the wavespeed has only variation in the lateral direction. We indicate the wavespeed in the first layer with $c_1(\mathbf{x}_T)$, where $\mathbf{x}_T$ is the transverse position vector in the x_1, x_2-direction. On the right-hand side of Figure 1 we show the situation in state 1 where the inhomogeneous wavespeed $c_1(\mathbf{x}_T)$ in the first layer has been replaced by the wavespeed of the upper

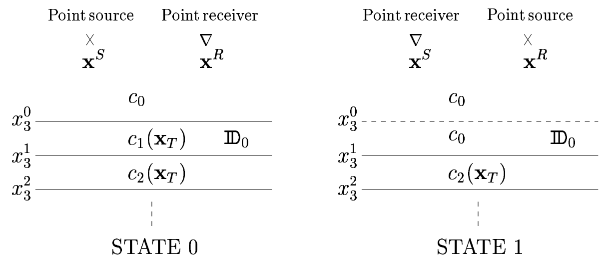

Fig. 1: *Model configuration.*

halfspace c_0. Then application of the reciprocity theorem to $\mathbb{R}^3$, considering the states 0 and 1, and reversing the positions of the sources and receivers in state 1 with respect to state 0 leads to the following interaction relation between the wave fields in the two states

$$\int_{\mathbf{x}\in\mathbb{D}^0} s^2 K(\mathbf{x}_T)\hat{P}^0(\mathbf{x}|\mathbf{x}^S)\hat{P}^1(\mathbf{x}|\mathbf{x}^R)\mathrm{d}V = \hat{W}\left[\hat{P}^0(\mathbf{x}^R|\mathbf{x}^S) - \hat{P}^1(\mathbf{x}^S|\mathbf{x}^R)\right], \tag{1}$$

where we have omitted the explicit dependence on s in the wavefields. The contrast function K is given by

$$K(\mathbf{x}_T) = \frac{1}{c_0^2} - \frac{1}{c_1^2(\mathbf{x}_T)}. \tag{2}$$

The wavefield in state 0, $\hat{P}^0$, can for $x_3 \leq x_3^0$ be decomposed into a downgoing incident wavefield $\hat{P}^{0,i}$ and an upgoing reflected wavefield $\hat{P}^{0,r}$ according to

$$\hat{P}^0(\mathbf{x}^R|\mathbf{x}^S) = \hat{P}^{0,i}(\mathbf{x}^R|\mathbf{x}^S) + \hat{P}^{0,r}(\mathbf{x}^R|\mathbf{x}^S), \quad \text{for } x_3^R \leq x_3^0, \tag{3}$$

in which $\hat{P}^{0,i}$ is given by

$$\hat{P}^{0,i}(\mathbf{x}^R|\mathbf{x}^S) = \frac{\hat{W}\exp\left(-\frac{s}{c_0}|\mathbf{x}^R - \mathbf{x}^S|\right)}{4\pi|\mathbf{x}^R - \mathbf{x}^S|} \tag{4}$$

and where $\hat{W}$ denotes the spectrum of the source wavelet. In a similar way the wavefield in state 1, $\hat{P}^1$, allows for such a decomposition in the domain $x_3 \leq x_3^1$. We write

$$\hat{P}^1(\mathbf{x}^S|\mathbf{x}^R) = \hat{P}^{1,i}(\mathbf{x}^S|\mathbf{x}^R) + \hat{P}^{1,r}(\mathbf{x}^S|\mathbf{x}^R), \quad \text{for } x_3^S \leq x_3^1. \tag{5}$$

Of course the incident wavefield in state 1 is the same as in state 0, only activated and recorded in reversed positions as in state 0. We have

$$\hat{P}^{1,i}(\mathbf{x}^S|\mathbf{x}^R) = \hat{P}^{0,i}(\mathbf{x}^R|\mathbf{x}^S). \tag{6}$$

When we use Parseval's theorem equation (1) can be rewritten as

$$\frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}_T\in\mathbb{R}^2}\mathrm{d}A\int_{x_3^0}^{x_3^1}\bar{P}^1(-js\boldsymbol{\alpha}_T,x_3|\mathbf{x}^R)\ \times$$
$$s^2\mathcal{K}(js\boldsymbol{\alpha}_T)\bar{P}^0(js\boldsymbol{\alpha}_T,x_3|\mathbf{x}^S)\mathrm{d}x_3$$
$$=\hat{W}\Big[\hat{P}^0(\mathbf{x}^R|\mathbf{x}^S)-\hat{P}^1(\mathbf{x}^S|\mathbf{x}^R)\Big]. \tag{7}$$

In equation (7) the spatial Fourier transformations of the wavefields are defined as

$$\bar{P}^0(js\boldsymbol{\alpha}_T,x_3|\mathbf{x}^S)=\int_{\mathbf{x}_T\in\mathbb{R}^2}\exp(js\boldsymbol{\alpha}_T\cdot\mathbf{x}_T)\hat{P}^0(\mathbf{x}|\mathbf{x}^S)\mathrm{d}A \tag{8}$$

and

$$\bar{P}^1(-js\boldsymbol{\alpha}_T,x_3|\mathbf{x}^R)=\int_{\mathbf{x}_T\in\mathbb{R}^2}\exp(-js\boldsymbol{\alpha}_T\cdot\mathbf{x}_T)\hat{P}^1(\mathbf{x}|\mathbf{x}^R)\mathrm{d}A. \tag{9}$$

The operator expression $\mathcal{K}\bar{P}^0$ is a compact notation for the convolution in the transformed domain given by

$$\mathcal{K}(js\boldsymbol{\alpha}_T)\bar{P}^0(js\boldsymbol{\alpha}_T,x_3|\mathbf{x}^S)= \tag{10}$$
$$\frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}'_T\in\mathbb{R}^2}\bar{K}(js\boldsymbol{\alpha}_T-js\boldsymbol{\alpha}'_T)\bar{P}^0(js\boldsymbol{\alpha}'_T,x_3|\mathbf{x}^S)\mathrm{d}A$$

and

$$\bar{K}(js\boldsymbol{\alpha}_T)=\int_{\mathbf{x}_T\in\mathbb{R}^2}\exp(js\boldsymbol{\alpha}_T\cdot\mathbf{x}_T)K(\mathbf{x}_T)\mathrm{d}A. \tag{11}$$

Next we apply the operator

$$\int_{\mathbf{x}_T^R\in\mathbb{R}^2}\exp(js\boldsymbol{\alpha}_T^R\cdot\mathbf{x}_T^R)\int_{\mathbf{x}_T^S\in\mathbb{R}^2}\exp(-js\boldsymbol{\alpha}_T^S\cdot\mathbf{x}_T^S)\cdots\mathrm{d}A\mathrm{d}A \tag{12}$$

to both sides of equation (7), yielding

$$\frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}_T\in\mathbb{R}^2}\mathrm{d}A\int_{x_3^0}^{x_3^1}\bar{\bar{P}}^1(-js\boldsymbol{\alpha}_T,x_3|js\boldsymbol{\alpha}_T^R,x_3^R)\ \times$$
$$s^2\mathcal{K}(js\boldsymbol{\alpha}_T)\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T,x_3|-js\boldsymbol{\alpha}_T^S,x_3^S)\mathrm{d}x_3= \tag{13}$$
$$\hat{W}\Big[\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R,x_3^R|-js\boldsymbol{\alpha}_T^S,x_3^S)-\bar{\bar{P}}^1(-js\boldsymbol{\alpha}_T^S,x_3^S|js\boldsymbol{\alpha}_T^R,x_3^R)\Big]$$

where the double overbar indicates the double spatial Fourier transformation of the relevant quantity with respect to the source and receiver coordinates. Note that exponential transformation kernels for the source and receiver coordinates have opposite signs. Next using the physical reciprocity property in the transformed domain for the wavefield in state 1

$$\bar{\bar{P}}^1(-js\boldsymbol{\alpha}_T^S,x_3^S|js\boldsymbol{\alpha}_T^R,x_3^R)=\bar{\bar{P}}^1(js\boldsymbol{\alpha}_T^R,x_3^R|-js\boldsymbol{\alpha}_T^S,x_3^S), \tag{14}$$

we can rewrite equation (13) as

$$\frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}_T\in\mathbb{R}^2}\mathrm{d}A\int_{x_3^0}^{x_3^1}\bar{\bar{P}}^1(js\boldsymbol{\alpha}_T^R,x_3^R|-js\boldsymbol{\alpha}_T,x_3)\ \times$$
$$s^2\mathcal{K}(js\boldsymbol{\alpha}_T)\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T,x_3|-js\boldsymbol{\alpha}_T^S,x_3^S)\mathrm{d}x_3= \tag{15}$$
$$\hat{W}\Big[\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R,x_3^R|-js\boldsymbol{\alpha}_T^S,x_3^S)-\bar{\bar{P}}^1(js\boldsymbol{\alpha}_T^R,x_3^R|-js\boldsymbol{\alpha}_T^S,x_3^S)\Big].$$

Equation (15) constitutes the integral equation that can be used to obtain the wavefield in state 1 ($\bar{\bar{P}}^1$) from the wavefield in state 0 ($\bar{\bar{P}}^0$). However, to achieve this, we have to know $\bar{\bar{P}}^0$ in the inhomogeneous slab $x_3^0\le x_3\le x_3^1$. To achieve this we evaluate equation (15) at $x_3=x_3^0$ and $x_3=x_3^1$. Next we decompose the total wavefields into incident and reflected wavefield constituents. Then comparing the two results of our evaluation and noting that any level x_3 in the homogeneous slab suffices, we readily arrive at the following result

$$\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R,x_3|-js\boldsymbol{\alpha}_T^S,x_3^S)= \tag{16}$$
$$\bar{\bar{P}}^{0,i}(js\boldsymbol{\alpha}_T^R,x_3^0|-js\boldsymbol{\alpha}_T^S,x_3^S)\exp\big(-s\Gamma_0^R(x_3-x_3^0)\big)$$
$$+\bar{\bar{P}}^{0,r}(js\boldsymbol{\alpha}_T^R,x_3^0|-js\boldsymbol{\alpha}_T^S,x_3^S)\exp\big(s\Gamma_0^R(x_3-x_3^0)\big)$$
$$+\int_{x_3^0}^{x_3}\frac{\sinh\big(s\Gamma_0^R(z-x_3)\big)}{s\Gamma_0^R}s^2\mathcal{K}(js\boldsymbol{\alpha}_T^R)\ \times$$
$$\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R,z|-js\boldsymbol{\alpha}_T^S,x_3^S)\mathrm{d}z,\quad \text{for } x_3^0\le x_3\le x_3^1,$$

where

$$\bar{\bar{P}}^{0,i}(js\boldsymbol{\alpha}_T^R,x_3^0|-js\boldsymbol{\alpha}_T^S,x_3^S)= \tag{17}$$
$$(2\pi)^2\frac{\hat{W}}{2s\Gamma_0^R}\delta(s\boldsymbol{\alpha}_T^R-s\boldsymbol{\alpha}_T^S)\exp\Big(-s\Gamma_0^R|x_3^0-x_3^S|\Big),$$

with the vertical wavenumber Γ_0^S

$$\Gamma_0^R=\Big(\frac{1}{c_0^2}+\boldsymbol{\alpha}_T^R\cdot\boldsymbol{\alpha}_T^R\Big)^{1/2},\quad \mathrm{Re}\{\Gamma_0^R\}\ge 0. \tag{18}$$

We observe that in equation (16) the contribution of the wavefield in state 1 does not play any role. Therefore in the next section we propose to employ this equation to determine the wavefield in state 0 in the inhomogeneous slab, the so-called wavefield extrapolation of the wavefield in state 0. Note that when the slab has no contrast, i.e. $\mathcal{K}=0$, the first two terms on the right-hand side represent the standard wavefield extrapolation in homogeneous domains realized by exponential incident and reflected wavefield operators.

Wavefield extrapolation

In order to come to the required result of extrapolation we employ a Taylor expansion of $\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R,x_3|-js\boldsymbol{\alpha}_T^S,x_3^S)$ in terms of $(x_3-x_3^0)$. Then substitution of this expansion into equation (16), carrying out the integration in the right-hand side and comparing the result in a consistent way concerning the orders of $(x_3 - x_3^0)$ with the original expansion yields

$$\begin{aligned}&\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3| - js\boldsymbol{\alpha}_T^S, x_3^S) = \\ &\Big[1 + \frac{s^2}{2}(x_3 - x_3^0)^2\{(\Gamma_0^R)^2 - \mathcal{K}(js\boldsymbol{\alpha}_T^R)\} + \\ &\frac{s^4}{24}(x_3 - x_3^0)^4\{(\Gamma_0^R)^2 - \mathcal{K}(js\boldsymbol{\alpha}_T^R)\}^2 \cdots\Big] \times \\ &\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3^0| - js\boldsymbol{\alpha}_T^S, x_3^S) + \\ &\Big[(x_3 - x_3^0) + \frac{s^2}{6}(x_3 - x_3^0)^3\{(\Gamma_0^R)^2 - \mathcal{K}(js\boldsymbol{\alpha}_T^R)\} + \\ &\frac{s^4}{120}(x_3 - x_3^0)^5\{(\Gamma_0^R)^2 - \mathcal{K}(js\boldsymbol{\alpha}_T^R)\}^2 \cdots\Big] \times \\ &\partial_3\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3^0| - js\boldsymbol{\alpha}_T^S, x_3^S), \quad \text{for } x_3^0 \le x_3 \le x_3^1.\end{aligned} \tag{19}$$

Observe that $\{(\Gamma_0^R)^2 - \mathcal{K}\}^2\bar{\bar{P}}^0$ equals $\{(\Gamma_0^R)^4 - 2(\Gamma_0^R)^2\mathcal{K} + \mathcal{K}^2\}\bar{\bar{P}}^0$, signifying that $\mathcal{K}$ and $\mathcal{K}^2$ solely operate on $\bar{\bar{P}}^0$. Inspection of the right-hand side of equation (19) learns us that the expansion can be readily extended to include all orders. Next we introduce the square-root operator $\mathcal{F}$ according to

$$\mathcal{F}(js\boldsymbol{\alpha}_T^R) = \sqrt{(\Gamma_0^R)^2 - \mathcal{K}(js\boldsymbol{\alpha}_T^R)}. \tag{20}$$

In fact $\mathcal{F}$ constitutes a pseudo-differential operator in the transformed domain. This allows us to write equation (19) in the compact notation

$$\begin{aligned}&\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3| - js\boldsymbol{\alpha}_T^S, x_3^S) = \\ &\cosh(s\mathcal{F}(js\boldsymbol{\alpha}_T^R)(x_3 - x_3^0))\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3^0| - js\boldsymbol{\alpha}_T^S, x_3^S) + \\ &\frac{\sinh(s\mathcal{F}(js\boldsymbol{\alpha}_T^R)(x_3 - x_3^0))}{s\mathcal{F}(js\boldsymbol{\alpha}_T^R)}\partial_3\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3^0| - js\boldsymbol{\alpha}_T^S, x_3^S).\end{aligned} \tag{21}$$

The expression of equation (21) is used in the next section to carry out the velocity replacement.

Velocity replacement

To operationalize the velocity replacement we rewrite equation (15), using the representation of the wavefields at $x_3 = x_3^0$ in the following form

$$\begin{aligned}&X(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) + \frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}_T \in \mathbb{R}^2} L(js\boldsymbol{\alpha}_T, js\boldsymbol{\alpha}_T^S) \times \\ &X(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T)\mathrm{d}A = Y(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S),\end{aligned} \tag{22}$$

with the unknown term

$$X(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) = \bar{\bar{P}}^1(js\boldsymbol{\alpha}_T^R, x_3^0| - js\boldsymbol{\alpha}_T^S, x_3^S), \tag{23}$$

the known term

$$\begin{aligned}&Y(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) = \bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3^0| - js\boldsymbol{\alpha}_T^S, x_3^S) + \\ &\int_0^{\Delta x_3}\frac{\sinh\big(s\Gamma_0^R x_3\big)}{s\Gamma_0^R}s^2\mathcal{K}(js\boldsymbol{\alpha}_T^R)\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3|-js\boldsymbol{\alpha}_T^S, x_3^S)\mathrm{d}x_3\end{aligned} \tag{24}$$

and the kernel function

$$\begin{aligned}&L(js\boldsymbol{\alpha}_T, js\boldsymbol{\alpha}_T^S) = \exp\big(s\Gamma_0(x_3^0 - x_3^S)\big) \times \\ &\int_0^{\Delta x_3}\exp\big(s\Gamma_0 x_3\big)s^2\mathcal{K}(js\boldsymbol{\alpha}_T)\frac{\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T, x_3|-js\boldsymbol{\alpha}_T^S, x_3^S)}{\hat{W}}\mathrm{d}x_3.\end{aligned} \tag{25}$$

In equations (24) and (25) $\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T, x_3| - js\boldsymbol{\alpha}_T^S, x_3^S)$ follows from equation (21). Equation (22) constitutes our final result yielding the total wavefield at $x_3 = x_3^0$ in state 1. As is clear from our representation, the procedure outlined sofar is recursive in nature and can be continued at the level $x_3 = x_3^1$. To solve equation (22) we propose to use a Neumann expansion of the solution. To that end we rewrite equation (22) as

$$\begin{aligned}&X(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) = Y(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) - \\ &\frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}_T \in \mathbb{R}^2} L(js\boldsymbol{\alpha}_T, js\boldsymbol{\alpha}_T^S)X(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T)\mathrm{d}A,\end{aligned} \tag{26}$$

which admits the following Neumann iterative solution

$$\begin{aligned}&X^{(n)}(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) = Y(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) - \\ &\frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}_T \in \mathbb{R}^2} L(js\boldsymbol{\alpha}_T, js\boldsymbol{\alpha}_T^S)X^{(n-1)}(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T)\mathrm{d}A \\ &\qquad n = 1, 2, \cdots\end{aligned} \tag{27}$$

and

$$X^{(0)}(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) = Y(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S), \tag{28}$$

where $X^{(n)}$ denotes the approximate solution after n iterations. We observe that equation (26) has a similar form as the operational expression that is used in the removal of surface-related multiples for marine seismic data [Verschuur et al. [3], Fokkema and van den Berg [1], van Borselen [2]]. From the correspondence with the solution of the surface-related multiple problem we conclude that the iterative terms in our solution constitute the removal of the internal multiples in the slab from the data in state 0. Elsewhere we argued [1] that the convergence of the Neumann series expansion is guaranteed by the causal nature in time of the multiples. Furthermore, as in the surface-multiple problem the kernel has to be deconvolved for the wavelet. This allows us to estimate the wavelet after every recursion step using the minimum energy norm introduced by [3]. This re-estimation of the wavelet after every recursion step could be profitable to stabilize our replacement process.

In the case we are dealing with horizontally layered media the pertinent equations are simplified using the following relations

$$X(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) = (2\pi)^2\delta(s\boldsymbol{\alpha}_T^R - s\boldsymbol{\alpha}_T^S)X(js\boldsymbol{\alpha}_T^R), \tag{29}$$

$$Y(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) = (2\pi)^2\delta(s\boldsymbol{\alpha}_T^R - s\boldsymbol{\alpha}_T^S)Y(js\boldsymbol{\alpha}_T^R), \tag{30}$$

$$L(js\boldsymbol{\alpha}_T, js\boldsymbol{\alpha}_T^S) = (2\pi)^2\delta(s\boldsymbol{\alpha}_T - s\boldsymbol{\alpha}_T^S)L(js\boldsymbol{\alpha}_T) \tag{31}$$

and the operator $\mathcal{K}$ and the pseudo-differential operator $\mathcal{F}$ simplify to simple algebraic operations according to

$$\mathcal{K}(js\boldsymbol{\alpha}_T) \quad \to \quad K, \tag{32}$$

$$\mathcal{F}(js\boldsymbol{\alpha}_T) \quad \to \quad \sqrt{\Gamma_0^2 - K} = \sqrt{\frac{1}{c_1^2} + \boldsymbol{\alpha}_T \cdot \boldsymbol{\alpha}_T}, \tag{33}$$

where K is the original contrast function introduced in equation (2). Consequently for this class of simple media equation (26) is written as

$$X(js\boldsymbol{\alpha}_T^R) = Y(js\boldsymbol{\alpha}_T^R) - L(js\boldsymbol{\alpha}_T^R)X(js\boldsymbol{\alpha}_T^R). \tag{34}$$

Now a straightforward algebraic solution of X from equation (34) is possible, however, to consistently treat the solution we propose also in this case to approximate the final answer through the iteration

$$X^{(n)}(js\boldsymbol{\alpha}_T^R) = Y(js\boldsymbol{\alpha}_T^R) - L(js\boldsymbol{\alpha}_T^R)X^{(n-1)}(js\boldsymbol{\alpha}_T^R), \quad n = 1, 2, \cdots \tag{35}$$

and

$$X^{(0)}(js\boldsymbol{\alpha}_T^R) = Y(js\boldsymbol{\alpha}_T^R). \tag{36}$$

Figure 2 illustrates our procedure for a simple horizontally layered acoustic medium. In Figure 2a we show the 2-D acoustic response of three layers embedded in the background medium. Figure 2b shows the result after replacing the velocity of the first layer by that of the background medium. With this result we conclude our discussion on the replacement of velocities for lateral inhomogeneous and homogeneous media.

Conclusions

In this paper we have shown how Rayleigh's reciprocity theorem aids the velocity replacement in thin slabs of an inhomogeneous acoustic medium. This process is divided into two steps: first the extrapolation of the original wavefield into the inhomogeneous thin slab is carried out (equation 21). The resulting expression is formulated in terms of pseudo-differential operators that act on the original wavefield. The second step encompasses the actual removal of the influence of the thin slab on the original wavefield. This operation is furnished by an integral equation of the second kind (equation 26) and is in nature closely related to the process of surface-related multiple removal in marine seismic data processing. There the solution of the removal process is achieved by an iterative Neumann series expansion. Also in the case of velocity replacement we propose to use such an approach (equation 27). In a similar way the successive terms in the velocity replacement Neumann series correspond with the internal multiples in the thin slab. We have demonstrated that the procedure can be done in an iterative fashion so that we can successively peel off the inhomogeneous medium and in doing so replace it with the homogeneous background. Finally we showed that for the case of the horizontally layered medium this leads to a simple algebraic operation. Our future work will focus on the numerical stabilization of the process and application of the result in seismic imaging techniques.

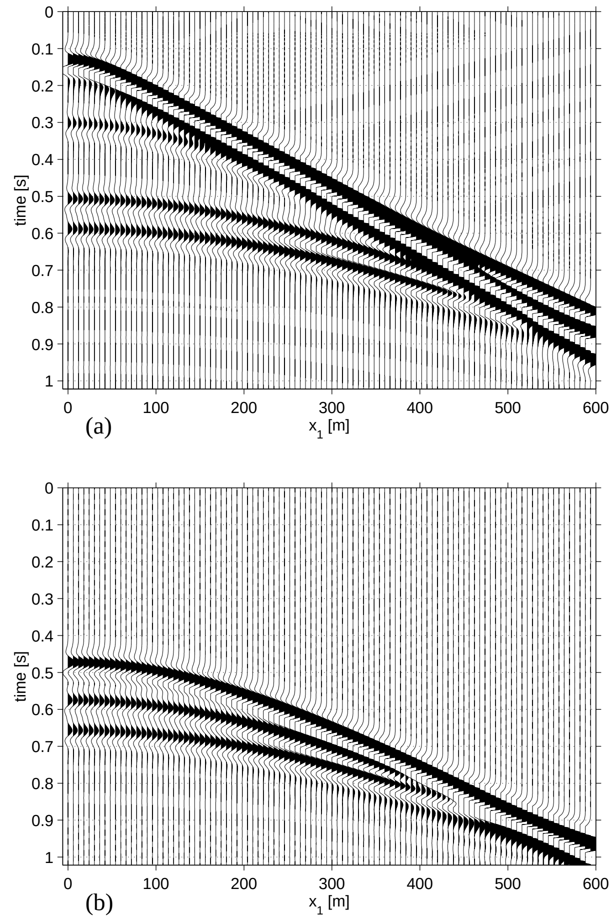

Fig. 2: (a) Acoustic response of three acoustic layers embedded in a homogeneous background. (b) Same configuration, but velocity in first layer replaced by background velocity.

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