

Decomposition of multicomponent ocean-bottom data in two steps

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Summary

The decomposition procedure for separating multicomponent ocean-bottom data into up- and downgoing P- and S-waves is based on a combination of the pressure, horizontal and vertical velocity components. This makes the decomposition method less easy to apply on field data – differences between the components not due to the earth properties (e.g. instrument response, coupling with the sea-bottom) have to be compensated for. In addition the medium parameters just below the sea-bottom are needed as input for the decomposition. Without a priori knowledge of at least some of these unknowns it is a difficult task to arrive at a correct decomposition result.

By performing the decomposition in two steps – first decomposition into up- and downgoing wavefields, then decomposition of the up- and downgoing wavefields, respectively, into P- and S-waves – the number of unknowns in each step is reduced. This offers possibilities for performing an elastic decomposition on field data without any a priori knowledge of medium parameters or coupling. An adaptive decomposition scheme, applicable to field data, is presented here.

Introduction

In order to get elastic information about the sub-sea-bottom, ocean-bottom data can be decomposed into up- and downgoing P- and converted S-waves ([1],[4]). The converted S-waves can then be separately processed (e.g. migration, inversion) from the P-waves.

There are certain conditions which the decomposed data have to satisfy, namely

- for the decomposition just above the bottom, no sub-bottom primaries should be present in the *downgoing* waves;
- for the decomposition just below the bottom, no direct source wave and no water bottom multiples should be present in the *upgoing* P- and S-waves.

These quality conditions are used to search for the unknown medium parameters as well as the unknown differences between the components (e.g. due to different coupling, instrument response, etc.).

Application of acoustic decomposition (just above the sea-bottom) into up- and downgoing pressure fields to field data is straightforward and has been demonstrated previously in [2]. The amount of unknowns to be evaluated in the elastic decomposition (just below the sea-bottom) makes its application to field data more problematic. As the quality conditions for the decomposition results do not distinguish between P- and S-waves,

a decomposition into merely up- and downgoing waves would suffice. It turns out that a decomposition in two steps (first up-/down separation and then P-/S separation) requires less data components at the same time, making it a more practical procedure for field data.

The operators derived for a two-step decomposition in a land acquisition setting ([3]) will be rewritten for the ocean-bottom case. Then an adaptive decomposition scheme applicable to real data, combining the one- and two-step decompositions, is explained.

One-step versus two-step decomposition

In the following “decomposition” stands for “elastic decomposition just below the sea-bottom”. Some decomposition results will be demonstrated with a synthetic dataset. The ocean-bottom model and the synthetic data that will be used are shown in Figure 1.

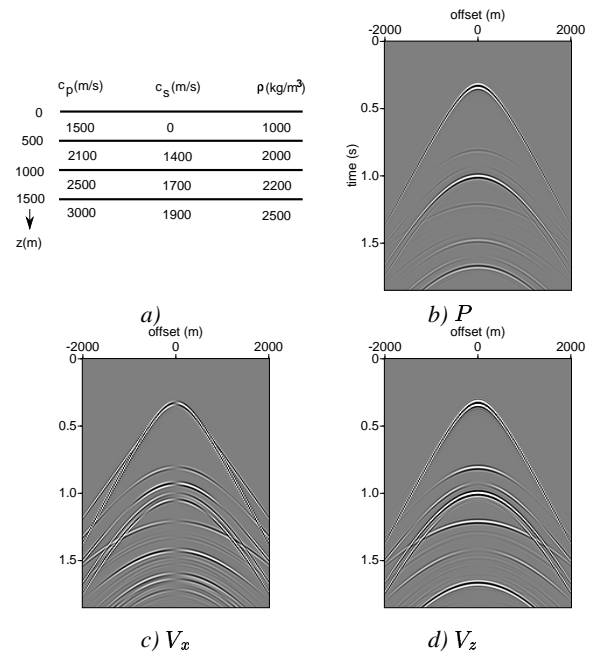


Fig. 1: a) Model used to obtain 3-component synthetic data at the sea-bottom (at 500 m depth). b) Pressure component. c) Horizontal velocity component V_x . d) Vertical velocity component V_z .

The general relations between the two-way and one-way wavefield vectors can be written symbolically in the space-frequency

Two-step decomposition of ocean-bottom data

domain as (see [4] for the operators)

$$\begin{pmatrix} -\vec{\tau}_z(z) \\ \vec{V}(z) \end{pmatrix} = \begin{pmatrix} \mathbf{L}_1^+(z) & \mathbf{L}_1^-(z) \\ \mathbf{L}_2^+(z) & \mathbf{L}_2^-(z) \end{pmatrix} \begin{pmatrix} \vec{D}^+(z) \\ \vec{D}^-(z) \end{pmatrix} \quad (1)$$

and

$$\begin{pmatrix} \vec{D}^+(z) \\ \vec{D}^-(z) \end{pmatrix} = \begin{pmatrix} \mathbf{N}_1^+(z) & \mathbf{N}_2^+(z) \\ \mathbf{N}_1^-(z) & \mathbf{N}_2^-(z) \end{pmatrix} \begin{pmatrix} -\vec{\tau}_z(z) \\ \vec{V}(z) \end{pmatrix}, \quad (2)$$

where

$$\vec{\tau}_z(z) = \begin{pmatrix} \vec{\tau}_{xz}(z) \\ \vec{\tau}_{yz}(z) \\ \vec{\tau}_{zz}(z) \end{pmatrix}, \quad \vec{V}(z) = \begin{pmatrix} \vec{V}_x(z) \\ \vec{V}_y(z) \\ \vec{V}_z(z) \end{pmatrix} \quad (3)$$

and

$$\vec{D}^+(z) = \begin{pmatrix} \vec{\Phi}^+(z) \\ \vec{\Psi}^+(z) \\ \vec{\Upsilon}^+(z) \end{pmatrix}, \quad \vec{D}^-(z) = \begin{pmatrix} \vec{\Phi}^-(z) \\ \vec{\Psi}^-(z) \\ \vec{\Upsilon}^-(z) \end{pmatrix}. \quad (4)$$

Here $\vec{\tau}_{iz}(z)$ and $\vec{V}_i(z)$ for $i = x, y, z$ contain the discretized two-way wave fields at depth level z in terms of stress and velocity components; $\vec{\Phi}^\pm(z)$, $\vec{\Psi}^\pm(z)$ and $\vec{\Upsilon}^\pm(z)$ contain the one-way wave fields in terms of downgoing and upgoing P - and S -waves.

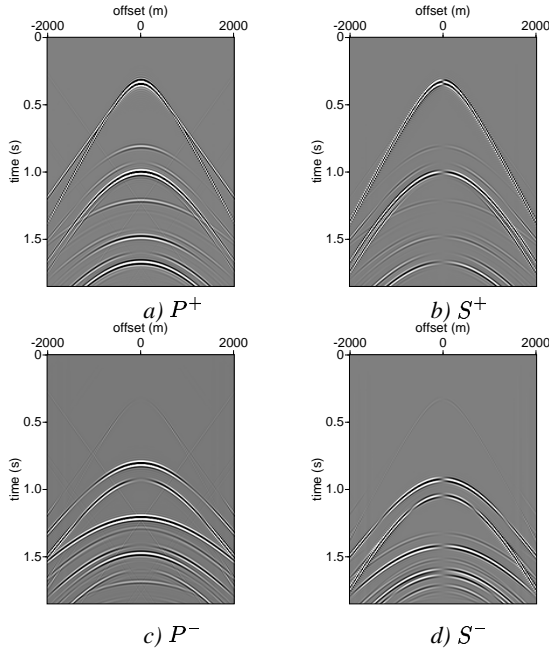


Fig. 2: One-step decomposition into up- and downgoing P - and S -waves just below the sea-bottom. a) Downgoing P -waves. b) Downgoing S -waves. c) Upgoing P -waves. d) Upgoing S -waves.

At the sea-bottom $z = z_1$ the shear-stresses vanish, whereas the normal stress is equal to minus the acoustic pressure just above

the sea-bottom. Hence, one-step decomposition is accomplished by applying equation (2) at $z = z_1$, with

$$-\vec{\tau}_z(z_1) = \begin{pmatrix} \vec{0} \\ \vec{0} \\ \vec{P}(z_1) \end{pmatrix}. \quad (5)$$

The one-step decomposition result is shown in Figure 2.

In the two-step decomposition the first decomposition step into up- and downgoing fields is expressed in terms of stresses. This choice is arbitrary; other wave field quantities could be chosen. However, with this choice the decomposition operators have a simple form. From equation (1) we obtain

$$-\vec{\tau}_z(z_1) = \underbrace{\mathbf{L}_1^+(z_1)\vec{D}^+(z_1)}_{-\vec{\tau}_z^+(z_1)} + \underbrace{\mathbf{L}_1^-(z_1)\vec{D}^-(z_1)}_{-\vec{\tau}_z^-(z_1)}, \quad (6)$$

or

$$\begin{pmatrix} -\vec{\tau}_z^+(z_1) \\ -\vec{\tau}_z^-(z_1) \end{pmatrix} = \begin{pmatrix} \mathbf{L}_1^+(z_1) & \mathbf{O} \\ \mathbf{O} & \mathbf{L}_1^-(z_1) \end{pmatrix} \begin{pmatrix} \vec{D}^+(z_1) \\ \vec{D}^-(z_1) \end{pmatrix}, \quad (7)$$

or, upon substitution of equation (2)

$$\begin{pmatrix} -\vec{\tau}_z^+(z_1) \\ -\vec{\tau}_z^-(z_1) \end{pmatrix} = \begin{pmatrix} \mathbf{M}_1^+(z_1) & \mathbf{M}_2^+(z_1) \\ \mathbf{M}_1^-(z_1) & \mathbf{M}_2^-(z_1) \end{pmatrix} \begin{pmatrix} -\vec{\tau}_z(z_1) \\ \vec{V}(z_1) \end{pmatrix}, \quad (8)$$

where the partial decomposition operators are defined as

$$\mathbf{M}_1^\pm(z_1) = \mathbf{L}_1^\pm(z_1)\mathbf{N}_1^\pm(z_1), \quad (9)$$

$$\mathbf{M}_2^\pm(z_1) = \mathbf{L}_1^\pm(z_1)\mathbf{N}_2^\pm(z_1). \quad (10)$$

For the second decomposition step into P - and S -waves equation (7) is merely inverted, yielding

$$\begin{pmatrix} \vec{D}^+(z_1) \\ \vec{D}^-(z_1) \end{pmatrix} = \begin{pmatrix} \{\mathbf{L}_1^+(z_1)\}^{-1} & \mathbf{O} \\ \mathbf{O} & \{\mathbf{L}_1^-(z_1)\}^{-1} \end{pmatrix} \begin{pmatrix} -\vec{\tau}_z^+(z_1) \\ -\vec{\tau}_z^-(z_1) \end{pmatrix}. \quad (11)$$

Next, the operators in the first step of the two-step decomposition are given in the rayparameter-frequency domain for a 2-D situation. This decomposition step is given in more detail because it is especially important for the adaptive decomposition scheme explained further on. From equation 8 we obtain

$$-\vec{\tau}_z^\pm(z_1) = -\tilde{\mathbf{M}}_1^\pm(z_1)\tilde{\tau}_z(z_1) + \tilde{\mathbf{M}}_2^\pm(z_1)\tilde{\vec{V}}(z_1), \quad (12)$$

or

$$\begin{pmatrix} -\tilde{\tau}_{xz}^\pm(z_1) \\ -\tilde{\tau}_{yz}^\pm(z_1) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \pm \frac{\gamma p}{q_S} \\ \mp \frac{\gamma p}{q_P} & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \vec{P}(z_1) \end{pmatrix} + \frac{\rho}{2} \begin{pmatrix} \pm \frac{\beta}{q_S} & 0 \\ 0 & \pm \frac{\beta}{q_P} \end{pmatrix} \begin{pmatrix} \tilde{V}_x(z_1) \\ \tilde{V}_z(z_1) \end{pmatrix}, \quad (13)$$

Two-step decomposition of ocean-bottom data

or

$$-\tilde{\tau}_{xz}^{\pm}(z_1) = \pm \frac{\gamma p}{2q_S} \tilde{P}(z_1) \pm \frac{\rho \beta}{2q_S} \tilde{V}_x(z_1), \quad (14)$$

$$-\tilde{\tau}_{zz}^{\pm}(z_1) = \frac{1}{2} \tilde{P}(z_1) \pm \frac{\rho \beta}{2q_P} \tilde{V}_z(z_1), \quad (15)$$

where

$$\beta = c_S^4 [4p^2 q_P q_S + (c_S^{-2} - 2p^2)^2], \quad (16)$$

$$\gamma = c_S^2 [2q_P q_S - (c_S^{-2} - 2p^2)], \quad (17)$$

$$q_P = \sqrt{c_P^{-2} - p^2}, \quad (18)$$

$$q_S = \sqrt{c_S^{-2} - p^2}. \quad (19)$$

Note the simple structure of equations (14) and (15): only two data-components are required simultaneously. Also note that equation (15) has the same form as the acoustic decomposition equation, apart from the factor β (this factor approaches one when c_S approaches zero). The result of the decomposition into up- and downgoing stresses is shown in Figure 3.

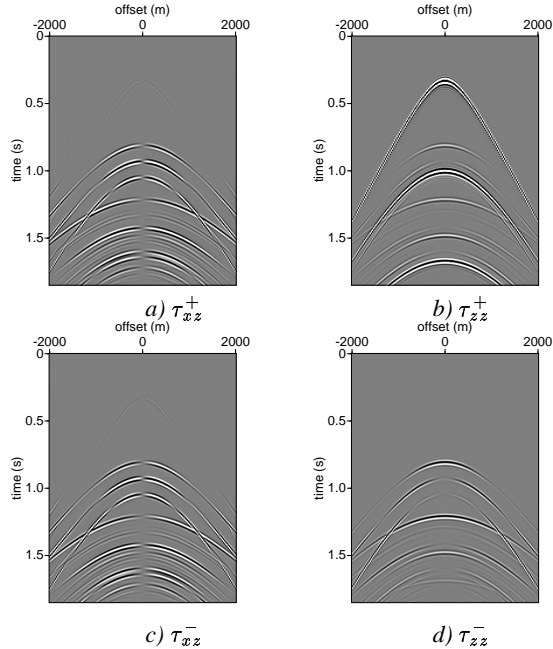


Fig. 3: Two-step decomposition, step 1: decomposition into up- and downgoing stresses a) Downgoing shear stress. b) Downgoing normal stress. c) Upgoing shear stress. d) Upgoing normal stress.

Energy analysis of decomposed result

The condition that there is no direct (transmitted) source wave in the upgoing wavefield below the sea-bottom provides a way to find the sub-sea-bottom velocities and density. The medium parameters construct a 3-D variable space. The decomposition

can be performed somewhere in this space. The energy is calculated in a tight window around the direct arrival. The correct parameters give a minimum energy. An optimization procedure should be used to find this minimum.

The minima of the decomposition results into up- and downgoing normal and shear stresses were analyzed for a synthetic dataset. Figures 4 and 5 show the minima for the upgoing normal and shear stressfields in the 3-D parameter space.

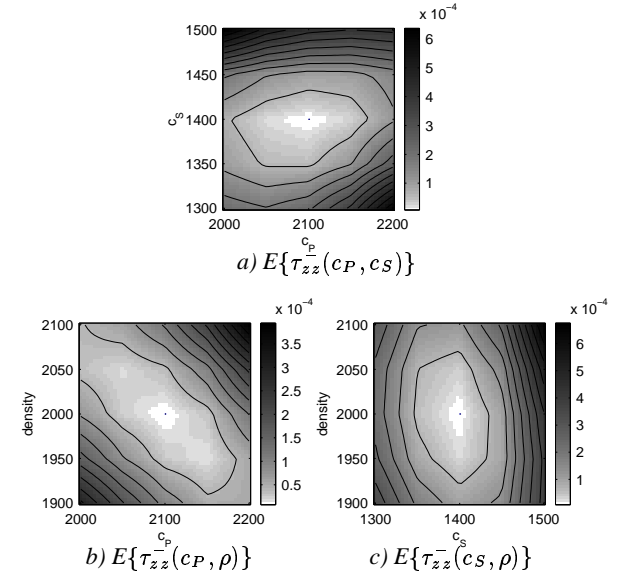


Fig. 4: Parameter-sensitivity in the output of step 1 of the two-step decomposition. The sensitivity for τ_{zz} is expressed by the amount of energy in the direct source wave as a function of (a) P - and S -velocity; (b) P -velocity and density and (c) S -velocity and density. As the energy was calculated from the upgoing τ_{zz} , minimal energy is expected where the parameters are correct (i.e. $c_P = 2100$ m/s, $c_S = 1400$ m/s and $\rho = 2000$ kg/m³).

An adaptive decomposition scheme

The two-step decomposition now provides the possibility to apply an adaptive elastic decomposition. No a priori knowledge of the medium parameters below the sea-bottom or the coupling parameters is needed. This can be accomplished with the following scheme:

- 1) **Acoustic one-step decomposition above the sea-bottom**
With P and $a(\omega)V_z$ components and the water parameters, the unknown coupling filter $a(\omega)$ of the vertical geophone can be found. The used criterion is that there should be no sub-bottom primaries in the downgoing wavefield above the bottom.
- 2) **Elastic two-step decomposition below the sea-bottom (step 1a)**

Two-step decomposition of ocean-bottom data

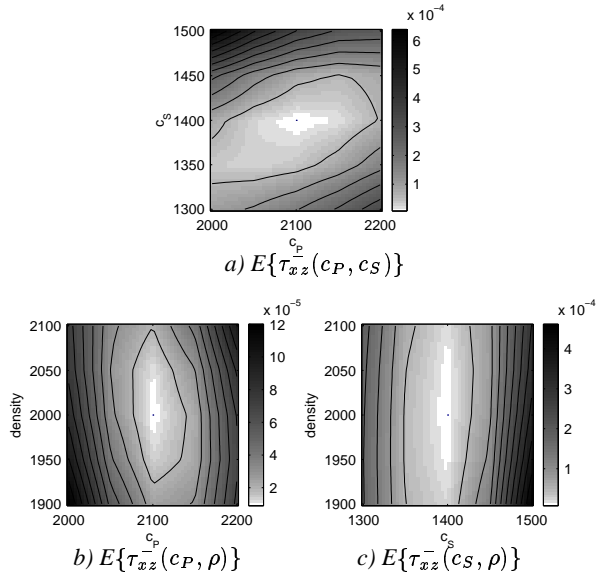


Fig. 5: Parameter-sensitivity in the output of step 1 of the two-step decomposition. The sensitivity for τ_{xz} is expressed by the amount of energy in the direct source wave as a function of the medium parameters, (a), (b) and (c) idem Figure 4.

Separation into up- and downgoing τ_{zz} ;

$$-\tilde{\tau}_{zz}^{\pm} = \frac{1}{2}\tilde{P} \pm \tilde{f}(c_P, c_S, \rho)a(\omega)\tilde{V}_z,$$

$$\text{where } \tilde{f}(c_P, c_S, \rho) = \varrho\beta/2q_P.$$

The used criterion is that there should be no direct source wave in the upgoing wavefield below the bottom. This is a minimization problem. In other words, the parameters c_P, c_S, ρ are estimated at those values where the energy of the first arrival is minimal in the upgoing wavefield below the bottom. Note that $b(\omega)\tilde{V}_x$ is not needed in this step.

3) Elastic two-step decomposition below the sea-bottom (step 1b)

Separation into up- and downgoing τ_{xz} ;

$$-\tilde{\tau}_{xz}^{\pm} = \pm \tilde{f}'(c_P, c_S, \rho)P \pm \tilde{f}''(c_P, c_S, \rho)b(\omega)\tilde{V}_x.$$

Using the medium parameters estimated in step 2 plus the criterion of step 2, the unknown coupling filter $b(\omega)$ of the horizontal inline geophone can be estimated.

4) Elastic decomposition below the sea-bottom into P- and S-waves

Now the elastic decomposition into P- and S-waves can be

performed with the values of $a(\omega), c_P, c_S, \rho$ and $b(\omega)$ estimated in steps 1,2 and 3.

Conclusions

Two approaches have been used to decompose multicomponent ocean-bottom data. The one-step decomposition for multicomponent ocean-bottom data works nicely on synthetic data. It is more difficult to apply to field data, as in practice the different data components are not well matched. Therefore, a decomposition in two steps - first up/down then P/S separation - has been introduced.

The two-step decomposition uses just two data components at a time, making it easier to match them. An adaptive decomposition procedure that combines the one- and two-step decomposition has been proposed. In this procedure all unknown parameters, necessary for performing a decomposition, can be estimated step by step.

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References

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