Characterization of reflectors by multi-scale amplitude and phase analysis of seismic data

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Summary

Strong reflections in seismic data can often be matched with sharp outliers in the velocity function of the subsurface. This velocity function is generally known by means of the sonic P-velocity log. The usual description of layering in the earth is by means of stepfunctions, by which only an approximated version of the outliers can be constructed. These outliers have different phase and amplitude effects on the angle dependent reflection response, compared with stepfunctions.

We propose an effective parameterization for these outliers, which is a generalization of stepfunctions, for which we have closed-form implicit expressions for their effect on the reflected wave field. Using this model, we want to extract both a singularity parameter, which characterizes the singularity "strength" of the outlier, as well as the contrast of the P-wave velocities above and below the singular point. For this characterization we use the amplitudes in the continuous wavelet transform of the angle dependent reflectivity as well as the angle dependent instantaneous phase of the reflected wavefield.

It is shown that it is possible to derive a consistent singularity parameter from both the well-log and the seismic reflection response, making use of the amplitudes in the continuous wavelet transform of the angle dependent reflectivity. Secondly, an inversion scheme is proposed, which makes use of both the former analysis and instantaneous phase information of the reflection event. With this scheme it is possible to derive the singularity parameter and the contrast of the P-wave velocities above and below the singular point from seismic reflection responses.

Introduction

In [2] we proposed a method to derive a singularity parameter α from seismic data making use of the following representation of outliers in the velocity,

$$c(z) = \begin{cases} c_1|z/z_1|^{\alpha_1} & \text{for } z < 0 \\ c_2|z/z_1|^{\alpha_2} & \text{for } z > 0, \end{cases} \qquad (1)$$

in which $\alpha_{1,2}$ are the singularity parameters, and $c_{1,2}$ are factors that define the velocity contrast. The density profile was taken constant throughout the whole depth interval.

In the case that we have $\alpha_1 = \alpha_2 = \alpha$, a function that can be seen in Figure 1a, these functions satisfy a relationship of self-similarity $c(\beta z) = \beta^\alpha c(z)$, so we will refer to velocity functions as in eq.(1) as *self similar singularities.* Using this relation, simple relationships for the extraction of parameter α from both the well-log and the seismic reflection response modeled in the well-log, can be derived. We make use of the continuous wavelet transform

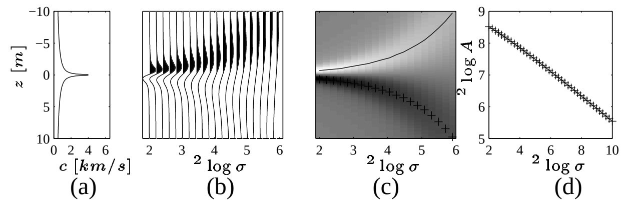

Fig. 1: *(a) well-log with singularity according to eq.(1) for which $\alpha_1 = \alpha_2 = \alpha = -0.4$, and $c_1 = c_2 = 1200$ m/s*
(b) continuous wavelet transform of (a)
(c) modulus maxima representation of (b)
(d) amplitude along modulus maxima lines: the crossed and the solid lines overlap completely, their slope (α) is -0.393

$$\mathcal{W}\{f,\psi\}(\sigma,z) = \check{f}(\sigma,z) = \frac{1}{\sigma^\mu}\int f(z')\psi(\frac{z'-z}{\sigma})dz', \quad (2)$$

in which $f(z)$ is the function to be transformed, σ the scale, $\psi(z)$ the analyzing wavelet and μ a parameter that is dependent on the actual use of the wavelet transform. When we are examining well-logs, we take $\mu = 1$; on the other hand for the analysis of reflection responses, discussed below, we will take $\mu = 0$. Figure 1b shows the continuous wavelet transform of the well-log; a very clear amplitude decay in the scale direction is visible. Mallat and Hwang [4] have shown that estimations of *local* regularity can be found by means of a *modulus maxima* representation. This is essentially the amplitude along lines which connect the maxima in the absolute value of the wavelet transformed section, for which an example is shown in figure 1c. The first application of this theory on well-logs was performed by Herrmann, see e.g. [3]. As the analyzing wavelet, we use the first derivative of the Gaussian function (which has only one vanishing moment) because it gives rise to the smallest amount of modulus maxima lines.

The solid *modulus maxima* line we can see in figure 1c, is the one connected to the maximum of the wavelet transformed section, the crossed line is the one connected to the minimum. If we now define the positions of the *modulus maxima* lines by $z_{\max}$, the following relation holds,

$$\log|\check{c}(\sigma,\sigma z_{\max})| = \alpha\log\sigma + \log|\check{c}(1,z_{\max})|, \qquad (3)$$

which shows that we can find the value of the parameter α from the slope of a log-log plot of the amplitude along the *modulus maxima* lines versus the scale σ. This has been performed in figure 1d, and we can see that the slope along both the *modulus* *maxima* lines is almost identical to the α with which we modeled the velocity function $c(z)$. In [2], we presented also the examples for a step-function ($\alpha = 0$) and a positive α. In [2], we showed that the same parameter α can also be found from the angle-dependent seismic reflection response $R(p, \tau)$. It can be proven that the same analysis can be performed on $R(p, z)$, i.e. the data migrated to the rayparameter-depth (p, z)-domain. The reason why we will give the preference to this domain is that when we perform the wavelet transform to the migrated reflection response, $\mathrm{R}(p, z) \rightarrow \check{\mathrm{R}}(p, \sigma, z)$, the automation of the tracking of the reflections in the p-direction will be much easier, as they will be at constant depth. The reason that we want to track the reflections, is that we have to construct *modulus maxima* planes in the cube $\check{\mathrm{R}}(p, \sigma, z)$, in the same way as in the well-log, but now also in the p-direction, see figure 2. The *modulus maxima* planes are created by connecting the maxima in the scale and rayparameter direction. Referring to [7], we can prove that the amplitude of $|\check{\mathrm{R}}(p, \sigma, z)|$ in a *modulus maxima* plane is constant along contours described by

$$p^{1-\alpha}\sigma^{\alpha} = \text{constant}. \tag{4}$$

After we have created the contours, we want to estimate α making use of the analytical contours described by eq. (4). We use the following 3-step inversion scheme for this purpose:
– Compute analytical contours using equation (4) for a relevant α-range. Normally this will be between $\alpha = -1$ and $\alpha = 0.5$.
– Along these contours, measure the value of $|\check{\mathrm{R}}(p, \sigma, z)|$ in the *modulus maxima* plane, and compute its standard deviation.
– The value of α we want to find is the one for which the smallest standard deviation is found.
This inversion scheme has the benefit that when the contours derived from the reflection response are not exactly equal to the analytical ones, we can still find a unique 'best fit' for a certain α. An example of this analysis on the contours in figure 2 can be seen in figure 3, which shows that an unique minimum is found at about -0.4. An example of this procedure on a reflection in a real well-log is shown in figure 4. Figure 4 shows the well-log and the cube $\check{\mathrm{R}}(p, \sigma, z)$. At a depth of 1265m the reflector in the well-log was analyzed, having an α=-0.50. Analysis of the contours in this figure showed an optimum value of α=-0.59. More examples on real well-logs can be found in [2] and in [7].

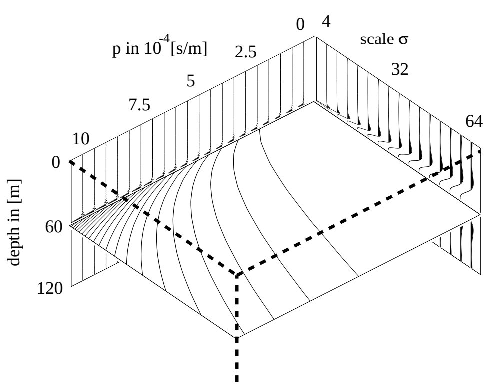

Fig. 2: *3-dimensional data cube, representing the continuous wavelet transform $\check{R}(p, \sigma, z)$ of the migrated reflection response $R(p, z)$, corresponding to the singularity in figure 1a, (but shifted to a depth of 60m).*

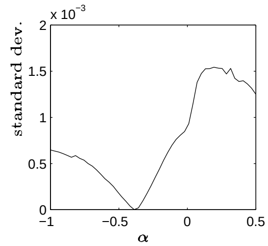

Fig. 3: *Standard deviation along analytical contours $p^{1-\alpha}\sigma^{\alpha}$ = constant, as a function of α for the contours in figure 2b.*

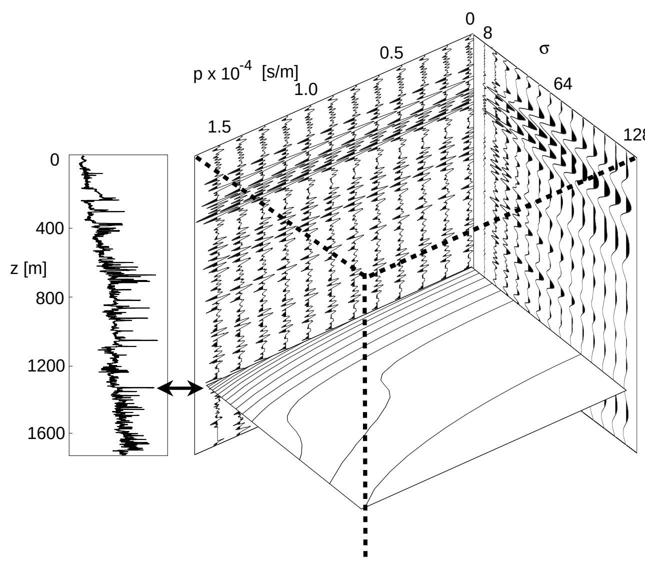

Fig. 4: *Well-log and its wavelet transformed reflections response $\check{\mathrm{R}}(p, \sigma, z)$. The arrow points at the reflector and the analyzed reflection.*

The analysis of reflection responses by amplitude and phase analysis

Until now we have only taken the amplitude along the contours in the (p-σ)-plane into account. However, singularities like equation (1) induce also a phase effect in the reflection, which can be easily derived from [6], eq.2 (if we take $\rho_1 = \rho_2$).

$$\mathrm{R}^{+} = j\left[\frac{\mathrm{e}^{-j\nu\pi}\, c_2^{2\nu} + \mathrm{e}^{j\nu\pi}\, c_1^{2\nu}}{c_2^{2\nu} + c_1^{2\nu}}\right], \tag{5}$$

in which R^+ is the reflection coefficient for the downgoing wavefield and $\nu = 1/(2-2\alpha)$. For clarification, this equation is decomposed into a real and an imaginary part, so we can analyze the effect of the parameters c_1, c_2 and α on the phase of R^+

$$\mathrm{R}^+ = \sin\nu\pi\left[\frac{1-\eta^{2\nu}}{1+\eta^{2\nu}}\right] + j\cos\nu\pi \qquad \text{with } \eta = \frac{c_1}{c_2}. \quad (6)$$

It is clear that the reflection coefficient is only dependent on the parameter α and the ratio η. It is evident that when $\eta = 1$ and $\alpha \neq 0$, we have a phase shift of exactly ∓ 90 degrees, dependent on the sign of α (R^+ is totally imaginary). In the case $\alpha = 0$ and η arbitrary we get the normal reflection coefficient of a step function: $(c_2 - c_1)/(c_2 + c_1)$. In the case that $\alpha \neq 0$ and $\eta \neq 1$, we will have 'non-standard' phase shifts.

Furthermore we can see that the amplitudes also are dependent on α. In Figure 5 we can see the effect for normal incidence reflection for different values of η, in the range α=[-1,0.5]. In figure 5a,b we have respectively the absolute value and the phase of the reflection coefficient R^+, using equation (6). Using only the $|\mathrm{R}^+|$, we cannot distinguish between η and $1/\eta$, but the phase of the event gives us the 'final verdict'.

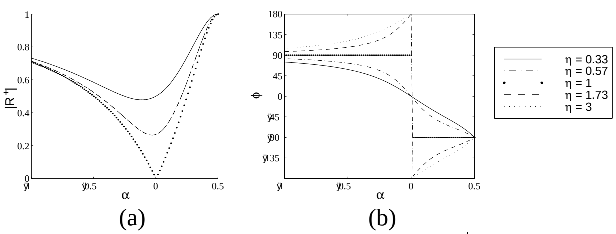

Fig. 5: *(a) absolute value of reflection coefficient* R^+ *using equation (6). The lines which represent* $\eta = 0.33, 3$ *and* $\eta = 0.57, 1.73$ *overlap (b) phase of the reflection coefficient* R^+ *using equation (6)*

Because we do not have closed form explicit expressions for the instantaneous phase ϕ as a function of rayparameter p, α and η, analyzing this effect has to be done by modeling angle dependent reflection data in a synthetic 'well-log' as in equation (1). The instantaneous phase will be extracted by the method described in [1]. The results for α=-0.3, α=0 and α=$+0.3$ can be found in figure 6. The figure shows that the phase of the reflection response is (almost) constant along the p-values, for each α and η. Note that in figure 6b the phase for η=$\frac{1}{2}$ is only constant up to the critical reflection angle, which is about 8×10^{-4}[s/m]. Beyond this point the phase will gradually change to a value of 180 degrees.

As has been shown by the examples in this section, it should be possible to use the instantaneous phase ϕ as a tool for deriving the parameters α and η from the seismic reflection response. Verhelst [5] on the contrary, uses the instantaneous phase as a tool in detecting lateral differences in facies on certain horizons, which is part of his geological characterization.

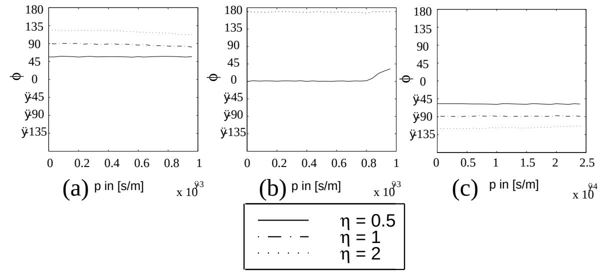

Fig. 6: *Phase* ϕ *as a function of rayparameter* p*, for different* η*, at (a)* $\alpha = -0.3$*, (b)* $\alpha = 0$ *and* $\alpha = 0.3$*. For* η*=.5 and* 1*,* c_2 *has been chosen 1200 m/s, for* η*=2,* c_2 *was chosen 600m/s.*

Estimation of α and η by inversion on the angle dependent reflection response

Before we can use the phase effect together with the amplitude in the wavelet transform domain, for purposes of characterization of η and α, we will have to:

– create and analyze a penalty function, which will yield an error surface with a unique minimum.

– make a choice for an inversion procedure, and test its performance.

Creation of the penalty function in inversion. For the amplitude effect we choose a penalty function that does not emphasize outliers in the data. Therefore a stable choice would be taking the ℓ_1- norm of the difference between data and modeled data

$$\mathrm{E}_A = \frac{1}{N}\sum_{i=1}^{N}|A_i^d - A_i^m|, \quad (7)$$

in which A^d represents the amplitudes measured in de *modulus maxima* plane of the data and $N = N_\sigma \times N_p$ the number of sample points in the planes. A^m is found by creating a synthetic velocity model, modeling reflection data in it and creating the *modulus maxima* planes in the data set. This can e.g. be done during the inversion process.

For the instantaneous phase ϕ, it has been shown, that the phase is constant for all rayparameter-values, as long as we are in the sub-critical reflection domain. So for this reflection function a suitable criterion would be the absolute difference between the phase of the data and the modeled data,

$$\mathrm{E}_\phi = |\bar{\phi}^d - \bar{\phi}^m|, \quad (8)$$

in which $\bar{\phi}$ is the mean of the instantaneous phases for all rayparameter values. If we take the mean of the phases, any misextraction of the instantaneous phase in one of the traces will be (partially) cancelled, and will have no disastrous effect on the penalty function. Together these two measures have to form the penalty function E, with a unique optimum solution. The penalty functions E_ϕ and E_A have to be summed in some

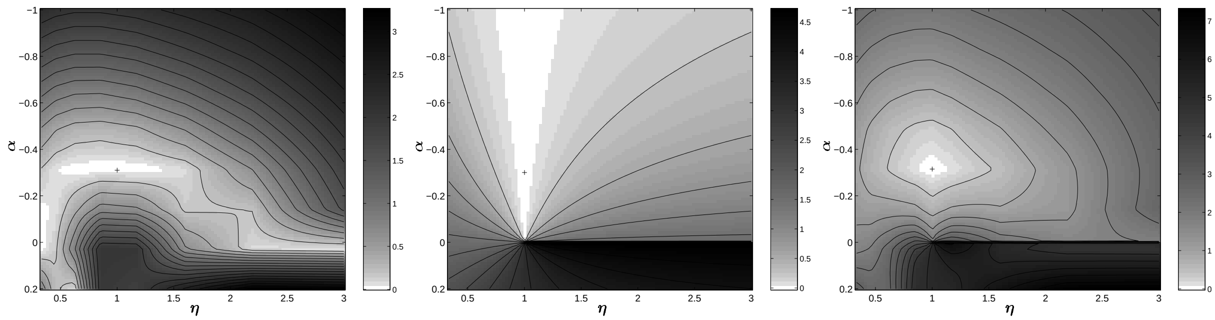

Fig. 7: contourplot of the penalty function (a) E_A, (b) E_ϕ and (c) E for the reflection on a singularity with $\eta_0 = 1$ and $\alpha_0 = -0.3$, the cross shows the desired minimum.

weighted manner, so that in E, defined as

$$\mathrm{E} = \frac{1}{W}\mathrm{E}_\phi + \mathrm{E}_A, \qquad (9)$$

E_ϕ and E_A have a weight, according to their sensitivity to the parameters α and η. For this reason the factor W in equation (9) has been introduced. This factor has to be found empirically, using an inversion problem on reflection data, for which we know the solution α_0 and η_0.

For this purpose we will compute the error surfaces $\mathrm{E}_{A,\phi}(\alpha, \eta)$ for synthetic reflection data, modeled in a velocity function as in eq. (1), with η_0=1 and α_0=-0.3. For this data the mean phase $\bar{\phi}$ is computed ($= 90$ degrees) and the (p-σ)-contours are constructed. For α in the range [-1,0.2] and η in the range [$\frac{1}{3}$,3] the errorsurfaces are computed for the penalty functions E_A and E_ϕ. Figure 7a shows the contours of the penalty function connected to the amplitude information E_A. The range around η=1 and α=-0.3 contains a minimum, but it is elongated. However, the most important feature of this plot is that the penalty function E_A is primarily sensitive to changes in α and very insensitive to changes in η. So inversion for η and α can not be performed stabily by means of the penalty function E_A alone. Figure 7b shows the same type of plot for the penalty function E_ϕ. This penalty function is primarily sensitive to changes in η, so the penalty function E might have a well defined minimum. In figure 7c we can see the sum of the two penalty functions, using a weighting factor W=1. It is clear that the error surface has a pronounced minimum at the optimal parameter values.

Inversion for α and ηi. An inversion on α and η was performed, with the data of the foregoing subsection, making use of the quasi-Newton method. Choosing quite arbitrarily starting values of α=-0.6 and η=1.7, the inversion method converged to the optimal solution in 6 iterations.

Conclusions

It has been shown that it is possible to derive a consistent singularity parameter α from both the well-log and the seismic reflection response, making use of the information from the amplitude along *modulus maxima* planes in the cube $\check{R}(p, \sigma, z)$. The results obtained from seismic data modeled in a real well-log, makes us to expect that the singularity exponent α might prove to be a useful seismic indicator.

Further, we have shown that in an extended parameterization of the self similar interfaces, with parameters η and α, these parameters can both be resolved from the reflection response. For this purpose an inversion scheme has been developed, which makes use of both amplitude and instantaneous phase information of the reflection event. A synthetic test showed that the inversion is stable.

Acknowledgement The work reported here was supported under a grant from the Dutch Science Foundation STW (DTN 44.3547).

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