Controlled-source seismic interferometry by multi-dimensional deconvolution applied to a laterally varying elastic model

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In controlled-source seismic interferometry (SI), one typical aim is to redatum an array of sources to a receiver location. In cross-correlation (CC) based SI this goal is achieved by cross-correlating registrations at two receiver locations and summation over sources. Although a closed boundary of multi-component sources surrounding the receiver array is required, applications generally involve one-sided illumination and single source types only, which can result in spurious artifacts, erroneous kinematics and incorrect amplitudes. Therefore it can be helpful in some cases to replace cross-correlation by multi-dimensional deconvolution (MDD). In MDD based SI the reflection response is retrieved by inverting a general integral equation, which we implement in a least-squares sense. Additional advantages include improved radiation characteristics of the retrieved (virtual) sources and a relaxation of some assumptions, including the absence of loss terms and knowledge of the source wavelet. Disadvantages include the need for accurate wavefield decomposition and instabilities that might occur in the matrix inversion that forms the core of MDD. We apply SI by both MDD and CC to an elastic model with lateral variations and report slight improvements of MDD compared to CC.

Introduction

Recently many researchers have focused on Cross-Correlation-based (CC) redatuming methods, like Reverse Time Acoustics (RTA), Interferometric Imaging and the Virtual Source (VS) method (Schuster & Zhou, 2006; Bakulin and Calvert, 2006). Most of these methods have been derived from time-reversal arguments or Seismic Interferometry (SI), requiring a closed boundary of sources to retrieve the Green's function between two receiver locations. In practice these methods are often applied to cases of one-sided illumination, which can yield an incomplete retrieval of the reflection response and the emergence of spurious events (Wapenaar, 2006; Snieder et al., 2006). Moreover, these CC-based methods generally require a lossless medium, which may not always be a valid assumption in practice. For these reasons it may in some cases be advantageous to replace Cross-Correlation by Multi-Dimensional deconvolution (MDD). We present the theory of SI by MDD and we demonstrate this methodology with a 2D synthetic elastic example.

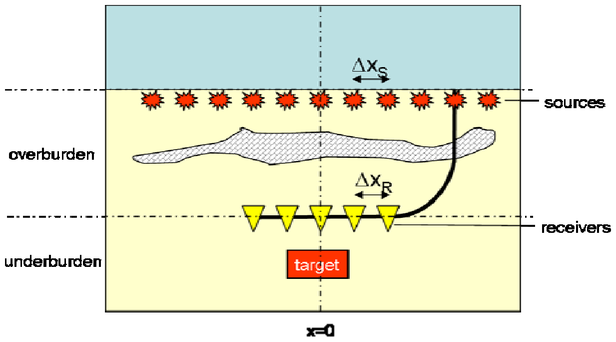

Figure 1: *Typical configuration for a redatuming problem as under discussion.*

Controlled-Source Seismic Interferometry by Multi-Dimensional Deconvolution

The configuration of a typical redatuming problem is shown in Figure 1. Aim is to redatum the shots at the surface to the receiver locations in a (horizontal) well without the need of an overburden velocity model, thus bringing the data closer to the target area, which could be advantageous if the overburden has a complex character. In our approach we collect the wavefields at the receiver level and decompose them into downgoing components $\hat{\mathbf{p}}^{+}(\mathbf{x}_R, \mathbf{x}_S; \omega)$ and upgoing components $\hat{\mathbf{p}}^{-}(\mathbf{x}_R, \mathbf{x}_S; \omega)$, where $\mathbf{x}_S$ and $\mathbf{x}_R$ denote the source and receiver coordinates, respectively. The hat indicates the frequency domain and ω is the angular frequency. The wavefields are powerflux-normalized and stored in 3-C vectors, holding P-, Sv and Sh-components. In this notation a general integral equation can be formulated (Wapenaar & Verschuur, 1996; Amundsen, 1999; Holvik & Amundsen, 2005; Schuster & Zhou, 2006; Wapenaar et al., 2008):

$$\hat{\mathbf{p}}^{-}(\mathbf{x}_A, \mathbf{x}_S, \omega) = \int_{\partial D_B} \hat{\mathbf{R}}_0^{+}(\mathbf{x}_A, \mathbf{x}_B, \omega)\hat{\mathbf{p}}^{+}(\mathbf{x}_B, \mathbf{x}_S, \omega) d\mathbf{x}_B \,. \tag{1}$$

The integral is carried out over a surface ∂D_B over the receiver coordinates $\mathbf{x}_B$ that should capture a fair representation of the downgoing wave field. Matrix $\hat{\mathbf{R}}_0^{+}(\mathbf{x}_A, \mathbf{x}_B, \omega)$ represents the multi-component reflection response of the half space below between ∂D_B receiver locations $\mathbf{x}_B$ and $\mathbf{x}_A$, with the upper half space replaced by a homogeneous medium. Various authors have shown how $\hat{\mathbf{R}}_0^{+}(\mathbf{x}_A, \mathbf{x}_B, \omega)$ can be retrieved by cross-correlation of the upgoing wave field at $\mathbf{x}_A$ with the downgoing wave field at $\mathbf{x}_B$ and summation over the source locations $\mathbf{x}_S$ (Schuster & Zhou, 2006; Mehta et al., 2007). We solve equation (1) by Multi-Dimensional Deconvolution (Wapenaar et al., 2008), which is closely related to least-squares redatuming (Schuster & Zhou, 2006).

We evaluate equation (1) for various source types and locations and rewrite the resulting equations in matrix form as

$$\hat{\mathsf{P}}^{-} = \hat{\mathsf{R}}_0^{+}\hat{\mathsf{P}}^{+} . \qquad (2)$$

Here $\hat{\mathsf{P}}^{+}$ is a matrix of vectors $\hat{\mathbf{p}}^{\pm}\left(\mathbf{x}_{A/B}, \mathbf{x}_S, \omega\right)$, where the columns have fixed source type and location but variable receiver type and location and the rows have fixed receiver type and location but variable source type and location. $\hat{\mathsf{R}}_0^{+}$ is a matrix of reflection matrices $\hat{\mathbf{R}}_0^{+}\left(\mathbf{x}_A, \mathbf{x}_B, \omega\right)$, holding the different wave mode reflections as its components. Equation (2) can be solved by least-squares inversion as

$$\hat{\mathsf{R}}_0^{+} \approx \hat{\mathsf{P}}^{-}\left\{\hat{\mathsf{P}}^{+}\right\}^{H}\left[\hat{\mathsf{P}}^{+}\left\{\hat{\mathsf{P}}^{+}\right\}^{H} + \varepsilon^2\mathsf{I}\right]^{-1}, \qquad (3)$$

where ε is introduced as a stabilization factor, superscript H denotes the complex-conjugate transpose and I is the identity matrix. In practice, the least-squares solution is updated with an iterative updating scheme. If the term between square brackets is approximated by the identity matrix, equation (3) converges to the CC-based solution:

$$\hat{\mathsf{R}}_0^{+} \approx \hat{\mathsf{P}}^{-}\left\{\hat{\mathsf{P}}^{+}\right\}^{H} . \qquad (4)$$

If we rewrite equation (4) in integral form we find:

$$\hat{\mathbf{R}}_0^{+}\left(\mathbf{x}_A, \mathbf{x}_B, \omega\right) = \int_{\partial D_S} \hat{\mathbf{p}}^{-}\left(\mathbf{x}_A, \mathbf{x}_S, \omega\right)\left\{\hat{\mathbf{p}}^{+}\left(\mathbf{x}_B, \mathbf{x}_S, \omega\right)\right\}^{H} d\mathbf{x}_S , \qquad (5)$$

where the integral is carried out over the source location $\mathbf{x}_S$. Equation 5 can be interpreted as a multi-component version of the Virtual Source (VS) method (Bakulin and Calvert, 2006). MDD can thus be seen as applying an addition filter $\left[\hat{\mathsf{P}}^{+}\left\{\hat{\mathsf{P}}^{+}\right\}^{H} + \varepsilon^2\mathsf{I}\right]^{-1}$ over existing Virtual Source data. This filter could enhance the amplitude recovery, improve kinematic behavior, correct for the presence of loss terms and handle non-uniform sampling of the sources in some cases.

Example

We now apply MDD and CC to a laterally varying elastic model – see Figure 2. 75 2-component sources are deployed at the surface with a spacing of 32m. A dense array of 128 receivers is deployed down the borehole with a spacing of 8m. Purpose is to redatum the sources to the receiver array without requiring a velocity model of the overburden. We do so by CC (equation 4) and MDD (equation 3) and compare results both in the time and frequency domain. In Figure 3 we show the results for the PP reflection response in the time domain. MDD has yield small improvements with respect to CC in terms of kinematics as well as a reduction of the spurious events below 0.6 seconds. Notice that the left flank of the first reflector could not be imaged correctly, due to insufficient illumination. Also for the PS converted reflection response, MDD shows a better kinematic match with the reference response – see Figure 4. If we observe the data in the frequency domain, the result of the MDD-filter is much more pronounced. For both PP and PS data, MDD results in a better retrieval of the frequency spectrum compared to CC as can be observed in Figures 5 and 6.

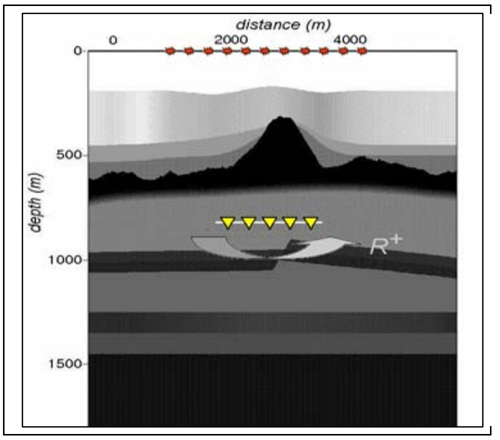

Figure 2: *Sketch of the P-wave velocity model of the 2D synthetic elastic example. 75 2-component sources are located at the surface with a spacing of 32m. A receiver array of 128 receivers is deployed down a borehole with a spacing of 8m at 800 meter depth. Our purpose is to image the reflectors below the receiver array without requiring information of the overburden.*

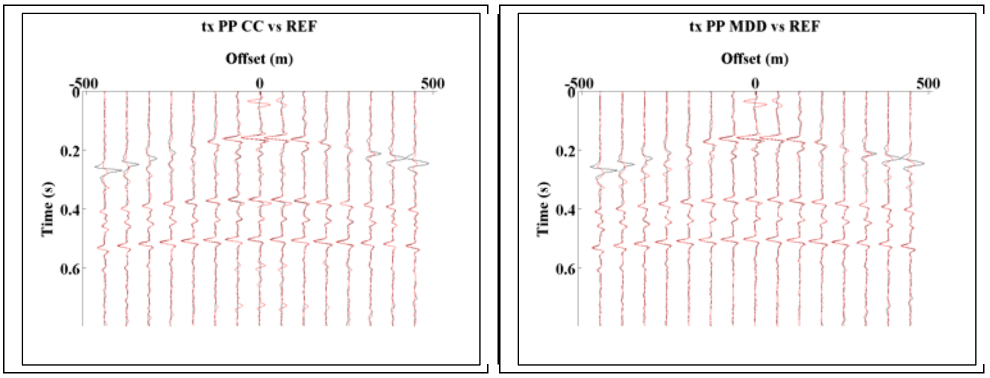

Figure 4: *left: PP reflection response by CC (red) versus reference response (black); right: PP reflection response by MDD (red) versus reference response (black).*

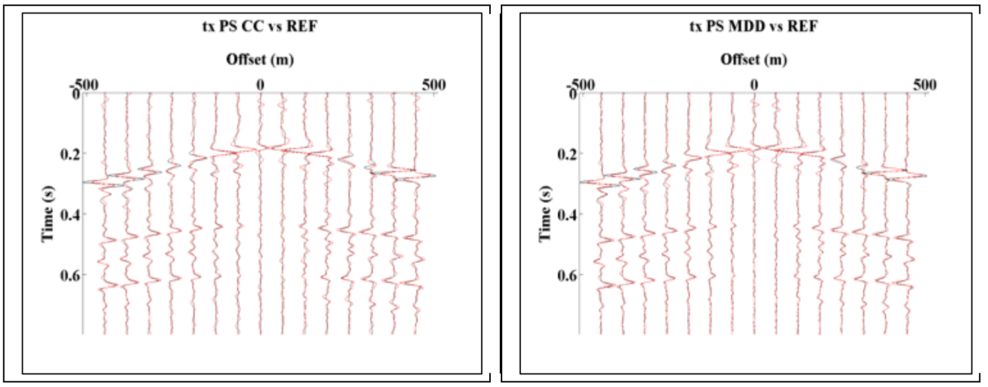

Figure 5: *left: PS reflection response by CC (red) versus reference response (black); right: PS reflection response by MDD (red) versus reference response (black).*

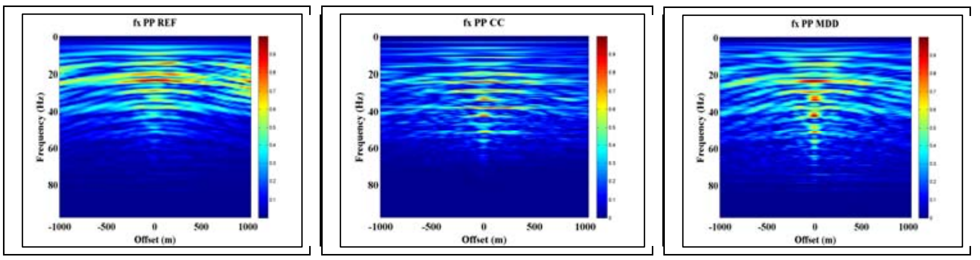

Figure 6: *FX-representation of the PP reflection response; reference response (left), CC (middle) and MDD (right).*

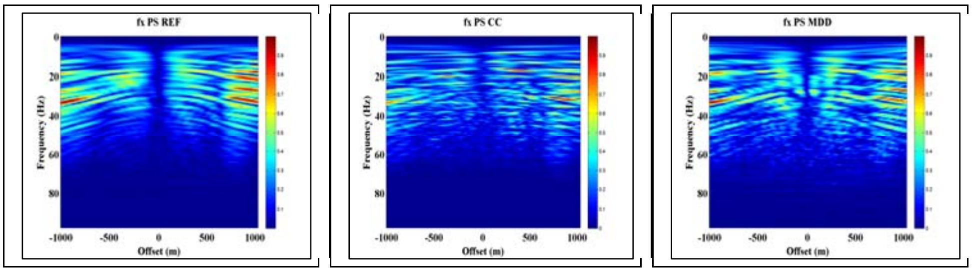

Figure 7: *FX-representation of the PS reflection response; reference response (left), CC (middle) and MDD (right).*

Conclusion and discussion

We have demonstrated that in some cases Seismic Interferometry by Multi-Dimensional Deconvolution (MDD) can accurately redatum sources at the earth surface to receivers in a borehole without requiring a velocity model of the overburden. In comparison to Seismic Interferometry by Cross-Correlation (CC), we have reported small visible improvements in the time domain but significant improvements of the retrieved frequency spectrum. Moreover, MDD has potentials to properly handle loss-terms and account for non-uniform source distributions. There are also some drawbacks of MDD. To correctly sample the wavefield, the receiver array needs to have sufficient aperture and to avoid spatial aliasing, dense sampling is required. Moreover, the inversion process can be hard to stabilize compared to CC.

Acknowledgements

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