REFLECTION AND TRANSMISSION OF WAVES AT A FLUID/POROUS-MEDIUM BOUNDARY

A.I.M. DENNEMAN, G.G. DRIJKONINGEN, D.M.J. SMEULDERS, AND C.P.A. WAPENAAR
Centre for Technical Geoscience
Delft University of Technology
P.O. Box 5028, 2600 GA Delft
The Netherlands

Abstract. Relatively simple closed-form expressions are derived for the reflection and transmission coefficients belonging to a fluid/porous-medium interface with open-pore boundary conditions. The wave propagation in the fluid-saturated porous-medium is described using Biot's theory and it is assumed that both the porous skeleton and the pore fluid are much more compressible than the skeletal solid grains themselves. The obtained results find their application in forward and inverse surface wave analysis.

1. Introduction

To calculate the reflection and transmission coefficients belonging to a fluid/porous-medium interface, one normally uses the boundary conditions of Deresiewicz and Skalak [1]. In a number of papers [2–7] it has been shown that these boundary conditions lead to a set of four linear equations with the reflection and transmission coefficients as the four unknowns.

It is rather easy to solve this set of equations numerically, while closed-form expressions for the reflection and transmission coefficients can be obtained by applying the well-known Cramer's rule (each coefficient is then equal to the ratio of two determinants of two different 4×4 matrices). Simply applying Cramer's rule results in closed-form expressions for the reflection and transmission coefficients that are very complicated. Due to this complexity it is rather difficult to acquire a good physical insight in the dependencies of these coefficients on the many measurable quantities defining the fluid/porous-medium interface.

We will show in this paper that it is possible to derive expressions for the reflection and transmission coefficients that are much simpler than the ones already known. We note, however, that we have assumed in this derivation that both the porous skeleton and the pore fluid are much more compressible than the skeletal solid grains themselves. The availability of simpler expressions for the reflection and transmission coefficients contributes to a better physical understanding of the reflection and transmission properties of waves at a fluid/porous-medium boundary.

We finally note that we restrict ourselves to a fluid/porous-medium interface with open-pore boundary conditions, while results associated with sealed-pore boundary conditions can be found in a different paper [8].

2. Waves at a fluid/porous-medium boundary

According to Biot's theory [9] three different types of waves may propagate through a porous material: a fast P-wave, a slow P-wave, and a S-wave. Consequently, at the fluid/porous-medium boundary an incident P-wave in the fluid is converted simultaneously into (i) a reflected P-wave, (ii) a transmitted fast P-wave, (iii) a transmitted slow P-wave, and (iv) a transmitted S-wave (see Fig. 1).

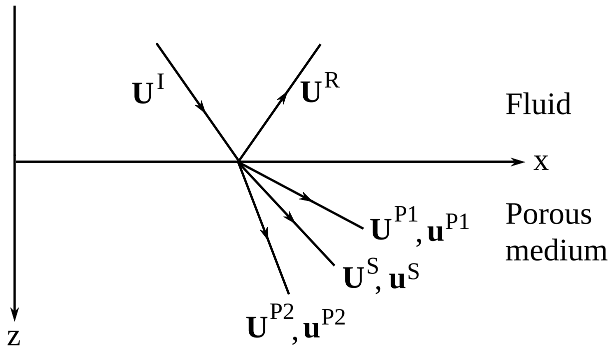

Figure 1. Wave conversion at a fluid/porous-medium boundary. The fluid displacements belonging to the incident and reflected P-wave are denoted by $\mathbf{U}^{\mathrm{I}}$ and $\mathbf{U}^{\mathrm{R}}$, respectively. The fluid displacements belonging to the fast P-wave, slow P-wave, and S-wave are denoted by $\mathbf{U}^{\mathrm{P1}}$, $\mathbf{U}^{\mathrm{P2}}$, and $\mathbf{U}^{\mathrm{S}}$, respectively, whereas the corresponding solid displacements are denoted by $\mathbf{u}^{\mathrm{P1}}$, $\mathbf{u}^{\mathrm{P2}}$, and $\mathbf{u}^{\mathrm{S}}$, respectively.

The fluid displacements in the x-z plane belonging to the incident and reflected P-wave are in the space-frequency domain given by

$$\mathbf{U}^{\mathrm{I}} = \begin{pmatrix} U_{\mathrm{x}}^{\mathrm{I}}(x,z,\omega) \\ U_{\mathrm{z}}^{\mathrm{I}}(x,z,\omega) \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} A^{\mathrm{I}} \exp\left[-j\omega(px+qz)\right], \tag{1}$$

$$\mathbf{U}^{\mathrm{R}} = \begin{pmatrix} U_{\mathrm{x}}^{\mathrm{R}}(x,z,\omega) \\ U_{\mathrm{z}}^{\mathrm{R}}(x,z,\omega) \end{pmatrix} = \begin{pmatrix} p \\ -q \end{pmatrix} A^{\mathrm{R}} \exp\left[-j\omega(px-qz)\right], \tag{2}$$

where ω is the angular frequency, p the horizontal slowness, q the vertical slowness with a positive real part and a negative imaginary part, and A^{I} and A^{R} are wave-amplitudes. The slownesses p and q are related to the propagation velocity c as

$$p^2 + q^2 = \frac{1}{c^2} = \frac{\rho}{K}, \tag{3}$$

where ρ is the fluid density and K the fluid bulk modulus. Furthermore, by combining the deformation equation $P = -K\boldsymbol{\nabla}\cdot(\mathbf{U}^{\mathrm{I}} + \mathbf{U}^{\mathrm{R}})$ with Eqs. (1)–(3) one finds that the fluid pressure P is given by

$$P(x, z, \omega) = j\omega\rho \left[A^{\mathrm{I}} \exp\left[-j\omega(px + qz)\right] + A^{\mathrm{R}} \exp\left[-j\omega(px - qz)\right]\right]. \tag{4}$$

The displacements of the solid skeleton in the x-z plane belonging to the fast P-wave, slow P-wave, and S-wave are given by

$$\mathbf{u}^{\mathrm{P1}} = \begin{pmatrix} u_{\mathrm{x}}^{\mathrm{P1}}(x, z, \omega) \\ u_{\mathrm{z}}^{\mathrm{P1}}(x, z, \omega) \end{pmatrix} = \begin{pmatrix} p \\ q_{\mathrm{P1}} \end{pmatrix} A^{\mathrm{P1}} \exp\left[-j\omega(px + q_{\mathrm{P1}}z)\right], \tag{5}$$

$$\mathbf{u}^{\mathrm{P2}} = \begin{pmatrix} u_{\mathrm{x}}^{\mathrm{P2}}(x, z, \omega) \\ u_{\mathrm{z}}^{\mathrm{P2}}(x, z, \omega) \end{pmatrix} = \begin{pmatrix} p \\ q_{\mathrm{P2}} \end{pmatrix} A^{\mathrm{P2}} \exp\left[-j\omega(px + q_{\mathrm{P2}}z)\right], \tag{6}$$

$$\mathbf{u}^{\mathrm{S}} = \begin{pmatrix} u_{\mathrm{x}}^{\mathrm{S}}(x, z, \omega) \\ u_{\mathrm{z}}^{\mathrm{S}}(x, z, \omega) \end{pmatrix} = \begin{pmatrix} -q_{\mathrm{S}} \\ p \end{pmatrix} A^{\mathrm{S}} \exp\left[-j\omega(px + q_{\mathrm{S}}z)\right], \tag{7}$$

where A^{P1}, A^{P2}, and A^{S} are wave-amplitudes. The vertical slownesses q_{P1}, q_{P2}, and q_{S} (all with a positive real part and a negative imaginary part) are related to the horizontal slowness p and the propagation velocities c_{P1}, c_{P2}, and c_{S} as

$$p^2 + q_{\mathrm{P1}}^2 = \frac{1}{c_{\mathrm{P1}}^2}, \qquad p^2 + q_{\mathrm{P2}}^2 = \frac{1}{c_{\mathrm{P2}}^2}, \qquad p^2 + q_{\mathrm{S}}^2 = \frac{1}{c_{\mathrm{S}}^2}. \tag{8}$$

According to Biot's theory [3, 9] the propagation velocities c_{P1}, c_{P2}, and c_{S} are given by

$$c_{\mathrm{P1}}^2 = \frac{c_1 + \sqrt{c_1^2 - 4c_0c_2}}{2c_0}, \qquad c_{\mathrm{P2}}^2 = \frac{c_1 - \sqrt{c_1^2 - 4c_0c_2}}{2c_0}, \qquad c_{\mathrm{S}}^2 = \frac{G\rho_{22}}{c_0} \tag{9}$$

with

$$c_0 = \rho_{11}\rho_{22} - \rho_{12}^2, \qquad c_2 = R(A + 2G) - Q^2, \tag{10}$$

$$c_1 = R\rho_{11} + (A + 2G)\rho_{22} - 2Q\rho_{12}, \tag{11}$$

in which G is the shear modulus of the porous material. The generalized elastic coefficients A, Q, and R are related to measurable quantities by the following expressions [3, 10]

$$A = \frac{(1-\phi)^2 K_s K_f - (1-\phi)K_b K_f + \phi K_s K_b}{K_f(1-\phi-K_b/K_s)+\phi K_s} - \frac{2}{3}G, \qquad (12)$$

$$Q = \frac{\phi K_f (K_s(1-\phi)-K_b)}{K_f(1-\phi-K_b/K_s)+\phi K_s}, \qquad (13)$$

$$R = \frac{\phi^2 K_f K_s}{K_f(1-\phi-K_b/K_s)+\phi K_s}, \qquad (14)$$

where K_s is the skeletal grain bulk modulus, K_f the pore fluid bulk modulus, K_b the "jacketed" bulk modulus of the porous material, and ϕ the porosity (pore fluid volume divided by bulk volume).

The density terms ρ_{11}, ρ_{22}, and ρ_{12} in Eqs. (9)–(11) are defined as

$$\rho_{11} = (1-\phi)\rho_s - \rho_{12}, \qquad \rho_{22} = \phi\rho_f - \rho_{12}, \qquad \rho_{12} = -(\alpha-1)\phi\rho_f, \quad (15)$$

where ρ_s and ρ_f are the densities of the solid skeleton and the pore fluid, respectively. According to Johnson *et al.* [11] the drag coefficient α belonging to a fluid-saturated porous material can be defined as

$$\alpha = \alpha_\infty \left(1 - j\frac{\omega_c}{\omega}\sqrt{1 + j\frac{M}{2}\frac{\omega}{\omega_c}}\right) \qquad \text{with} \qquad \omega_c = \frac{\eta\phi}{k_0 \rho_f \alpha_\infty}, \qquad (16)$$

where $M \approx 1$ is the so-called similarity parameter, α_∞ the inertial drag at infinite frequency, η the pore fluid viscosity, k_0 the permeability of the porous material, and the critical frequency ω_c is the frequency at which the inertial and viscous drag are of comparable magnitude.

According to Biot's theory the pore fluid displacements $\mathbf{U}^{P1}$, $\mathbf{U}^{P2}$, and $\mathbf{U}^{S}$ are related to the solid skeleton displacements $\mathbf{u}^{P1}$, $\mathbf{u}^{P2}$, and $\mathbf{u}^{S}$ as

$$\mathbf{U}^{P1} = G_{P1}\mathbf{u}^{P1} \quad \text{with} \quad G_{P1} = \frac{Q - c_{P1}^2\rho_{12}}{c_{P1}^2\rho_{22} - R} = \frac{A + 2G - c_{P1}^2\rho_{11}}{c_{P1}^2\rho_{12} - Q}, \quad (17)$$

$$\mathbf{U}^{P2} = G_{P2}\mathbf{u}^{P2} \quad \text{with} \quad G_{P2} = \frac{Q - c_{P2}^2\rho_{12}}{c_{P2}^2\rho_{22} - R} = \frac{A + 2G - c_{P2}^2\rho_{11}}{c_{P2}^2\rho_{12} - Q}, \quad (18)$$

$$\mathbf{U}^{S} = G_{S}\mathbf{u}^{S} \quad \text{with} \quad G_{S} = \frac{-\rho_{12}}{\rho_{22}} = \frac{\alpha - 1}{\alpha}. \quad (19)$$

The pore fluid stress $\boldsymbol{\tau}_f$ and solid skeleton stress $\boldsymbol{\tau}_s$ are defined as

$$\boldsymbol{\tau}_f = -\phi P_f \boldsymbol{\delta} = Q(\nabla\cdot\mathbf{u})\boldsymbol{\delta} + R(\nabla\cdot\mathbf{U})\boldsymbol{\delta}, \qquad (20)$$

$$\boldsymbol{\tau}_s = -\boldsymbol{\sigma} - (1-\phi)P_f\boldsymbol{\delta} = G\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right] + A(\nabla\cdot\mathbf{u})\boldsymbol{\delta} + Q(\nabla\cdot\mathbf{U})\boldsymbol{\delta} \quad (21)$$

with $\mathbf{u} = \mathbf{u}^{\mathrm{P1}} + \mathbf{u}^{\mathrm{P2}} + \mathbf{u}^{\mathrm{S}}$ and $\mathbf{U} = \mathbf{U}^{\mathrm{P1}} + \mathbf{U}^{\mathrm{P2}} + \mathbf{U}^{\mathrm{S}}$, and where P_{f} is the pore fluid pressure, $\boldsymbol{\delta}$ a unit tensor, and $\boldsymbol{\sigma}$ the intergranular stress tensor.

3. Reflection and transmission coefficients

The reflection and transmission coefficients R^{F}, T^{P1}, T^{P2}, and T^{S} are related to the wave-amplitudes A^{I}, A^{R}, A^{P1}, A^{P2}, and A^{S} as

$$A^{\mathrm{R}} = R^{\mathrm{F}} A^{\mathrm{I}}, \qquad A^{\mathrm{P1}} = T^{\mathrm{P1}} A^{\mathrm{I}}, \qquad A^{\mathrm{P2}} = T^{\mathrm{P2}} A^{\mathrm{I}}, \qquad A^{\mathrm{S}} = T^{\mathrm{S}} A^{\mathrm{I}}. \tag{22}$$

To solve R^{F}, T^{P1}, T^{P2}, and T^{S} we use boundary conditions that are valid for a fluid/porous-medium boundary with open pores [1, 3, 12]. Hence, at the boundary $z = 0$:

$$\phi(U_{\mathrm{z}}^{\mathrm{P1}} + U_{\mathrm{z}}^{\mathrm{P2}} + U_{\mathrm{z}}^{\mathrm{S}}) + (1-\phi)(u_{\mathrm{z}}^{\mathrm{P1}} + u_{\mathrm{z}}^{\mathrm{P2}} + u_{\mathrm{z}}^{\mathrm{S}}) = U_{\mathrm{z}}^{\mathrm{I}} + U_{\mathrm{z}}^{\mathrm{R}}, \tag{23}$$

$$P_{\mathrm{f}} = P, \qquad \sigma_{\mathrm{xz}} = 0, \qquad -\sigma_{\mathrm{zz}} - P_{\mathrm{f}} = -P. \tag{24}$$

By combining these boundary conditions with Eqs. (1), (2), (4)–(8), and (17)–(21) in an appropriate way one obtains the following set of equations

$$\begin{bmatrix} q & q_{\mathrm{P1}}(1-\phi+\phi G_{\mathrm{P1}}) & q_{\mathrm{P2}}(1-\phi+\phi G_{\mathrm{P2}}) & p(1-\phi+\phi G_{\mathrm{S}}) \\ -\rho\phi & \dfrac{Q+RG_{\mathrm{P1}}}{c_{\mathrm{P1}}^2} & \dfrac{Q+RG_{\mathrm{P2}}}{c_{\mathrm{P2}}^2} & 0 \\ 0 & pq_{\mathrm{P1}} & pq_{\mathrm{P2}} & \left(p^2 - \dfrac{1}{2c_{\mathrm{S}}^2}\right) \\ 0 & \left(p^2 - \dfrac{N_1}{2Gc_{\mathrm{P1}}^2}\right) & \left(p^2 - \dfrac{N_2}{2Gc_{\mathrm{P2}}^2}\right) & -pq_{\mathrm{S}} \end{bmatrix} \begin{bmatrix} R^{\mathrm{F}} \\ T^{\mathrm{P1}} \\ T^{\mathrm{P2}} \\ T^{\mathrm{S}} \end{bmatrix} = \begin{bmatrix} q \\ \rho\phi \\ 0 \\ 0 \end{bmatrix} \tag{25}$$

with

$$N_1 = A + 2G - \frac{1-\phi}{\phi}Q + G_{\mathrm{P1}}\left(Q - \frac{1-\phi}{\phi}R\right), \tag{26}$$

$$N_2 = A + 2G - \frac{1-\phi}{\phi}Q + G_{\mathrm{P2}}\left(Q - \frac{1-\phi}{\phi}R\right). \tag{27}$$

To obtain relatively simple closed-form expressions for R^{F}, T^{P1}, T^{P2}, and T^{S} we assume that both the porous skeleton and the pore fluid are much more compressible than the skeletal solid grains themselves. Consequently, the substitution of $K_{\mathrm{s}} \gg K_{\mathrm{b}}$ and $K_{\mathrm{s}} \gg K_{\mathrm{f}}$ in Eqs. (12)–(14) leads to

$$A = \frac{(1-\phi)^2}{\phi}K_{\mathrm{f}} + K_{\mathrm{b}} - \frac{2}{3}G, \qquad Q = (1-\phi)K_{\mathrm{f}}, \qquad R = \phi K_{\mathrm{f}}. \tag{28}$$

By combining Eq. (28) with Eqs. (9), (10), (15), (17)–(19), (26), and (27) one obtains

$$1 - \phi + \phi G_{\mathrm{S}} = \gamma_0 = 1 - \frac{\phi}{\alpha}, \tag{29}$$

$$1-\phi+\phi G_{\mathrm{P1}}=\gamma_0(1+\gamma_1) \quad \text{with} \quad \gamma_1=\frac{K_{\mathrm{f}}}{\rho_{\mathrm{f}}\alpha c_{\mathrm{P1}}^2-K_{\mathrm{f}}}, \tag{30}$$

$$1-\phi+\phi G_{\mathrm{P2}}=\gamma_0(1+\gamma_2) \quad \text{with} \quad \gamma_2=\frac{K_{\mathrm{f}}}{\rho_{\mathrm{f}}\alpha c_{\mathrm{P2}}^2-K_{\mathrm{f}}}, \tag{31}$$

$$c_{\mathrm{P1}}^2 c_{\mathrm{P2}}^2=\frac{K_{\mathrm{p}}K_{\mathrm{f}}}{G\alpha\rho_{\mathrm{f}}}c_{\mathrm{S}}^2 \quad \text{with} \quad K_{\mathrm{p}}=N_1=N_2=K_{\mathrm{b}}+\frac{4}{3}G, \tag{32}$$

where K_{p} is the constrained modulus. The substitution of Eqs. (28)–(32) in the four boundary conditions represented by Eq. (25) leads to

$$\begin{bmatrix} q & q_{\mathrm{P1}}\gamma_0(1+\gamma_1) & q_{\mathrm{P2}}\gamma_0(1+\gamma_2) & p\gamma_0 \\ -\rho\phi & \rho_{\mathrm{f}}(\alpha-\phi)\gamma_1 & \rho_{\mathrm{f}}(\alpha-\phi)\gamma_2 & 0 \\ 0 & pq_{\mathrm{P1}} & pq_{\mathrm{P2}} & \left(p^2-\frac{1}{2c_{\mathrm{S}}^2}\right) \\ 0 & \left(p^2-\frac{K_{\mathrm{p}}}{2Gc_{\mathrm{P1}}^2}\right) & \left(p^2-\frac{K_{\mathrm{p}}}{2Gc_{\mathrm{P2}}^2}\right) & -pq_{\mathrm{S}} \end{bmatrix} \begin{bmatrix} R^{\mathrm{F}} \\ T^{\mathrm{P1}} \\ T^{\mathrm{P2}} \\ T^{\mathrm{S}} \end{bmatrix} = \begin{bmatrix} q \\ \rho\phi \\ 0 \\ 0 \end{bmatrix}. \tag{33}$$

By solving this set of linear equations one finds that the closed-form expressions for R^{F}, T^{P1}, T^{P2}, and T^{S} are given by

$$R^{\mathrm{F}}=\frac{R_1-R_2}{R_1+R_2}, \qquad T^{\mathrm{P1}}=\frac{\Delta_{\mathrm{R2}}^{\mathrm{S,P2}}}{R_1+R_2}, \qquad T^{\mathrm{P2}}=\frac{-\Delta_{\mathrm{R1}}^{\mathrm{S,P1}}}{R_1+R_2}, \tag{34}$$

$$T^{\mathrm{S}}=\frac{pq_{\mathrm{P2}}\Delta_{\mathrm{R1}}^{\mathrm{S,P1}}-pq_{\mathrm{P1}}\Delta_{\mathrm{R2}}^{\mathrm{S,P2}}}{R_1+R_2}\left(p^2-\frac{1}{2c_{\mathrm{S}}^2}\right)^{-1}, \tag{35}$$

where R_1 and R_2 are defined as

$$\begin{aligned} R_1 &= \frac{\rho_{\mathrm{f}}}{2\rho}\left(1-\frac{\alpha}{\phi}\right)(\gamma_2\Delta_{\mathrm{R1}}-\gamma_1\Delta_{\mathrm{R2}}), \\ &= \frac{\rho_{\mathrm{f}}}{2\rho}\left(1-\frac{\alpha}{\phi}\right)(\gamma_2\Delta_{\mathrm{R1}}^{\mathrm{S,P1}}-\gamma_1\Delta_{\mathrm{R2}}^{\mathrm{S,P2}}), \end{aligned} \tag{36}$$

$$R_2 = \frac{1}{2q}\left(\frac{\phi}{\alpha}-1\right)(q_{\mathrm{P2}}\gamma_2\Delta_{\mathrm{R1}}^{\mathrm{P1,P1}}-q_{\mathrm{P1}}\gamma_1\Delta_{\mathrm{R2}}^{\mathrm{P2,P2}}), \tag{37}$$

while Δ_{R1}, $\Delta_{\mathrm{R1}}^{\mathrm{S,P1}}$, $\Delta_{\mathrm{R1}}^{\mathrm{P1,P1}}$, Δ_{R2}, $\Delta_{\mathrm{R2}}^{\mathrm{S,P2}}$, and $\Delta_{\mathrm{R2}}^{\mathrm{P2,P2}}$ are given by

$$\Delta_{\mathrm{R1}} = p^2 q_{\mathrm{S}}q_{\mathrm{P1}}+\left(p^2-\frac{1}{2c_{\mathrm{S}}^2}\right)^2, \tag{38}$$

$$\Delta_{\mathrm{R1}}^{\mathrm{S,P1}} = p^2 q_{\mathrm{S}}q_{\mathrm{P1}}+\left(p^2-\frac{1}{2c_{\mathrm{S}}^2}\right)\left(p^2-\frac{K_{\mathrm{p}}}{2Gc_{\mathrm{P1}}^2}\right), \tag{39}$$

$$\Delta_{\mathrm{R1}}^{\mathrm{P1,P1}} = p^2 q_{\mathrm{S}}q_{\mathrm{P1}}+\left(p^2-\frac{K_{\mathrm{p}}}{2Gc_{\mathrm{P1}}^2}\right)^2, \tag{40}$$

$$\Delta_{\mathrm{R2}} = p^2 q_{\mathrm{S}} q_{\mathrm{P2}} + \left(p^2 - \frac{1}{2c_{\mathrm{S}}^2}\right)^2, \tag{41}$$

$$\Delta_{\mathrm{R2}}^{\mathrm{S,P2}} = p^2 q_{\mathrm{S}} q_{\mathrm{P2}} + \left(p^2 - \frac{1}{2c_{\mathrm{S}}^2}\right)\left(p^2 - \frac{K_{\mathrm{p}}}{2Gc_{\mathrm{P2}}^2}\right), \tag{42}$$

$$\Delta_{\mathrm{R2}}^{\mathrm{P2,P2}} = p^2 q_{\mathrm{S}} q_{\mathrm{P2}} + \left(p^2 - \frac{K_{\mathrm{p}}}{2Gc_{\mathrm{P2}}^2}\right)^2. \tag{43}$$

Note that Δ_{R1} (multiplied by 4) is the Rayleigh function $\mathsf{R}(p)$, which is associated with surface waves traveling along a solid/vacuum boundary [13].

In Fig. 2 results are shown belonging to an interface between water and a water-saturated sand-layer. These results can be easily transformed into figures showing R^{F}, T^{P1}, T^{P2}, and T^{S} as a function of the incident angle θ by using the relation $\theta = \arcsin(pc)$ for the region $|pc| \leq 1$. In the region $|pc| > 1$ the P-waves in the fluid are evanescent; in this region one observes that the reflection and transmission coefficients are very large for $pc \approx 1.44$. The reciprocal of this p-value is equal to the propagation velocity of the surface wave traveling along the interface between water and a water-saturated sand layer, i.e., it is 0.7 times the P-wave velocity in water.

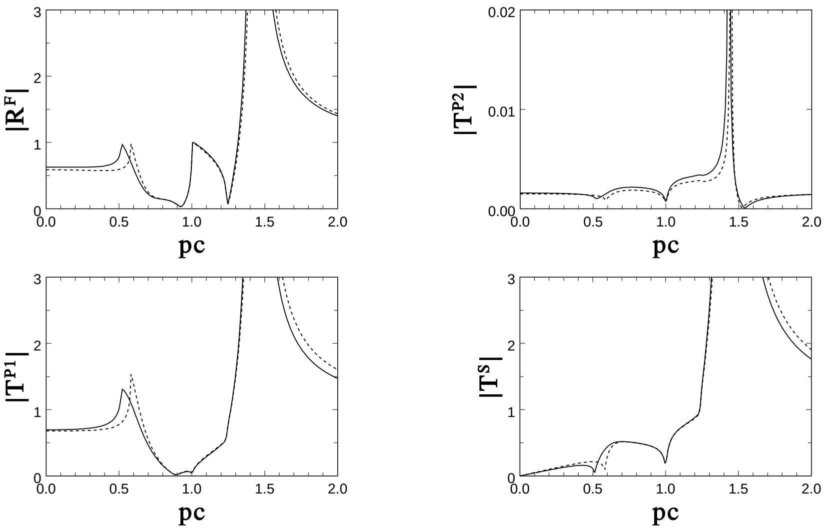

Figure 2. The coefficients R^{F}, T^{P1}, T^{P2}, and T^{S} as a function of the horizontal slowness p times the propagation velocity c of the P-wave in the fluid. The solid lines correspond to the case $K_{\mathrm{s}}/K_{\mathrm{b}} = \infty$: the coefficients R^{F}, T^{P1}, T^{P2}, and T^{S} are calculated by using Eqs. (34)–(43). The dashed lines correspond to the case $K_{\mathrm{s}}/K_{\mathrm{b}} = 5$: the coefficients R^{F}, T^{P1}, T^{P2}, and T^{S} are obtained by solving numerically the set of equations given by Eq. (25) and by using the expressions for A, Q, and R given by Eqs. (12)–(14) instead of the ones given in Eq. (28). The parameters used to obtain these results are: $\omega = 10000$ rad s^{-1}, $\phi = 0.24$, $\eta = 0.001$ Pa s, $k_0 = 0.39\ 10^{-12}$ m^2, $\rho = \rho_{\mathrm{f}} = 1000$ kg m^{-3}, $\rho_{\mathrm{s}} = 2760$ kg m^{-3}, $K = K_{\mathrm{f}} = 2.22$ G Pa, $K_{\mathrm{b}} = 5.8$ G Pa, $G = 3.4$ G Pa, $\alpha_\infty = 2.3$, and $M = 1$.

4. Concluding remarks

In general, the surface wave velocity can be obtained as follows: find the horizontal slowness p for which the denominator R_1+R_2 of the reflection and transmission coefficients given by Eqs. (34) and (35) is zero. Note, however, that the obtained $p=p_0$ for which $R_1+R_2=0$ is complex-valued. Actually, $\mathrm{Re}(p_0)^{-1}$ is the propagation velocity of the surface wave traveling along the fluid/porous-medium interface, whereas $\mathrm{Im}(-\omega p_0)$ is its attenuation in the propagation direction.

It is clear that the availability of closed-form expressions for R_1 and R_2, as given by Eqs. (36) and (37), will facilitate our research in solving the inverse problem: "given a measured surface wave, find the physical parameters describing the porous medium".

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