C032 ONE-WAY OPERATORS IN LATERALLY VARYING MEDIA

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Introduction
Currently, in constructing one-way operators for recursive wave field extrapolation, it is assumed that the medium parameters are constant within the lateral extent of the operator. This means that for each gridpoint an operator is used which is related only to the local medium parameters at that gridpoint. In this abstract we call this approach "standard". For laterally smoothly varying media (i.e. smooth compared to the wavelength) the standard method leads to fairly accurate results. However, when the order of the lateral variations approaches the wave length scale, the results become unreliable and may even exhibit unstable behaviour (Etgen,1994).

Determining the primary propagator
For notational convenience we make use of Berkhout's matrix notation (Berkhout, 1982). Our efforts to account correctly for lateral variations are directed towards solving the 2-D primary propagator $\underset{\sim}{\mathbf{W}}^+(z,z')$, from the one-way wave equation. For simplicity we assume that the medium parameters locally (i.e. between z and z') have no depth dependence:

$$\frac{\partial \underset{\sim}{\mathbf{W}}^+(z,z')}{\partial z} = -j\ \underset{\sim}{\mathbf{H}}_1\ \underset{\sim}{\mathbf{W}}^+(z,z'), \tag{1}$$

where the square-root operator $\underset{\sim}{\mathbf{H}}_1$ is implicitly defined by

$$\underset{\sim}{\mathbf{H}}_2 = \underset{\sim}{\mathbf{H}}_1\ \underset{\sim}{\mathbf{H}}_1. \tag{2}$$

$\underset{\sim}{\mathbf{H}}_2$ is the operator appearing in the two-way wave equation

$$\frac{\partial^2 \mathbf{P}(z)}{\partial z^2} = -\underset{\sim}{\mathbf{H}}_2\ \mathbf{P}(z) = -\Big[\underset{\sim}{\mathbf{C}} + \underset{\sim}{\mathbf{D}}_2\Big]\ \mathbf{P}(z), \tag{3}$$

where $\mathbf{P}(z)$ is a vector, containing the discretized monochromatic wavefield at depth level z, $\underset{\sim}{\mathbf{C}}$ is a diagonal matrix containing the discretized version of $\{\omega/c(x)\}^2$ and $\underset{\sim}{\mathbf{D}}_2$ contains the discretized version of the second order differentiation filter $d_2(x)$. In order to obtain the primary propagator $\underset{\sim}{\mathbf{W}}^+(z,z')$ we must determine the square-root operator from equation (2). We therefore perform an eigenvalue (*modal*) decomposition to $\underset{\sim}{\mathbf{H}}_2$ (Blok, 1993).

$$\underset{\sim}{\mathbf{H}}_2 = \underset{\sim}{\mathbf{L}}\ \underset{\sim}{\mathbf{\Lambda}}\ \underset{\sim}{\mathbf{L}}^{-1}. \tag{4}$$

The matrix $\underset{\sim}{\mathbf{L}}$ contains the eigenfunctions (*eigenmodes* or *wave modes*) on its columns. Because the matrix $\underset{\sim}{\mathbf{H}}_2$ is symmetrical, the eigenvalues which are stored in the diagonal matrix $\underset{\sim}{\mathbf{\Lambda}}$, are all real valued. By substituting an arbitrary column of $\underset{\sim}{\mathbf{L}}$ in the two-way wave equation (3) it can be shown that the columns of $\underset{\sim}{\mathbf{L}}$ represent wave field constituents having a vertical wavenumber k_z, directly related to the square-root of the corresponding eigenvalue. Furthermore, on using a proper scaling of the eigenvectors, $\underset{\sim}{\mathbf{L}}^{-1}$ equals $\underset{\sim}{\mathbf{L}}^H$, where ^{H} denotes transposition and complex conjugation. We thus obtain

$$\underset{\sim}{\mathbf{H}}_1 = \underset{\sim}{\mathbf{L}}\ \underset{\sim}{\mathbf{\Lambda}}^{1/2}\ \underset{\sim}{\mathbf{L}}^H, \tag{5}$$

where $\underset{\sim}{\mathbf{\Lambda}}^{1/2}$ contains on its diagonal the square-roots of the corresponding diagonal elements of $\underset{\sim}{\mathbf{\Lambda}}$. On physical grounds we chose these square-roots to be either positive real or negative imaginary. Finally, we can deduce a closed expression for $\underset{\sim}{\mathbf{W}}^+(z,z')$ by substituting equation (5) in the one-way wave equation (1)

$$\underset{\sim}{\mathbf{W}}^+(z,z') = \underset{\sim}{\mathbf{L}}\ \exp\{-j\ (z-z')\ \underset{\sim}{\mathbf{\Lambda}}^{1/2}\}\ \underset{\sim}{\mathbf{L}}^H. \tag{6}$$

Here, the exponential operation on $\underset{\sim}{\mathbf{\Lambda}}^{1/2}$ can be replaced by the same operation on each element of $\underset{\sim}{\mathbf{\Lambda}}^{1/2}$.

Results and conclusions

The contents of the matrices in equation (6) are schematically depicted in Figure 1. The matrix $\underset{\sim}{\mathbf{L}}$ contains the eigenmodes as columns, whereas $\underset{\sim}{\mathbf{L}}^H$ contains them as rows. In the diagonal matrix only eigenvalues which correspond to propagating wave constituents are visible. In Figure 2 these eigenvalues are depicted together with the eigenvalues of the standard primary propagator. Comparing the eigenvalue spectra of both operators, it is clear that the standard method causes irregular amplification of wavemodes, whereas the modal decomposition method remains fully stable.

These observations are confirmed by the plane wave response example in Figure 3, where finite difference modeling is compared to both the standard as the modal decomposition method. The standard method shows amplification of the wave field in the low-velocity region. The results of the modal decomposition method exhibit no instabillities and show excellent handling of high-angle wave constituents.

$$\underset{\sim}{\mathbf{W}}^{\pm}(z, z') \quad = \quad \underset{\sim}{\mathbf{L}} \quad \exp\{-j\,(z - z')\,\underset{\sim}{\Lambda}^{1/2}\} \quad \underset{\sim}{\mathbf{L}}^H$$

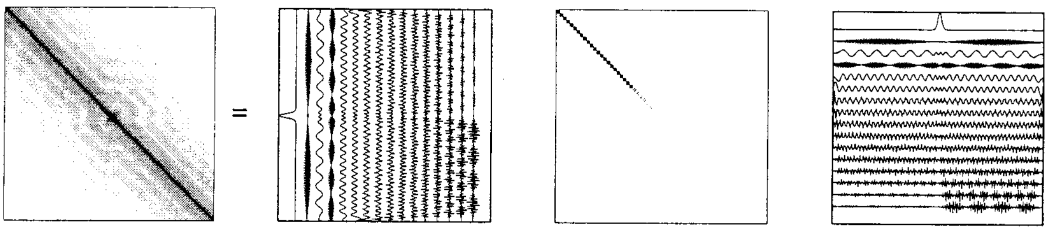

Figure 1: *Schematical representation of the construction of the primary propagator in matrix form. The medium is laterally variant.*

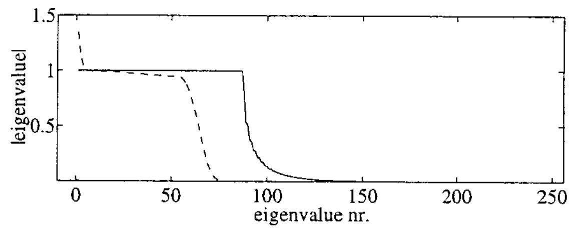

Figure 2: *Absolute value of the eigenvalues of the primary propagator in descending order. Modal decomposition (solid) and standard method (dashed). The extrapolation distance* $|z - z'|$*=50m, the medium corresponds to the example in Figure 1.*

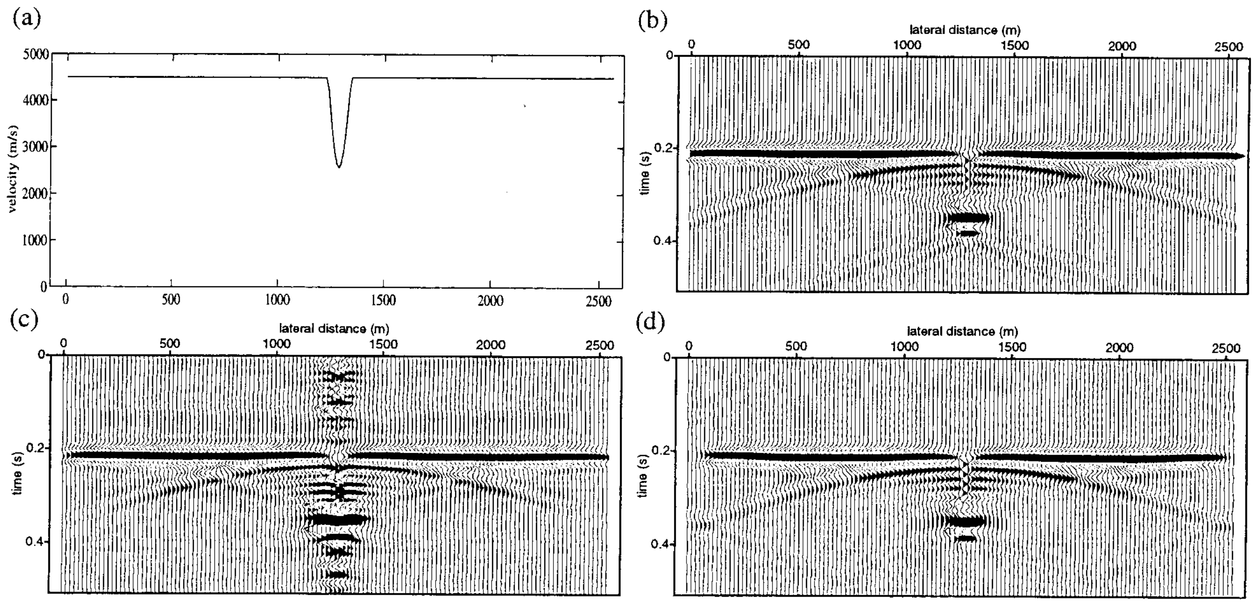

Figure 3: *Plane wave response of a laterally variant medium. (a) Lateral velocity profile of the medium. Results of 1000m extrapolation as a result of (b) finite difference (c) standard method (d) modal decomposition.*

References

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Blok, 1993, Theory of electromagnetic waveguides - part I, lecture notes, Delft University of Technology.

Etgen, 1994, Stability of explicit depth extrapolation through laterally varying media: 64^{th} SEG meeting, Los Angeles, 1266-1269.