G010 THE GENERALIZED PRIMARY REPRESENTATION OF 3D SEISMIC DATA FROM HIGHLY HETEROGENEOUS MEDIA

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Introduction

In seismic modeling as well as in seismic imaging it is common use to ignore the rapid spatial variations of the medium parameters at a scale much smaller than the seismic wavelength. In particular, for 2-D or 3-D heterogeneous media the following approach is frequently used:

- Replace the true medium by a 'macro model' that contains piecewise smoothly varying medium parameters and a 'contrast model' that is a *spatially band-limited* of the difference between the true medium and the macro model.
- Formulate an implicit integral equation for the wave field in which the macro model describes the 'propagation effects' (in terms of Green's functions) and in which the contrast model describes the 'scattering effects'.
- Solve this equation iteratively. This yields an explicit integral representation of the wave field that is *non-linear* in the contrast parameters.

Because the contrast parameters appear non-linearly in this representation, the aforementioned spatial band-limitation of these parameters should be chosen with utmost care. When the contrast parameters are small, the integral representation may be linearized and the spatial bandwidth of the parameters may be chosen in accordance with the temporal bandwidth of the seismic sources. This is another way of saying that for small contrast parameters it is indeed justified to ignore the rapid spatial variations of the medium parameters at a scale smaller to much smaller than the wavelength. However, in many cases the linearization is not justified and therefore the spatial bandwidth of the contrast model must be chosen much larger. It has been recognized by many researchers that the small scale variations may seriously affect the propagation properties of the seismic wave field. In particular, in the last three decades much research has been done on wave propagation through finely layered media (the 1-D problem). These studies have shown that the small scale variations manifest themselves as an anisotropic dispersion of the transmitted wave field (due to 'scattering losses'). Accounting for these effects in 2-D or 3-D numerical schemes, following the approach described above, requires a very fine discretization of the medium and many iterations, which is very unattractive from a computational point of view.

The generalized primary representation

As an alternative to the non-linear representation discussed in the introduction, the following approach is proposed:

- Decompose the wave equation into coupled 'one-way wave equations' for downgoing waves $\boldsymbol{P}^+(\boldsymbol{x})$ and upgoing waves $\boldsymbol{P}^-(\boldsymbol{x})$, where $\boldsymbol{x} = (x, y, z)$. For the elastodynamic situation the one-way wave vectors may be partitioned as follows

$$\boldsymbol{P}^+(\boldsymbol{x}) = \begin{pmatrix} \Phi^+ \\ \Psi^+ \\ \Upsilon^+ \end{pmatrix}(\boldsymbol{x}) \quad \text{and} \quad \boldsymbol{P}^-(\boldsymbol{x}) = \begin{pmatrix} \Phi^- \\ \Psi^- \\ \Upsilon^- \end{pmatrix}(\boldsymbol{x}), \qquad (1a, b)$$

where $\Phi^\pm$, $\Psi^\pm$ and $\Upsilon^\pm$ represent the downgoing and upgoing quasi-P, quasi-S1 and quasi-S2 waves, respectively.

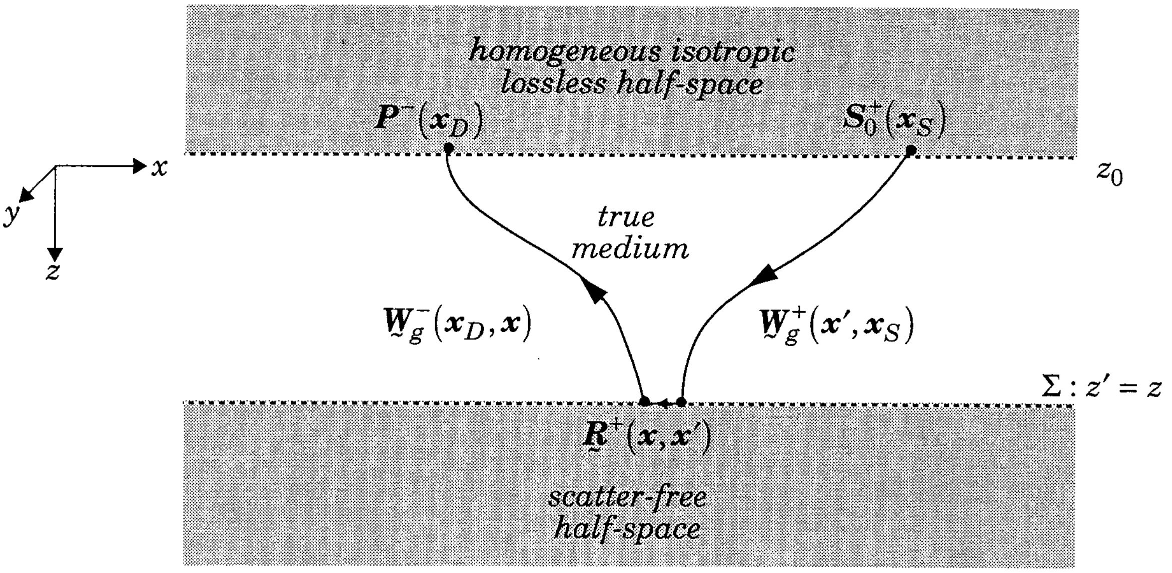

Figure 1: Schematic presentation of the vectors and matrices appearing in equation (2). $\mathbf{W}_g^{\pm}$ is defined in the true medium between z_0 and Σ. The depth of Σ changes during the integration from z_0 to ∞.

- Formulate an explicit integral representation of the wave field in which the true medium describes the propagation effects (in terms of one-way 'generalized primary Green's functions' $\underset{\sim}{W}_g^{\pm}$) and in which a 'one-way reflection operator' $\underset{\sim}{R}^+$ describes the scattering effects, according to

$$P^-(x_D) = \int_{\Omega}\Big(\int_{\Sigma} \underset{\sim}{W}_g^-(x_D, x)\underset{\sim}{R}^+(x, x')\underset{\sim}{W}_g^+(x', x_S)S_0^+(x_S)d^2x'\Big)d^3x, \tag{2}$$

  where Ω denotes the lower half-space $z > z_0$ and Σ denotes a horizontal depth level defined by $z' = z$, see Figure 1. This representation (hereafter referred to as the generalized primary representation) is *linear* in the reflection operator. This is fundamentally different from the linear*ized* approximation for small contrast parameters, as discussed in the introduction. For the mathematical derivation, see J. Seism. Expl., **2**, p 247-255.

- Define an 'extended macro model' that contains piecewise smoothly varying *effective* medium parameters (with anisotropic anelastic losses) in such a way that a seismic one-way wave field propagating through this extended macro model inherits the same anisotropic dispersion effects as a wave field propagating through the true medium. Hence, the (anisotropic) scattering losses mentioned in the introduction are mimicked by (anisotropic) anelastic losses.

- Reformulate the aforementioned generalized primary representation in terms of one-way generalized primary Green's functions in the extended macro model and in terms of a spatially *band-limited* reflection operator.

Discussion and conclusions

Because the reflection operator appears linearly in the generalized primary representation, the spatial bandwidth of this operator may now be chosen in accordance with the temporal bandwidth of the seismic sources. This is another way of saying that for the reflection operator it is justified to ignore the rapid spatial variations of the medium parameters at a scale smaller to much smaller than the wavelength, irrespective as to whether the contrasts are small or not (bear in mind that the extended macro model accounts for the effects of the rapid spatial variations). Hence, unlike for the approach discussed in the introduction, a modest discretization of the medium suffices for the numerical evaluation of equation (2). Moreover, since the generalized primary representation is linear in the reflection operator, only one 'iteration' is required. For the same reason, the generalized primary representation provides an excellent starting point for true amplitude migration taking fine-layering into account, which can now be formulated as a truly linear inversion of equation (2), see the 1993 EAEG abstract book, paper C041.