B019 WAVE EQUATION BASED SEISMIC PROCESSING: IN WHICH DOMAIN?

C.P.A. WAPENAAR
Delft University of Technology, Lab. of Seismics and Acoustics, PO Box 5046, 2600 GA Delft, The Netherlands

The forward model of prestack data that we use in our seismic inversion project DELPHI (Berkhout and Wapenaar, 1990) is based on a number of matrix multiplications. In its simplest form, (i.e. after pre-processing), it is given by

$$\mathbf{P}^- = \mathbf{W}^-\mathbf{R}\mathbf{W}^+\mathbf{S}^+, \tag{1}$$

where $\mathbf{S}^+$ contains the downgoing (source) wave fields at the surface, $\mathbf{W}^+$ describes downward propagation into the subsurface, $\mathbf{R}$ represents reflection in the subsurface, $\mathbf{W}^-$ describes upward propagation to the surface and, finally, $\mathbf{P}^-$ contains the upgoing wave fields at the surface.
Equation (1) is a monochromatic representation of the seismic data (i.e., one frequency component); the different elements in the matrices correspond to different spatial positions (Figure 1). Hence, the forward model based on equation (1) describes the seismic data in the *space-frequency* domain. This has proven to be a very flexible domain for wave equation based seismic processing.

In terms of multi-dimensional signal theory, the matrix multiplications in equation (1) represent *generalized* convolutions along the lateral space coordinate(s). By "generalized", we mean that the convolution kernel changes during the convolution process due to the lateral variations of the subsurface parameters.

In this paper we investigate the possibility of processing in the *wavenumber-frequency* domain in the presence of lateral variations of the subsurface parameters. We will show that without loss of generality equation (1) (or its extended version) may be transformed to the wavenumber-frequency domain, yielding

$$\tilde{\mathbf{P}}^- = \tilde{\mathbf{W}}^-\tilde{\mathbf{R}}\tilde{\mathbf{W}}^+\tilde{\mathbf{S}}^+, \tag{2}$$

where the tilde denotes a spatial Fourier transform along the rows and columns of the matrices.

Equation (2) is again a monochromatic representation of the seismic data; the different elements in the matrices correspond to different wavenumbers (Figure 2). The structure of the matrices in (2) depends largely on the complexity of the subsurface. In the special case of a laterally invariant macro subsurface model the matrices $\tilde{\mathbf{W}}^+$ and $\tilde{\mathbf{W}}^-$ are diagonal matrices which means that the processing (based on the inversion of this equation) can be done very efficiently. In the case of moderate lateral variations the matrices exhibit a narrow band structure and in the case of strong lateral variations the matrices are full matrices. In other words, the "bandwidth" of the propagation matrices in equation (2) is proportional to the magnitude of the lateral variations of the subsurface parameters.

Hence, the answer to the question which domain is the most efficient for wave equation based processing depends largely on the complexity of the macro subsurface model.

During the presentation we analyze the bandwidth of the matrices (and, hence, the efficiency of wave equation based seismic processing) for the 2-D and 3-D acoustic and full elastic situation. It appears that particularly 3-D prestack processing schemes may benefit significantly from the sparsity of the propagation matrices $\tilde{W}^+$ and $\tilde{W}^-$.

Reference
Berkhout, A.J., and Wapenaar, C.P.A., 1990, Delphi: Delft philosophy on acoustic and elastic inversion: The Leading Edge, 9(2), 30 - 33.

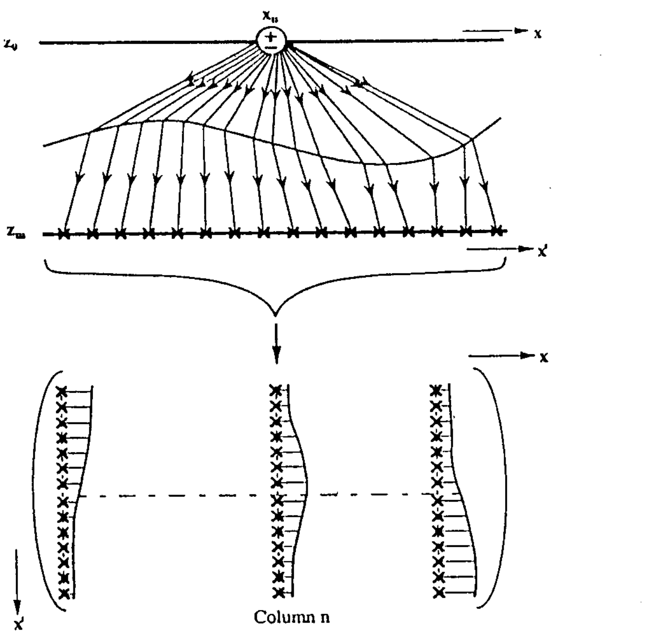

Figure 1 (left): *The propagation matrix $W^+(z_m|z_o)$. Column n contains the (monochromatic) discretized response (spatial wavelet as a function of x' at depth z_m for a dipole source at (x_n, z_o)(only the modulus is shown; the actual matrix is complex valued).*

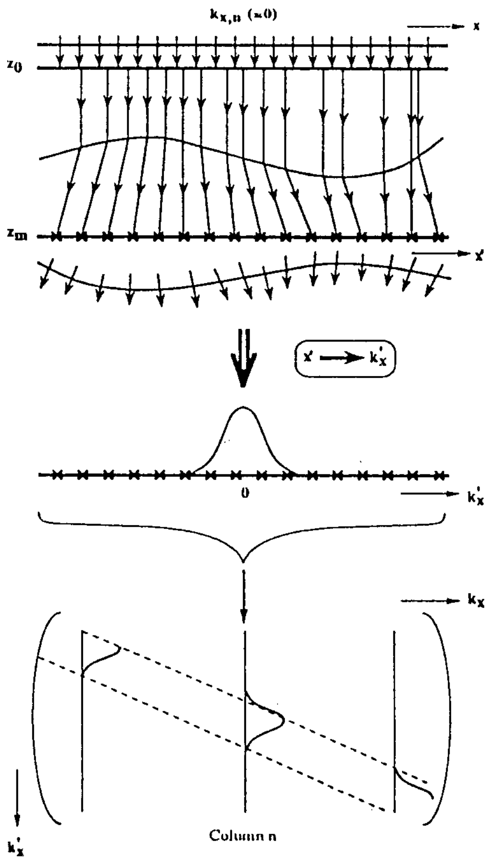

Figure 2 (right): *The propagation matrix $W^+(z_m|z_o)$. Column n contains the (monochromatic) Fourier transformed response as a function of k'_x at depth z_m for a plane wave source with wave number $k_{x,n}$ at depth z_o. The "bandwidth" of this matrix is proportional to the complexity of the macro model between z_o and z_m.*