C008 A UNIFIED FORMULATION OF WAVE FIELD DECOMPOSITION: THEORY AND EXAMPLES

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Introduction

Using a recording by multicomponent sources and receivers, it is possible to decompose the vector wave field (velocities and stresses) into its scalar components (P and S waves) at the source and receiver side. The fully decomposed data set can constitute the starting point of a new procedure to process multicomponent data, as has been described by Berkhout and Wapenaar (1990). In this scheme further processing is applied to the P and S potentials and no more on the velocities and stresses. A correct definition of those quantities is important. From their definition we can determine the decomposition operator that enables to decompose the vector wave field into its scalar components, as well as the composition operator that enables to reconstruct the vector wave field from its scalar components.

Surface seismic data

Dankbaar (1985) and Greenhalgh et al.(1990) have developed decomposition operators at the receiver side (in the (ω,k_x) and (τ,p) domain respectively) that enable to obtain the upgoing P and S waves reaching a stress free surface from the measured particle velocities (for 2D seismic wave fields). This operator is obtained from the particle velocity impulse response of an upgoing P and S plane wave at a stress free surface. As pointed out by Wapenaar et al.(1990) this problem is a particular case of the more general problem of separating the one-way components (upgoing and downgoing P and S waves) from the two-way seismic wavefield (velocities and stresses). Knowing the relations between the one-way and the two-way wave fields, the decomposition process can be extended to a larger number of situations. In its most general form the *composition* operation that relates the two way components to the one way components of the seismic wave field is given by the well known matrix equation :

$$\begin{pmatrix} \mathbf{V} \\ \mathbf{T} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_1^+ & \mathbf{L}_1^- \\ \mathbf{L}_2^+ & \mathbf{L}_2^- \end{pmatrix} \begin{pmatrix} \mathbf{P}^+ \\ \mathbf{P}^- \end{pmatrix} \qquad (1a)$$

The inverse constitutes the *decomposition* operation :

$$\begin{pmatrix} \mathbf{P}^+ \\ \mathbf{P}^- \end{pmatrix} = \begin{pmatrix} \mathbf{M}_1^+ & \mathbf{M}_2^+ \\ \mathbf{M}_1^- & \mathbf{M}_2^- \end{pmatrix} \begin{pmatrix} \mathbf{V} \\ \mathbf{T} \end{pmatrix} \qquad (1b)$$

$\mathbf{V}$: particle velocities $\mathbf{V} = (V_x,V_y,V_z)^T$

$\mathbf{T}$: stress $\mathbf{T} = (T_{xz},T_{yz},T_{zz})^T$

$\mathbf{P}^+$: downgoing waves $\mathbf{P}^+ = (\Phi^+,\Psi_x^+,\Psi_y^+)^T$

$\mathbf{P}^-$: upgoing waves $\mathbf{P}^- = (\Phi^-,\Psi_x^-,\Psi_y^-)^T$

Φ : P wave scalar potential

Ψ : S wave vector potential, $\Psi=(\Psi_x,\Psi_y,\Psi_z)^T$, $\nabla.\Psi = 0$

The decomposition process (1b) enables to obtain the up and downgoing waves from the particle velocities and stresses of an arbitrary 3D seismic wave field. In the case of a rigid ($\mathbf{V}$=0) or stress free ($\mathbf{T}$=0) acquisition surface the operator (1b) simplifies. Due to the general form of operator (1b) it is possible to achieve a modified wave field decomposition at the receiver side also for buried detectors. Furthermore it is possible to remove undesired near surface layer effects. Finally, writing the decomposition operator in cylindrical coordinates it is possible to decompose an elastic wave field with known azimuthal symmetry that is measured along a radius.

Similar to (1b), giving us the one way components of a seismic wave field from its two way components, equation (1a) enables to obtain the stress and velocity distribution associated to a distribution of up and downgoing P and S waves. This relation can be used to apply a wave field decomposition at the source side, which results in simulating a pure downgoing P or S wave source by combining several stress sources. In the case of a stress source applied on a stress free surface, only downgoing waves are emitted ($\mathbf{P}^- = 0$), hence we have $\mathbf{T}=\mathbf{L}_2^+\mathbf{P}^+$. Depending on the value given to the different components of the vector $\mathbf{P}^+$, a pure P or S wave source can be simulated, and the associated stress distribution is given by the operator $\mathbf{L}_2^+$. This stress distribution could in principle be directly applied in the field, but is preferably simulated during the processing by reorganizing the data into common detector gathers. It is also possible to simulate a pure P or S wave source not at the surface but at a certain depth. This possibility is handy to control the wave conversions generated by an undesired near surface layer.

VSP data

Devaney and Oristaglio (1986) and Dankbaar (1987) have developed decomposition operators at the receiver side in the (ω,k_z) domain that enable to determine the P and S wave fields crossing the VSP borehole from the measured particle velocities, making the assumption that there are only waves propagating in the source-borehole direction. The decomposition operator has been determined by computing the particle velocity impulse response of a P and S plane wave crossing the well. Once more this problem is part of the more general problem of separating the one way components (left and right propagating P and S waves) from the two way seismic wavefield (velocities and stresses). Using an approach similar to the one previously presented, the two way and one way components of the seismic wavefield can be related using composition (1a) and decomposition (1b) operators. The operator elements are different for VSP and surface data, as well as the meaning of the **P**, **T** and **V** vectors. For the VSP situation we have :

V : particle velocities $\mathbf{V} = (V_r, V_\theta, V_z)^T$

T : stress vector $\mathbf{T} = (T_{rr}, T_{\theta r}, T_{zr})^T$

Ψ : S wave vector $\Psi=(\Psi_r, \Psi_\theta, \Psi_z)^T$

$\mathbf{P}^+$: *right* propagating waves $\mathbf{P}^+ = (\Phi^+, \Psi_\theta^+, \Psi_z^+)^T$

$\mathbf{P}^-$: *left* propagating waves $\mathbf{P}^- = (\Phi^-, \Psi_\theta^-, \Psi_z^-)^T$

In the hypothetical case that velocities and stresses are measured along the well, operator (1b) can be used to determine the right and left propagating waves crossing the well. When only the velocities are measured, the wave field decomposition can only be done exactly if either $\mathbf{P}^-$ or $\mathbf{P}^+$ is known. In the case the source is located at the right of the borehole we assume that $\mathbf{P}^+$ can be neglected, then $\mathbf{P}^- = (\mathbf{L}_1^-)^{-1}\mathbf{V}$ and $(\mathbf{L}_1^-)^{-1}$ constitutes the decomposition operator at the receiver side.

Conclusion

For surface as well as VSP data, knowledge of both the particle velocities and the stresses is required to make a unique decomposition possible (equation 1b). For surface data decomposition, we can make advantageously use of the free surface boundary conditions, stating that the stresses are zero. For VSP data there is no free surface boundary condition and, if only velocities are measured, an additional assumption need be made to make the decomposition process unique. By assuming that all waves arrive at the well from one side (source side) we can derive VSP decomposition operators. For surface as well as VSP data, wave field decomposition is the first step of our elastic stepwise inversion scheme. Application occurs in the space domain to cope with inhomogeneous media.

References

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