REDUCING THE OVERBURDEN-RELATED ARTIFACTS IN TARGET-ORIENTED LEAST-SQUARES MIGRATION BY MARCHENKO DOUBLE-FOCUSING

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Summary

Least-squares reverse time migration (LSRTM) is a common imaging technique that geophysicists have been using to obtain high-resolution images. Nevertheless, the high computational cost shifted the focus of researchers to the target-oriented approach. In this approach, by limiting the computational grid to a relatively smaller region, the computational cost of the LSRTM is significantly reduced. However, without an accurate model of the overburden, which can model all orders of overburden-generated multiples, the image produced by this approach suffers from overburden-related artifacts. Recently, Marchenko double-focusing presented itself as a powerful data-driven tool that can focus the recorded wavefield above the target region and eliminate the effects of the overburden-related multiple reflections. This paper proposes a forward modeling and inversion algorithm based on Marchenko double-focusing for target-oriented LSRTM to produce artifact-free high-resolution images.

Reducing the overburden-related artifacts in target-oriented least-squares migration by Marchenko double-focusing

Introduction

Seismic imaging is a methodology that seismologists utilize to produce images of the subsurface. One of the most popular imaging algorithms is Reverse Time Migration (RTM) (Baysal et al., 1983). In this algorithm, the subsurface model is divided into two scales: 1) A smooth background (m_0) which is assumed as a known parameter, and 2) A perturbation (δm) which is the unknown parameter. The perturbation (δm) is assumed to be related to the data by a Born model and that image is constructed by applying the adjoint of the Born modeling operator to the data (Schuster, 2017).

An RTM image can suffer from a number of issues such as low resolution. To improve the resolution of RTM images, researchers have employed a least-squares algorithm which is known as least-squares RTM (LSRTM). LSRTM aims to minimize the L2-norm of the error between observed data and predicted data by updating δm (Schuster, 2017). LSRTM is also known as linear waveform inversion.

However, LSRTM is computationally expensive and its images are usually affected by internal multiple reflections. One way to reduce the computational cost of LSRTM is to limit our computations to a small target region of interest. This can be done by dividing the medium into an overburden and target region, and then bringing down the acquisition surface to a boundary just above the target region. This process is called redatuming (Berryhill, 1984). Despite the benefits of conventional LSRTM, the issue of multiple reflections is not addressed in this approach.

Traditionally, redatuming is done by applying the adjoint of the transmission response between the target boundary and the acquisition surface. This transmission response is computed in a smooth background medium (Berryhill, 1984), which does not include the overburden multiple reflections effects. These overburden generated multiple reflections negatively affect the image of the target, so if we can eliminate them, we will get a clearer image.

Recently, Marchenko redatuming has emerged as a powerful tool which can predict all orders of multiple reflections that are generated inside the overburden, and compute receiver-side redatumed Green's function (Wapenaar et al., 2014). In addition, another result of solving Marchenko equations is the downgoing part of the focusing function, which is the inverse of the transmission response of the overburden. This (downgoing) focusing function can help us in redatuming the sources and create double-focused data with virtual sources and receivers at a boundary just above the target region (Staring et al., 2018).

In this paper, we use Marchenko double-focusing to develop a target-oriented LSRTM algorithm. In our method, we consider the double-focused upgoing Green's function as the observed data. Then, we construct the predicted data by modeling the response of the target area, where we use the double-focused downgoing Green's function as the source term.

Least-squares reverse time migration

LSRTM is an inverse problem which minimizes the L2-norm of the error between predicted data and observed data to get an estimation of perturbation (δm). The cost function of this optimization problem is

$$C(\delta m) = \frac{1}{2}\left\|P^{scat}_{pred}(\delta m) - P^{scat}_{obs}\right\|^2_2. \tag{1}$$

Here, P^{scat}_{obs} is the recorded scattered pressure field, and P^{scat}_{pred} is

$$P^{scat}_{pred}(\mathbf{x}_r,\mathbf{x}_s,\omega) = \omega^2 \int_v G_0(\mathbf{x}_r,\mathbf{x},\omega)\delta m(\mathbf{x})G_0(\mathbf{x},\mathbf{x}_s,\omega)W(\omega)\,d\mathbf{x}, \tag{2}$$

where ω is the angular frequency, $W(\omega)$ is the source signature, and G_0 is the Green's function of the background medium (m_0). Moreover, $\mathbf{x}$ is a location inside the medium, and $\mathbf{x}_r$ and $\mathbf{x}_s$ are locations of receivers and sources, respectively. To minimize the cost function (Eq. 1) we use a conjugate gradient algorithm.

Target-oriented LSRTM by Marchenko double-focusing

To formulate target-oriented LSRTM, we first need to do a redatuming of P^{scat}_{obs} from the acquisition surface to a boundary just above the target area, which requires the inverse of the transmission response (including multiple reflections) of the overburden. Marchenko equations provide us with a mathematical tool for conducting receiver-side redatuming and computing the downgoing focusing function, which is the inverse of the overburden's transmission response (Wapenaar et al., 2014). The only requirement for Marchenko redatuming is a smooth background velocity of the overburden. After computing the Marchenko Green's functions (i.e. receiver-side redatumed data), we convolve them with downgoing focusing functions to get a double-focused wavefield on a boundary just above the target (Staring et al., 2018)

$$G^{-,+}_{df}(\mathbf{x}_v, \mathbf{x}'_v, \omega) = \int_{\mathbb{D}_{acq}} G^{-}_{Mar}(\mathbf{x}_v, \mathbf{x}_s, \omega) \mathcal{F}^{+}_{1}(\mathbf{x}_s, \mathbf{x}'_v, \omega)\, d\mathbf{x}_s, \tag{3}$$

and

$$G^{+,+}_{df}(\mathbf{x}_v, \mathbf{x}'_v, \omega) = \int_{\mathbb{D}_{acq}} G^{+}_{Mar}(\mathbf{x}_v, \mathbf{x}_s, \omega) \mathcal{F}^{+}_{1}(\mathbf{x}_s, \mathbf{x}'_v, \omega)\, d\mathbf{x}_s. \tag{4}$$

Here, on the left-hand side, the second superscript shows the direction of propagation at the source-side, the left superscript shows the direction of propagation at the receiver-side, and "df" stands for the "double-focused". On the right-hand side, $G^{-}_{Mar}(\mathbf{x}_v, \mathbf{x}_s, \omega)$ and $G^{+}_{Mar}(\mathbf{x}_v, \mathbf{x}_s, \omega)$ are upgoing and downgoing parts of the receiver-side redatumed Green's functions, $\mathcal{F}^{+}_{1}(\mathbf{x}_s, \mathbf{x}'_v, \omega)$ is the spatial derivative of the downgoing focusing function in the vertical direction at the source location, $\mathbb{D}_{acq}$ is acquisition surface, and "s" and "v" stand for source and an arbitrary virtual location, respectively.

Next, we consider the following equation as the starting point for connecting the "double-focused" Green's functions to the dipole reflection response of the target region ($\mathcal{R}^{tar}$):

$$G^{-,+}(\mathbf{x}_{vr}, \mathbf{x}'_{vs}, \omega) = \int_{\mathbb{D}_{tar}} \mathcal{R}^{tar}(\mathbf{x}_{vr}, \mathbf{x}_{vs}, \omega) G^{+,+}(\mathbf{x}_{vs}, \mathbf{x}'_{vs}, \omega)\, d\mathbf{x}_{vs}, \tag{5}$$

where $\mathbb{D}_{tar}$ is target boundary, $G^{-,+}(\mathbf{x}_{vr}, \mathbf{x}'_{vs}, \omega)$ and $G^{+,+}(\mathbf{x}_{vs}, \mathbf{x}'_{vs}, \omega)$ are Green's functions due to a downgoing monopole point source. Moreover, we added the subscripts "s" and "r" to the previous "v" to specify the desired virtual locations for this problem.

In the real world, where we cannot put real point sources above the target area, we approximate $G^{+,+}$ with $G^{+,+}_{df}$, and $G^{-,+}$ with $G^{-,+}_{df}$. However, the double-focused Green's functions are spatially band-limited, which is especially problematic at the receiver side. To take this bandlimitation into account, we assume that $G^{-,+}_{df}$ is related to $G^{-,+}$ by

$$G^{-,+}_{df}(\mathbf{x}'_{vr}, \mathbf{x}'_{vs}, \omega) = \int_{\mathbb{D}_{tar}} \Gamma(\mathbf{x}'_{vr}, \mathbf{x}_{vr}, \omega) G^{-,+}(\mathbf{x}_{vr}, \mathbf{x}'_{vs}, \omega)\, d\mathbf{x}_{vr}. \tag{6}$$

Here

$$\Gamma(\mathbf{x}'_{vr}, \mathbf{x}_{vr}, \omega) = \int_{\mathbb{D}_{tar}} \mathcal{T}_d(\mathbf{x}'_{vr}, \mathbf{x}_s, \omega)^{-1} G^{+}_{d}(\mathbf{x}_{vr}, \mathbf{x}_s, \omega)\, d\mathbf{x}_s \tag{7}$$

is a point-spread function (PSF) that imposes a spatial and temporal band limitation on $G^{-,+}$, $\mathcal{T}_d(\mathbf{x}'_{vr}, \mathbf{x}_s, \omega)$ is the direct arrival of vertical spatial derivative of the dipole transmission response of the overburden at the virtual receiver location, and $G^{+}_{d}(\mathbf{x}_{vr}, \mathbf{x}_s, \omega)$ is the direct arrival part of the monopole transmission response. Equation 6 implies that we have to convolve both sides of the Equation 5 with PSF (Eq. 7). By doing so, and approximating $G^{+,+}$ with $G^{+,+}_{df}$, Equation 5 transforms into

$$G^{-,+}_{df}(\mathbf{x}'_{vr}, \mathbf{x}'_{vs}, \omega) = \int_{\mathbb{D}_{tar}} \int_{\mathbb{D}_{tar}} \Gamma(\mathbf{x}'_{vr}, \mathbf{x}_{vr}, \omega) \mathcal{R}^{tar}(\mathbf{x}_{vr}, \mathbf{x}_{vs}, \omega) G^{+,+}_{df}(\mathbf{x}_{vs}, \mathbf{x}'_{vs}, \omega)\, d\mathbf{x}_{vr} d\mathbf{x}_{vs}. \tag{8}$$

Finally, to complete our target-oriented LSRTM we take

$$P_{obs}^{scat}(\mathbf{x}'_{vr}, \mathbf{x}'_{vs}, \omega) = G_{df}^{-,+}(\mathbf{x}'_{vr}, \mathbf{x}'_{vs}, \omega), \tag{9}$$

and

$$P_{pred}^{scat}(\mathbf{x}'_{vr}, \mathbf{x}'_{vs}, \omega, \delta m) = \int_{\mathbb{D}_{tar}} \int_{\mathbb{D}_{tar}} \Gamma(\mathbf{x}'_{vr}, \mathbf{x}_{vr}, \omega) \mathcal{R}_{pred}^{tar}(\mathbf{x}_{vr}, \mathbf{x}_{vs}, \delta m, \omega) G_{df}^{+,+}(\mathbf{x}_{vs}, \mathbf{x}'_{vs}, \omega)\, d\mathbf{x}_{vr} d\mathbf{x}_{vs}, \tag{10}$$

where

$$\mathcal{R}_{pred}^{tar}(\mathbf{x}_{vr}, \mathbf{x}_{vs}, \delta m, \omega) = \omega^2 \int_{v} G_0(\mathbf{x}_{vr}, \mathbf{x}, \omega) \delta m(\mathbf{x}) (\frac{\partial_{3,vs} G_0(\mathbf{x}, \mathbf{x}_{vs}, \omega)}{\frac{1}{2} i \omega \rho(\mathbf{x}_{vs})}) W(\omega)\, d\mathbf{x}, \tag{11}$$

and we minimize the same cost function of Equation 1.

Numerical results

To test our algorithm, we define a velocity model (Fig. 1) with constant density. Then, we implement target-oriented RTM and LSRTM one time with conventional double-redatuming and one time with Marchenko double-focusing. In Figure 1, blue stars indicate the source and receiver locations at the acquisition surface, and red dots indicate the virtual source/receiver locations above the target area.

Figure 2 shows the target-oriented RTM image of this model, with conventional double-redatuming (Fig. 2a) and Marchenko double-focusing (Fig. 2b). It is clearly visible that conventional double-redatuming is not able to remove overburden-related multiple reflections, whereas Marchenko double-focusing significantly weakens these reflections. Still, the RTM image resolution needs improvement. The target-oriented LSRTM image (Fig. 3) that is retrieved by Marchenko double-focused data (Fig. 3b) is more clear compared to the image retrieved by conventional double-redatuming (Fig. 3a) and has a higher resolution compared to the RTM image (Fig. 2).

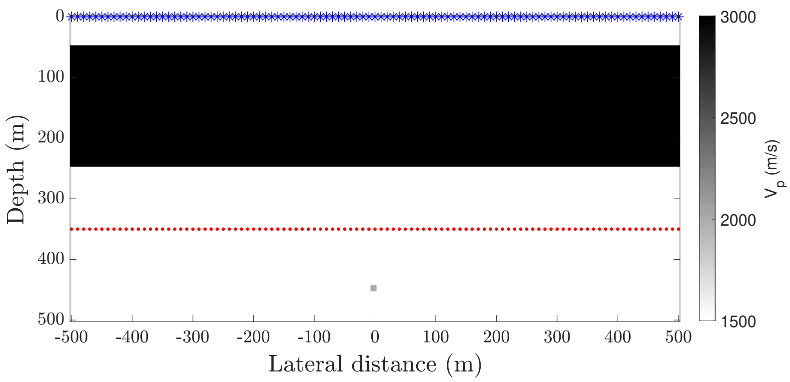
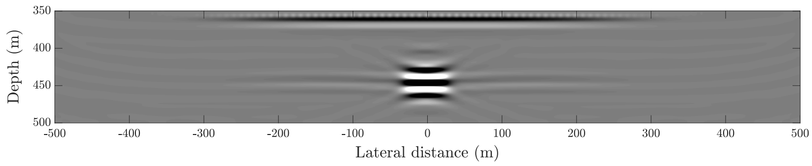


Figure 1 *Velocity model*

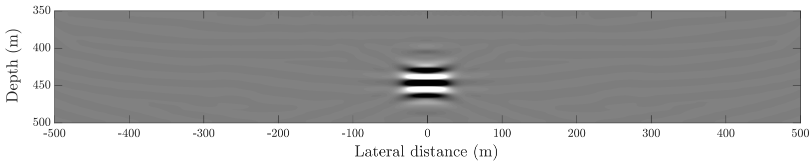
Conclusions

In this paper, we formulated a target-oriented LSRTM algorithm by Marchenko double-focusing that is able to reduce the effects of overburden-related multiple reflections on the image of the target. To attain this, we first conduct receiver-side Marchenko redatuming and then we redatum the sources with the help of downgoing focusing functions. Then, we predict data with a Born model of the target area, which are matched with the double-focused data by convolving it with the double-focused downgoing Green's function and a point-spread function.

Geophysicists require methods which are able to produce images with high resolution and free of artefacts related to multiple scattering. Our target-oriented LSRTM algorithm, as proposed in this paper, has the potential to produce high-resolution images free of spurious events caused by the overburden.

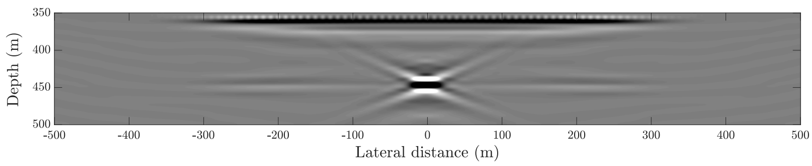


(a) Conventional double-redatuming

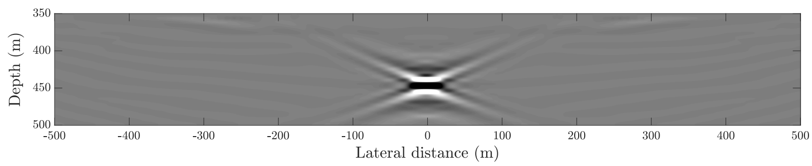


(b) Marchenko double-focusing

Figure 2 *Retrieved RTM image of the target. a) With conventional double-redatuming. b) With Marchenko double-focusing*



(a) Conventional double-redatuming



(b) Marchenko double-focusing

Figure 3 *Retrieved LSRTM image of the target after 10 iterations. a) With conventional double-redatuming. b) With Marchenko double-focusing*

Acknowledgements

This research was funded by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No: 742703).

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