

Introduction

Marchenko-equation-based multiple-elimination strategies aim to remove all overburden-generated internal multiples. The scattering relations used by the Marchenko method are identical for acoustic and elastodynamic waves. However, due to differences in the underlying physics, the elastodynamic case reveals previously-hidden details and violates assumptions made in the acoustic scheme. In this abstract, firstly, we contrast the acoustic and elastodynamic Marchenko schemes with an emphasis on the multiple-elimination step. Secondly, we highlight that nearly-identical assumptions are required for the elastic formulations of the Marchenko method and the inverse scattering series (ISS). Thirdly, we show how the elastic scheme with augmentation leads to a non-trivial minimum-phase reconstruction problem. Finally, we discuss a geological setting where the acoustic Marchenko method can suffice.

Acoustic vs. elastodynamic Marchenko method

In geophysics, Marchenko-equation-based multiple-elimination methods can be broadly divided in two categories: *redatuming* (Wapenaar et al., 2014) and *multiple-elimination* (van der Neut and Wapenaar, 2016). Both approaches deconvolve up- and downgoing fields (Green's functions G^\pm for the former, and U^\pm for the latter) to obtain a target reflection response without overburden-related multiples. Prior to the multiple-elimination step, the Green's functions must be retrieved. Representation theorems are scattering relations between focusing (f_1^\pm or v^\pm) and Green's functions (G^\pm or U^\pm) that are independent of the wavefield type. The solution strategy for this problem with four unknowns and two equations is inspired by the underlying physics: the Green's functions are suppressed with a temporal mute, except for *two overlaps* with the focusing functions. The first one (f_1^+/G^+ , or v^+/U^+) is unavoidable and required by the scheme as initial estimate, whereas the second one (f_1^-/G^- , or v^-/U^-) is assumed to vanish.

These assumptions can be reasonable for sufficiently-simple acoustic media where the first overlap (χ_+) can be estimated, thus fully constraining the problem. However, in elastic redatuming, χ_+ contains a *forward-scattered transmission coda*, which is an overburden-generated transmission, including conversions between compressional and shear modes (see Fig. 1a and left panel in Fig. 1b), but also so-called fast multiples (Reinicke, 2020). So far, the forward-scattered transmission cannot be estimated without detailed knowledge of the overburden. This issue vanishes in the multiple-elimination scheme, where χ_+ simplifies to an identity, for both acoustic and elastodynamic fields. Nevertheless, in contrast to redatuming, which recovers measurements just above the target and eliminates the impact of the overburden, multiple-elimination only removes overburden-related reflections: events that result from a change in up/down propagation direction such as internal multiples. Elison et al. (2020) present an acoustic example where the latter strategy retrieves a dataset that is associated with a smooth scattering-free overburden. In elastic media, however, multiple-elimination calculates the target reflection response dressed with the overburden-generated forward-scattered transmission coda. We expect a similar effect for complex acoustic media with strong forward-scattering. Moreover, while simplifying χ_+ , the multiple-elimination scheme increases the risk of a non-zero second overlap, which is already almost-inevitable in elastic media due to the large difference between P- and S-wave velocities (Reinicke et al., 2019). The elastic ISS has a similar limitation known as a monotonicity (Sun and Innanen, 2019). Quantifying these assumptions reveals that they are nearly identical, and thus, we decided to also use the term *monotonicity* in anticipation that the limitation is caused by the underlying physics and is not algorithm dependent.

Assuming monotonicity is not violated, χ_+ may be retrieved by the elastic Marchenko scheme augmented with energy conservation and a minimum-phase condition, which is mathematically similar to the short-period multiple problem. Initial developments for 1.5D acoustic media (Dukalski et al., 2019; Elison et al., 2020) use the Kolmogorov method on angle gathers to recover f_1^+ from the scalar autocorrelation $|\hat{f}_1^+|^2$ (hats denote frequency domain). In 1.5D elastic media, the minimum-phase reconstruction of the focusing function can be maximally reduced to 2×2 matrix factorization as the fields are no longer scalars,

$$f_1^+ \xrightarrow[\text{generalization}]{\text{Elastic}} \begin{pmatrix} f_{1,pp}^+ & f_{1,ps}^+ \\ f_{1,sp}^+ & f_{1,ss}^+ \end{pmatrix}, \quad \text{and}, \quad |\hat{f}_1^+|^2 \xrightarrow[\text{generalization}]{\text{Elastic}} \hat{f}_1^+ \hat{f}_1^{+\dagger}. \quad (1)$$

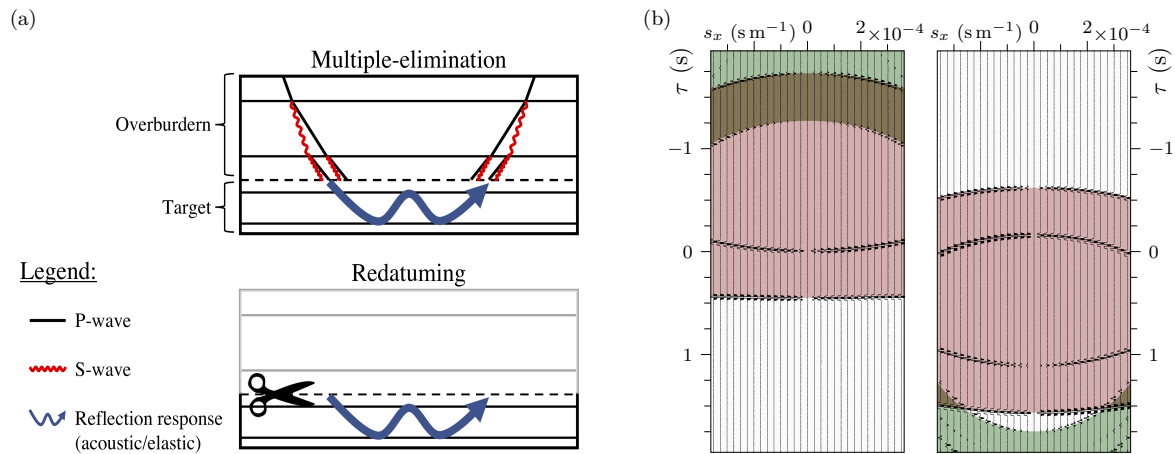


Figure 1: (a) Multiple-elimination vs. redatuming. The former one does not remove the forward-scattered transmission coda. (b) Visualization of two terms of the elastodynamic representation theorems (sp component) in the horizontal-slowness (s_x) intercept-time (τ) domain. The focusing (red) and Green's (green) functions share an unavoidable (first) and a conditional (second) temporal overlap, which are visible in the left and right panels, respectively.

The subscripts p and s correspond to compressional and shear wave sources/receivers, respectively, and the superscript \dagger denotes a complex-conjugate transpose. Matrix generalization of minimum-phase factorization is needed and is currently under investigation (Reinicke, 2020).

Considering the above, it is desirable to understand where we can still use an acoustic approximation. A promising candidate is marine reflection data, because, firstly, only the acoustic component is measured, and secondly, true-amplitude reflections are more likely to be obtained with high-end pre-processing. Using a synthetic (elastic) Arabian Gulf model, we demonstrated that acoustic Marchenko multiple-elimination can suffice for structural imaging (Reinicke, 2020). Similar to other elastic media monotonicity is violated. However, the events that violate monotonicity are associated with mode conversions, and hence, their contributions are minor at small angles of incidence. Moreover, in nearly-horizontally layered geological settings akin to the Middle East, small angles of incidence are often sufficient for structural imaging.

Conclusions and outlook

Apart from the availability of high-quality multi-component data, the success of the elastodynamic Marchenko method hinges on our management of the two field overlaps. Augmentation might help to retrieve the first overlap, given availability of an elastic minimum-phase reconstruction algorithm. The second overlap needs to be predicted, or handled by other means.

References

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