Retrieving higher-mode surface waves using seismic interferometry by multidimensional deconvolution

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Summary

Virtual-source surface-wave responses can be retrieved using the crosscorrelation of wavefields observed at two receivers. Higher-mode surface waves cannot be properly retrieved when there is a lack of subsurface sources, which is often the case. In this paper, we present a multidimensional-deconvolution scheme that introduces an additional processing step in which the crosscorrelation result is deconvolved by a point-spread function. The scheme is based on an approximate convolution theorem that includes point-force responses only, which is advantageous for applications with contemporary field-acquisition geometries. The point-spread function captures the imprint of the lack of subsurface sources and quantifies the associated smearing of the virtual source in space and time. The function can be calculated from the same wavefields used in the correlation method, provided that one or more vertical arrays of subsurface receivers are present and the illumination is from one side. We show that the retrieved surface-wave response, including the higher modes, becomes much more accurate. The waveforms are properly reconstructed and there is only a small amplitude error, which is due to non-canceling cross terms in the employed approximate convolution theorem. The improved retrieval of the multi-mode surface waves can facilitate dispersion analyses and near-surface inversion algorithms.

Introduction

The crosscorrelation of observations at two different seismic receivers is known to reconstruct the Green's function as if one of the receivers were a source (i.e., the virtual-source response) (e.g., Shapiro et al., 2005; Wapenaar & Fokkema, 2006). The reconstruction/retrieval of surface waves has received relatively wide attention, which can be explained by the fact that wavefield recordings are often dominated by surface waves. Surface-wave responses can be retrieved by crosscorrelating wavefields excited by noise sources (e.g., ocean storms) or by controlled sources. Applications exist on different scales, and the retrieved responses are, among others, used for removing surface waves from seismic data (Halliday et al., 2007) and for obtaining information about the subsurface structure (Shapiro et al., 2005).

For accurate retrieval of the virtual-source response, one needs sources along a contour enclosing the receivers whose observations are crosscorrelated, and the source distribution needs to be regular (Wapenaar & Fokkema, 2006). Higher-mode surface waves cannot be correctly estimated when subsurface sources are absent (Halliday & Curtis, 2008). The associated spurious arrivals, coming from non-cancelling cross-mode correlations, might even overwhelm the retrieved higher modes (Kimman & Trampert, 2010). The spurious arrivals can be suppressed when there is a homogeneous distribution of sources at the surface, but inaccuracies remain.

Nevertheless, the correct retrieval of higher-mode surface waves can be very beneficial. They provide information about greater depth ranges than the fundamental mode and enhance the depth resolution of the estimated near-surface velocity models. The technique called "seismic interferometry by multidimensional deconvolution (MDD)" (Wapenaar et al., 2011a) has the potential to improve the retrieval of the higher modes. In heuristic terms, this technique introduces an extra processing step compared to the crosscorrelation (CC) method: its result is deconvolved by the so-called point-spread function. This function quantifies the spatial and temporal smearing of the virtual source due to non-uniform illumination and anelastic effects. The deconvolution can partially correct for the smearing of the virtual source; the retrieved virtual-source response then becomes more accurate. The point-spread function can be calculated from the same wavefields as used in the CC method. MDD has been shown to improve the retrieval of the fundamental surface wavemode for non-ideal surface-source distribution (Wapenaar et al., 2011b), but has not yet been applied to improve the retrieval of higher-mode surface waves. Therefore, in this paper we apply MDD to retrieve the full surface-wave response. We use a modified MDD scheme that is particularly suited for surface waves. It is based on an approximate convolution theorem that contains point-force responses only, which is advantageous for potential field applications. The performance of the MDD scheme is illustrated using a numerical example.

Approximate convolution theorem

In the space-frequency domain, the exact elastodynamic reciprocity theorem of the convolution type reads (e.g., Aki & Richards, 2002):

$$\hat{G}_{im}(\mathbf{x}_R,\mathbf{x}_S) = \int_S \left[\hat{G}_{in}(\mathbf{x}_R,\mathbf{x}) n_j c_{njkl}(\mathbf{x}) \partial_k \hat{G}_{lm}(\mathbf{x},\mathbf{x}_S) - n_j c_{njkl}(\mathbf{x}) \partial_k \hat{G}_{il}(\mathbf{x}_R,\mathbf{x}) \hat{G}_{nm}(\mathbf{x},\mathbf{x}_S)\right] dS, \qquad (1)$$

where the partial derivative operators act on $\mathbf{x}$. The involved Green's tensors (e.g., $\hat{G}_{im}$) are force-source Green's tensors for particle displacement, where the first and second arguments denote receiver and source coordinates, respectively, and c_{njkl} denotes the elasticity tensor. The integral describes forward propagation of a wavefield, which is excited outside a closed volume V at $\mathbf{x}_S$, from observations at $\mathbf{x}$ to receiver $\mathbf{x}_R$ located inside V. This volume has receivers $\mathbf{x}$ all over its enclosing boundary S, which has outward pointing unit normal n_j (see Figure 1, where V is cylindrical in view of the analysis below, and $\mathbf{x}'$ is an auxiliary coordinate). We only consider the surface-wave part of the Green's tensors. For horizontally layered media with a free surface and a half-space below the layering, the force-source surface-wave Green's tensors can be represented in the far field by a sum over horizontally propagating

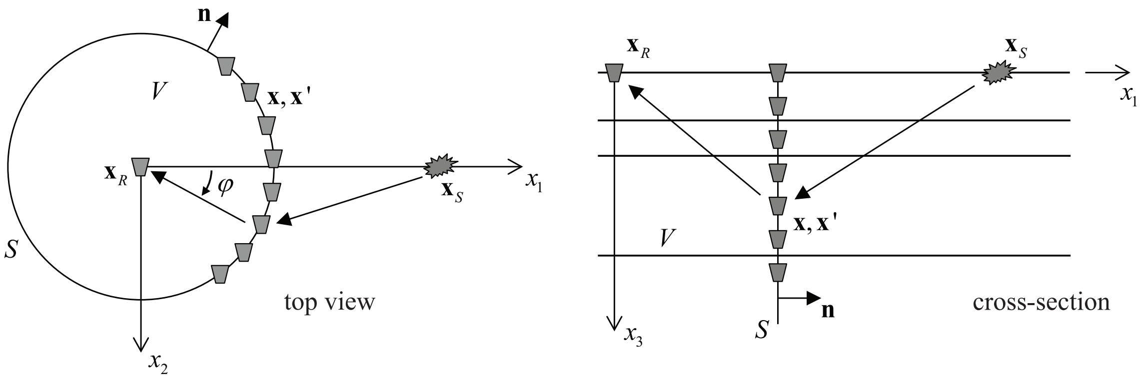

Figure 1: Cylinder V enclosed by surface S, having outward pointing unit normal $\mathbf{n}$, used for forward propagation of a wavefield from $\mathbf{x}_S$ via $\mathbf{x}$ to $\mathbf{x}_R$, and as a basis for the MDD scheme; $\mathbf{x}'$ denotes an auxiliary coordinate. The medium is horizontally layered. S is covered with receivers.

modes ν (Aki & Richards, 2002) [$\hat{G}^{\nu}_{nm}(\mathbf{x},\mathbf{x}_S)$ denotes a single-mode Green's function]:

$$\hat{G}_{nm}(\mathbf{x},\mathbf{x}_S) = \sum_{\nu} \hat{G}^{\nu}_{nm}(\mathbf{x},\mathbf{x}_S). \tag{2}$$

Here, we circumvent the need of spatial derivatives of the point-force responses at $\mathbf{x}$ [see Eq. (1)] by introducing an approximate convolution theorem that incorporates modal scale factors $\hat{N}_\nu$ instead. It is similar to the approximate correlation theorem proposed by Halliday & Curtis (2008) and reads

$$\hat{G}^{\nu}_{im}(\mathbf{x}_R,\mathbf{x}_S) \cong \int_S -\mathrm{i}k_\nu \hat{N}_\nu \hat{G}^{\nu}_{in}(\mathbf{x}_R,\mathbf{x}) \hat{G}^{\nu}_{nm}(\mathbf{x},\mathbf{x}_S)\, dS, \tag{3}$$

where $\mathrm{i} = \sqrt{-1}$ and k_ν the modal wavenumber (the summation convention does not hold for ν). The theorem assumes one-sided illumination and holds for the modes separately. In practice, however, only the summation of the modes is detected. Therefore, we take all modes together, ignore the cross-mode contributions, and define an approximate propagator $\hat{W}_{in}(\mathbf{x}_R,\mathbf{x})$ that brings the wavefield from $\mathbf{x}$ to $\mathbf{x}_R$:

$$\hat{G}_{im}(\mathbf{x}_R,\mathbf{x}_S) \cong \int_S \hat{W}_{in}(\mathbf{x}_R,\mathbf{x}) \hat{G}_{nm}(\mathbf{x},\mathbf{x}_S)\, dS, \quad \hat{W}_{in}(\mathbf{x}_R,\mathbf{x}) = \sum_{\nu} -\mathrm{i}k_\nu \hat{N}_\nu \hat{G}^{\nu}_{in}(\mathbf{x}_R,\mathbf{x}). \tag{4}$$

We determine the modal scale factors by requiring that, for each ν, the integral in Eq. (4) gives the same result as that in Eq. (1). After substituting the point-force Green's tensors, the integrals over φ are approximated using the stationary-phase method; the stationary point lies at $\varphi = 0$. After the depth integration, the result of Eq. (1) can be expressed in terms of the phase velocity c_ν, the group velocity U_ν, and the modal kinetic energy I^{ν}_1. Eq. (4) should give the same, and hence we find

$$\hat{N}_\nu = -2c_\nu U_\nu I^{\nu}_1 / J^{\nu}_1, \tag{5}$$

where J^{ν}_1 has the same expression as I^{ν}_1 (Aki & Richards, 2002) apart from the missing material density.

In the right-hand side of Eq. (1), cross-mode convolutions do not contribute due to orthogonality of the involved surface-wave eigenfunctions (Aki & Richards, 2002). Therefore, we obtain a single summation over modes (like in the left-hand side). In Eq. (4), however, cross-mode contributions do not necessarily vanish. Hence, the forward propagation described by Eq. (4) is not exact.

Multidimensional deconvolution scheme for surface waves

We derive a MDD scheme for the retrieval of surface waves. Now, we assume that more sources are present to the right of V (cf. Figure 1; not necessarily at the surface). As a starting point, we consider

Eq. (4) for each of the sources ($\mathbf{x}_S \rightarrow \mathbf{x}_S^{(j)}$). We correlate both the left- and the right-hand sides with the observations at S by introducing $\mathbf{x}'$ to distinguish it from the integration coordinate $\mathbf{x}$, and sum over source types. Taking the equations of all sources together, we obtain

$$\hat{C}_{ik}(\mathbf{x}_R,\mathbf{x}') \cong \int_S \hat{W}_{in}(\mathbf{x}_R,\mathbf{x})\hat{P}_{nk}(\mathbf{x},\mathbf{x}')\,dS, \tag{6}$$

$$\hat{C}_{ik}(\mathbf{x}_R,\mathbf{x}') = \sum_j \hat{G}_{im}(\mathbf{x}_R,\mathbf{x}_S^{(j)})\hat{G}^*_{km}(\mathbf{x}',\mathbf{x}_S^{(j)}), \quad \hat{P}_{nk}(\mathbf{x},\mathbf{x}') = \sum_j \hat{G}_{nm}(\mathbf{x},\mathbf{x}_S^{(j)})\hat{G}^*_{km}(\mathbf{x}',\mathbf{x}_S^{(j)}). \tag{7}$$

We address $\hat{C}_{ik}$ as the correlation functions because they are very similar to the corresponding integrals in the approximate correlation theorem of Halliday & Curtis (2008), apart from the associated modal scale factors [similar to those in Eq. (4)]. $\hat{P}_{nk}$ are the point-spread functions (Wapenaar et al., 2011a). $\hat{C}_{ik}$ and $\hat{P}_{nk}$ can be calculated from the wavefield observations, provided that all required receivers are present: along surface S and one inside V. $\hat{W}_{in}$ can be solved from Eq. (6) by deconvolution (Wapenaar et al., 2011a). Using multicomponent sources and observations, the following discretized system of 9 equations and unknowns is obtained (in 3D), which can be inverted in a stabilized way (ε^2 is small):

$$\hat{\mathbf{C}} \cong \hat{\mathbf{W}}\hat{\mathbf{P}} \quad \rightarrow \quad \hat{\mathbf{W}} \cong \hat{\mathbf{C}}\left[\hat{\mathbf{P}}+\varepsilon^2\mathbf{I}\right]^{-1}. \tag{8}$$

The correlation matrix $\hat{\mathbf{C}}$, the propagator matrix $\hat{\mathbf{W}}$, and the point-spread matrix $\hat{\mathbf{P}}$ are organized as in Wapenaar et al. (2011a). The deconvolution in Eq. (8) is referred to as multidimensional because it employs sources and observations in different directions. $\hat{\mathbf{W}}$ contains the virtual-source responses $\hat{W}_{in}$.

The correlation functions $\hat{C}_{ik}$ are related to Green's functions $\hat{G}_{ik}$ (see Halliday & Curtis, 2008). According to Eq. (6), a specific $\hat{C}_{ik}$ is obtained from a generalized convolution over space and time of propagators and associated point-spread functions, where the latter represent virtual sources (i.e., components). A poor crosscorrelation (CC) result can therefore be explicitly linked to smeared point-spread functions: the more this smearing, the poorer the obtained $\hat{G}_{ik}$ (Wapenaar et al., 2011a). Halliday & Curtis (2008) and Kimman & Trampert (2010) showed that especially higher-mode surface waves are poorly retrieved when there is a lack of (physical) subsurface sources. In terms of the current analysis, this implies relatively strong smearing of the point-spread functions. MDD is expected to correct for this smearing so that the virtual sources become more ideal in space and time; the retrieved virtual-source responses $\hat{W}_{in}$, including the higher-mode surface waves, are then expected to become more accurate.

Numerical example

Now, we show the results of applying the MDD scheme in a 2D situation (x_2 vanishes). To ensure relatively strong higher-mode surface waves, we chose the strongly layered medium used by Halliday & Curtis (2008). We modelled band-limited surface-wave responses (10 Hz Ricker pulse) for 5 surface sources (20 m spacing), with a minimum distance of 600 m between $\mathbf{x}_R$ and $\mathbf{x}_S$ (cf. Figure 1). The receivers along S were placed at 400 m distance from $\mathbf{x}_R$ (between $\mathbf{x}_R$ and $\mathbf{x}_S$); there were 34 receivers with 1.5 m spacing. We computed all required components [for $i,k,n = 1,3$; see Eqs. (6) and (7)] to form the correlation and point-spread matrices. The virtual-source response $W_{33}(\mathbf{x}_R,\mathbf{x},t)$ obtained using MDD is shown in Figure 2a together with its directly modelled equivalent [Eq. (4)]; the virtual source is located at the surface: $\mathbf{x} = [x,0]$. The virtual-source response G_{33} obtained using the CC method (Halliday & Curtis, 2008) is shown in Figure 2b for comparison; the directly modelled response is also included. G_{33} is obtained by applying a time derivative to the correlation function C_{33}; we then ignore the unknown modal scale factors of the approximate correlation theorem, but this is often done in practice.

The retrieved W_{33} (MDD scheme) is quite accurate. There is some amplitude mismatch compared with the true W_{33}, but the phase is correct and the waveform is properly retrieved. The cross terms thus have small influence, even in the case of relatively strong higher modes. The retrieved G_{33} (CC method) is less accurate [cf. the findings of Halliday & Curtis (2008)]: the amplitude mismatch is stronger, and the waveform is not fully reconstructed.

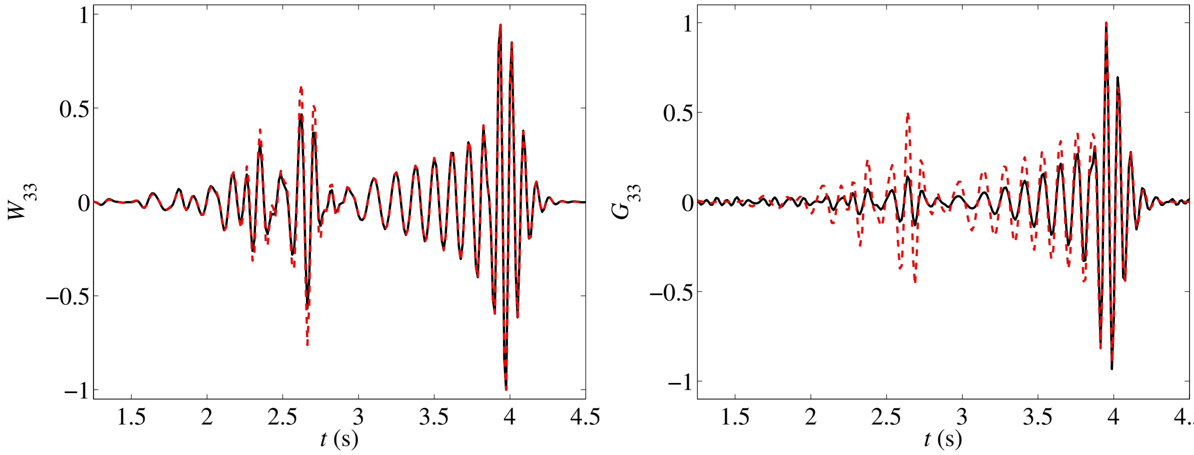

Figure 2: Retrieved (–) and directly modelled (– –) virtual-source responses (a) $W_{33}(\mathbf{x}_R, \mathbf{x}, t)$ (MDD scheme) and (b) $G_{33}(\mathbf{x}_R, \mathbf{x}, t)$ (CC method). The associated frequency-domain spectra have been multiplied by the power spectrum of the Ricker pulse. Units are arbitrary.

Conclusions

We proposed an approximate convolution theorem for forward propagation of surface waves in layered media and determined the involved modal scale factors. Based on this theorem, we derived a multidimensional deconvolution (MDD) scheme for the retrieval of surface waves. Numerical results show that the virtual-source response, including the higher-mode surface waves, is retrieved quite accurately; there is only some amplitude mismatch. The MDD scheme contains point-force responses only, which is advantageous for applications with contemporary field-acquisition geometries, and assumes illumination from one side. The scheme also requires one or more shallow vertical arrays of geophones, which may be made available by the presence of buried arrays, boreholes, or a seismic cone penetrometer (Ghose, 2012). We presented a numerical example with controlled sources having the same spectrum, but in principle the source spectra can be different. The improved retrieval of multi-mode surface waves is expected to facilitate dispersion analyses and near-surface inversion algorithms.

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