C020

On the Relation between Seismic Interferometry and the Simultaneous-source Method

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SUMMARY

In seismic interferometry the response of a virtual source is created from responses of sequential transient or simultaneous noise sources. In the simultaneous-source method (also known as blended acquisition), overlapping responses of sources with small time delays are recorded. Clearly seismic interferometry and the simultaneous-source method are related. We make this relation explicit by deriving deblending as a form of seismic interferometry by MDD. The discussed representation for the simultaneous-source method has two interesting limiting cases. When each source group consists of a single source, then we obtain the original expressions for interferometry with sequential transient sources. On the other hand, when there is only one source group containing all sources and when the source wavelets are replaced by mutually uncorrelated noise signals, then we obtain the expressions for interferometry with simultaneous noise sources.

Introduction

In seismic interferometry the response of a virtual source is created from responses of sequential transient or simultaneous noise sources. Most methods use crosscorrelation, but recently seismic interferometry by multidimensional deconvolution (MDD) has been proposed as well.

In the simultaneous-source method (also known as blended acquisition), overlapping responses of sources with small time delays are recorded (Beasley *et al.* , 1998). The crosstalk that occurs in imaging of simultaneous-source data can be reduced by using phase-encoded sources (Bagaini, 2006) or simultaneous noise sources (Howe *et al.* , 2007), by randomizing the time interval between the shots (Stefani *et al.* , 2007), or by inverting the blending operator (Berkhout, 2008).

Clearly seismic interferometry and the simultaneous-source method are related. In this paper we make this relation explicit by deriving deblending as a form of seismic interferometry by MDD.

Basic expression for seismic interferometry by multidimensional deconvolution

We consider an arbitrary inhomogeneous dissipative acoustic medium. In this medium we define a surface $\mathbb{S}$ enclosing a volume $\mathbb{V}$ (Figure 1). Outside $\mathbb{V}$ there is a source located at $\mathbf{x}_S$ and inside $\mathbb{V}$ we consider a receiver at $\mathbf{x}_B$. The Green's function between this source and receiver is defined as $G(\mathbf{x}_B, \mathbf{x}_S, t)$. For this configuration we consider the following convolutional Green's function representation (Wapenaar and van der Neut, 2010)

$$G(\mathbf{x}_B, \mathbf{x}_S, t) = \oint_{\mathbb{S}} \bar{G}_{\mathrm{d}}(\mathbf{x}_B, \mathbf{x}, t) * G^{\mathrm{in}}(\mathbf{x}, \mathbf{x}_S, t)\, \mathrm{d}\mathbf{x}. \tag{1}$$

$G^{\mathrm{in}}(\mathbf{x}, \mathbf{x}_S, t)$ is the part of the Green's function $G(\mathbf{x}, \mathbf{x}_S, t)$ that propagates inward into $\mathbb{V}$. $\bar{G}_{\mathrm{d}}(\mathbf{x}_B, \mathbf{x}, t)$ is the response of a dipole source at $\mathbf{x}$ (indicated by the subscript 'd'). The bar denotes that $\bar{G}_{\mathrm{d}}(\mathbf{x}_B, \mathbf{x}, t)$ is defined in a reference medium which is identical to the actual medium in $\mathbb{V}$ and which is homogeneous outside $\mathbb{S}$ (hence, $\mathbb{S}$ is an absorbing boundary for $\bar{G}_{\mathrm{d}}(\mathbf{x}_B, \mathbf{x}, t)$).

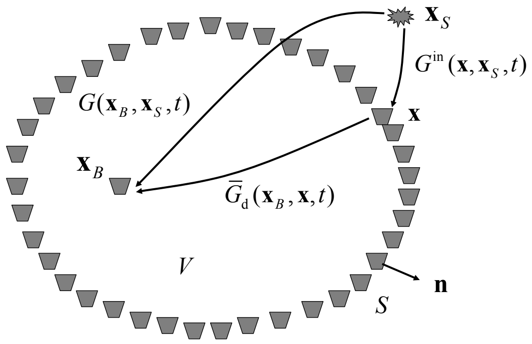

Figure 1: *Configuration for the convolution-type Green's function representation (equation 1). The medium does not need to be lossless. The rays represent full responses, including primary and multiple scattering due to inhomogeneities inside as well as outside* $\mathbb{S}$.

If we consider $\bar{G}_{\mathrm{d}}(\mathbf{x}_B, \mathbf{x}, t)$ as the unknown quantity, then equation (1) is an implicit representation of the convolution type for $\bar{G}_{\mathrm{d}}(\mathbf{x}_B, \mathbf{x}, t)$. If it were a single equation, the inverse problem would be ill-posed. However, equation (1) holds for each source position $\mathbf{x}_S$ (outside $\mathbb{V}$), which we will denote from here onward by $\mathbf{x}_S^{(i)}$, where i denotes the source number. Solving the ensemble of equations for $\bar{G}_{\mathrm{d}}(\mathbf{x}_B, \mathbf{x}, t)$ involves MDD.

In the following we modify the configuration to that of the virtual-source method, see Figure 2a. The closed surface $\mathbb{S}$ is replaced by an open surface. This is allowed as long as the source positions $\mathbf{x}_S^{(i)}$ are located at the appropriate side of $\mathbb{S}$ (i.e., outside $\mathbb{V}$). We choose $\mathbf{x}_B$ just below $\mathbb{S}$ and apply decomposition at $\mathbf{x}_B$ on both sides of equation (1), according to

$$G^{\mathrm{out}}(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) = \int_{\mathbb{S}} \bar{G}_{\mathrm{d}}^{\mathrm{out}}(\mathbf{x}_B, \mathbf{x}, t) * G^{\mathrm{in}}(\mathbf{x}, \mathbf{x}_S^{(i)}, t)\, \mathrm{d}\mathbf{x}, \tag{2}$$

where the superscript 'out' denotes waves propagating outward from $\mathbb{V}$.

For practical applications the Green's functions G^{in} and G^{out} in equation (2) should be replaced by responses of real sources, i.e., Green's functions convolved with source functions. In the following we consider three kinds of acquisition (sequential transient sources, noise sources and the simultaneous-source method) and discuss the inversion of equation (2) for those situations.

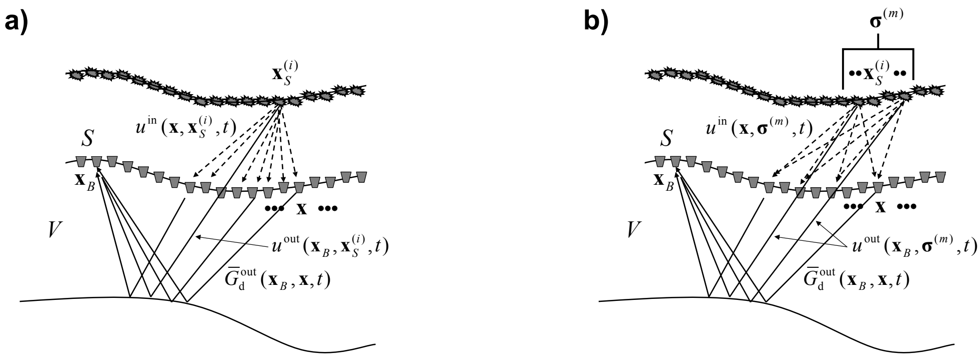

Figure 2: *(a) Convolutional model for the virtual-source method. (b) Convolutional model for the simultaneous-source method.*

Interferometry by MDD for sequential transient sources

For sequential transient sources we may write for the (non-overlapping) responses at $\mathbf{x}$ and $\mathbf{x}_B$

$$u^{\text{in}}(\mathbf{x},\mathbf{x}_S^{(i)},t) = G^{\text{in}}(\mathbf{x},\mathbf{x}_S^{(i)},t) * s^{(i)}(t), \tag{3}$$

$$u^{\text{out}}(\mathbf{x}_B,\mathbf{x}_S^{(i)},t) = G^{\text{out}}(\mathbf{x}_B,\mathbf{x}_S^{(i)},t) * s^{(i)}(t). \tag{4}$$

Hence, by convolving both sides of equation (2) with $s^{(i)}(t)$ we obtain

$$u^{\text{out}}(\mathbf{x}_B,\mathbf{x}_S^{(i)},t) = \int_{\mathbb{S}} \bar{G}_{\text{d}}^{\text{out}}(\mathbf{x}_B,\mathbf{x},t) * u^{\text{in}}(\mathbf{x},\mathbf{x}_S^{(i)},t)\,\text{d}\mathbf{x}. \tag{5}$$

Solving equation (5) in a least-squares sense is equivalent to solving its normal equation (Menke, 1989). We obtain the normal equation by crosscorrelating both sides of equation (5) with $u^{\text{in}}(\mathbf{x}_A,\mathbf{x}_S^{(i)},t)$ (with $\mathbf{x}_A$ on $\mathbb{S}$) and taking the sum over all sources (van der Neut *et al.* , 2010). This gives

$$C_{\text{seq}}(\mathbf{x}_B,\mathbf{x}_A,t) = \int_{\mathbb{S}} \bar{G}_{\text{d}}^{\text{out}}(\mathbf{x}_B,\mathbf{x},t) * \Gamma_{\text{seq}}(\mathbf{x},\mathbf{x}_A,t)\ \text{d}\mathbf{x}, \tag{6}$$

(subscript 'seq' standing for 'sequential') where

$$\begin{aligned} C_{\text{seq}}(\mathbf{x}_B,\mathbf{x}_A,t) &= \sum_i u^{\text{out}}(\mathbf{x}_B,\mathbf{x}_S^{(i)},t) * u^{\text{in}}(\mathbf{x}_A,\mathbf{x}_S^{(i)},-t) && (7)\\ &= \sum_i G^{\text{out}}(\mathbf{x}_B,\mathbf{x}_S^{(i)},t) * G^{\text{in}}(\mathbf{x}_A,\mathbf{x}_S^{(i)},-t) * S^{(i)}(t), \\ \Gamma_{\text{seq}}(\mathbf{x},\mathbf{x}_A,t) &= \sum_i u^{\text{in}}(\mathbf{x},\mathbf{x}_S^{(i)},t) * u^{\text{in}}(\mathbf{x}_A,\mathbf{x}_S^{(i)},-t) && (8)\\ &= \sum_i G^{\text{in}}(\mathbf{x},\mathbf{x}_S^{(i)},t) * G^{\text{in}}(\mathbf{x}_A,\mathbf{x}_S^{(i)},-t) * S^{(i)}(t), \end{aligned}$$

with

$$S^{(i)}(t) = s^{(i)}(t) * s^{(i)}(-t). \tag{9}$$

Note that $C_{\text{seq}}(\mathbf{x}_B,\mathbf{x}_A,t)$ as defined in equation (7) is a correlation function with a similar form as the one used by Mehta *et al.* (2007) in their virtual-source method by wave field decomposition. $\Gamma_{\text{seq}}(\mathbf{x}_B,\mathbf{x}_A,t)$ as defined in equation (8) is what we call the point-spread function. Equation (6) shows that the sequential transient correlation function $C_{\text{seq}}(\mathbf{x}_B,\mathbf{x}_A,t)$ is proportional to the sought Green's function $\bar{G}_{\text{d}}^{\text{out}}(\mathbf{x}_B,\mathbf{x},t)$ with its source smeared in space and time by the sequential point-spread function $\Gamma_{\text{seq}}(\mathbf{x},\mathbf{x}_A,t)$. MDD involves inverting equation (6). Ideally, this removes the distorting effects of the point-spread function $\Gamma_{\text{seq}}(\mathbf{x},\mathbf{x}_A,t)$ (including the so-called spurious multiples) from the correlation function $C_{\text{seq}}(\mathbf{x}_B,\mathbf{x}_A,t)$ and thus yields an estimate of the Green's function $\bar{G}_{\text{d}}^{\text{out}}(\mathbf{x}_B,\mathbf{x},t)$.

Interferometry by MDD for simultaneous noise sources

For simultaneous noise sources we write for the responses at $\mathbf{x}$ and $\mathbf{x}_B$

$$u^{\text{in}}(\mathbf{x},t) = \sum_i G^{\text{in}}(\mathbf{x},\mathbf{x}_S^{(i)},t) * N^{(i)}(t), \tag{10}$$

$$u^{\text{out}}(\mathbf{x}_B,t) = \sum_j G^{\text{out}}(\mathbf{x}_B,\mathbf{x}_S^{(j)},t) * N^{(j)}(t). \tag{11}$$

We define the noise correlation function and the noise point-spread function, respectively, as

$$
\begin{aligned}
C_{\text{noise}}(\mathbf{x}_B, \mathbf{x}_A, t) &= \langle u^{\text{out}}(\mathbf{x}_B, t) * u^{\text{in}}(\mathbf{x}_A, -t)\rangle, && (12)\\
\Gamma_{\text{noise}}(\mathbf{x}, \mathbf{x}_A, t) &= \langle u^{\text{in}}(\mathbf{x}, t) * u^{\text{in}}(\mathbf{x}_A, -t)\rangle. && (13)
\end{aligned}
$$

Here $\langle\cdot\rangle$ denotes ensemble averaging. In practice the ensemble averaging is replaced by integrating over sufficiently long time and/or averaging over different time intervals. Upon substitution of equations (10) and (11) into equations (12) and (13), assuming the noise sources are mutually uncorrelated, according to

$$
\langle N^{(j)}(t) * N^{(i)}(-t)\rangle = \delta_{ij} S^{(i)}(t), \tag{14}
$$

it follows that the noise correlation and point-spread functions as defined in equations (12) and (13) converge to the sequential transient correlation and point-spread functions defined in equations (7) and (8). Hence, analogous to equation (6), interferometry by MDD for simultaneous noise sources comes to inverting

$$
C_{\text{noise}}(\mathbf{x}_B, \mathbf{x}_A, t) = \int_{\mathbb{S}} \bar{G}_{\text{d}}^{\text{out}}(\mathbf{x}_B, \mathbf{x}, t) * \Gamma_{\text{noise}}(\mathbf{x}, \mathbf{x}_A, t)\, \mathrm{d}\mathbf{x}. \tag{15}
$$

Interferometry by MDD for the simultaneous-source method

Consider the configuration depicted in Figure 2b, where $\boldsymbol{\sigma}^{(m)}$ denotes a group of source positions $\mathbf{x}_S^{(i)}$. Although the figure suggests that these sources are adjacent to each other, they may also be randomly selected from the total array of sources. Assuming the source at $\mathbf{x}_S^{(i)}$ emits a delayed source wavelet $s^{(i)}(t - t_i)$, the blended inward and outward propagating fields at $\mathbb{S}$ are given by

$$
\begin{aligned}
u^{\text{in}}(\mathbf{x}, \boldsymbol{\sigma}^{(m)}, t) &= \sum_{\mathbf{x}_S^{(i)} \in \boldsymbol{\sigma}^{(m)}} G^{\text{in}}(\mathbf{x}, \mathbf{x}_S^{(i)}, t) * s^{(i)}(t - t_i), && (16)\\
u^{\text{out}}(\mathbf{x}_B, \boldsymbol{\sigma}^{(m)}, t) &= \sum_{\mathbf{x}_S^{(i)} \in \boldsymbol{\sigma}^{(m)}} G^{\text{out}}(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) * s^{(i)}(t - t_i), && (17)
\end{aligned}
$$

where $\mathbf{x}_S^{(i)} \in \boldsymbol{\sigma}^{(m)}$ denotes that the summation takes place over all source positions $\mathbf{x}_S^{(i)}$ in group $\boldsymbol{\sigma}^{(m)}$. By convolving both sides of equation (2) with $s^{(i)}(t-t_i)$ and summing over all sources in $\boldsymbol{\sigma}^{(m)}$ we obtain, analogous to equation (5),

$$
u^{\text{out}}(\mathbf{x}_B, \boldsymbol{\sigma}^{(m)}, t) = \int_{\mathbb{S}} \bar{G}_{\text{d}}^{\text{out}}(\mathbf{x}_B, \mathbf{x}, t) * u^{\text{in}}(\mathbf{x}, \boldsymbol{\sigma}^{(m)}, t)\, \mathrm{d}\mathbf{x}. \tag{18}
$$

When the indicated receivers on $\mathbb{S}$ in Figure 2b are real receivers in a borehole, then $u^{\text{out}}(\mathbf{x}_B, \boldsymbol{\sigma}^{(m)}, t)$ and $u^{\text{in}}(\mathbf{x}, \boldsymbol{\sigma}^{(m)}, t)$ are the (decomposed) measured blended wave fields in the borehole. On the other hand, in case of surface data acquisition, $u^{\text{out}}(\mathbf{x}_B, \boldsymbol{\sigma}^{(m)}, t)$ represents the blended data after model-based receiver redatuming to $\mathbb{S}$ and $u^{\text{in}}(\mathbf{x}, \boldsymbol{\sigma}^{(m)}, t)$ represents the blended sources, forward extrapolated through the model to $\mathbb{S}$.

We obtain the normal equation by crosscorrelating both sides of equation (18) with $u^{\text{in}}(\mathbf{x}_A, \boldsymbol{\sigma}^{(m)}, t)$ (with $\mathbf{x}_A$ on $\mathbb{S}$) and taking the sum over all source groups $\boldsymbol{\sigma}^{(m)}$. This gives

$$
C_{\text{sim}}(\mathbf{x}_B, \mathbf{x}_A, t) = \int_{\mathbb{S}} \bar{G}_{\text{d}}^{\text{out}}(\mathbf{x}_B, \mathbf{x}, t) * \Gamma_{\text{sim}}(\mathbf{x}, \mathbf{x}_A, t)\, \mathrm{d}\mathbf{x}, \tag{19}
$$

(subscript 'sim' standing for 'simultaneous') where

$$
\begin{aligned}
C_{\text{sim}}(\mathbf{x}_B, \mathbf{x}_A, t) &= \sum_m u^{\text{out}}(\mathbf{x}_B, \boldsymbol{\sigma}^{(m)}, t) * u^{\text{in}}(\mathbf{x}_A, \boldsymbol{\sigma}^{(m)}, -t), && (20)\\
\Gamma_{\text{sim}}(\mathbf{x}, \mathbf{x}_A, t) &= \sum_m u^{\text{in}}(\mathbf{x}, \boldsymbol{\sigma}^{(m)}, t) * u^{\text{in}}(\mathbf{x}_A, \boldsymbol{\sigma}^{(m)}, -t). && (21)
\end{aligned}
$$

Substituting equations (16) and (17) into equations (20) and (21) gives

$$
C_{\text{sim}}(\mathbf{x}_B, \mathbf{x}_A, t) = C_{\text{seq}}(\mathbf{x}_B, \mathbf{x}_A, t) + \text{crosstalk}, \tag{22}
$$

$$
\Gamma_{\text{sim}}(\mathbf{x}, \mathbf{x}_A, t) = \Gamma_{\text{seq}}(\mathbf{x}, \mathbf{x}_A, t) + \text{crosstalk}. \tag{23}
$$

The contributions to the crosstalk have the form $G^{\rm out}(\mathbf{x}_B, \mathbf{x}_S^{(j)}, t) * G^{\rm in}(\mathbf{x}_A, \mathbf{x}_S^{(i)}, -t) * s^{(j)}(t-t_j) * s^{(i)}(-t-t_i)$ for $i \neq j$, where $\mathbf{x}_S^{(i)}$ and $\mathbf{x}_S^{(j)}$ are members of the same group. Deblending involves inverting equation (19) by MDD. Ideally this eliminates the crosstalk from the correlation function and gives the deblended virtual-source response $\bar{G}_{\rm d}^{\rm out}(\mathbf{x}_B, \mathbf{x}, t)$.

In Figure 3 we show two examples of the point-spread function for the simultaneous-source method. We consider a homogeneous overburden (propagation velocity 2000 m/s), and 256 sources at the surface with a lateral spacing of 20 m. We form 64 source groups $\boldsymbol{\sigma}^{(m)}$, each containing four adjacent sources which emit transient wavelets, 0.25 s after one another. The inward propagating field $u^{\rm in}(\mathbf{x}, \boldsymbol{\sigma}^{(m)}, t)$ at depth level $x_3 = 500$ m is defined by equation (16). The point-spread function $\Gamma_{\rm sim}(\mathbf{x}, \mathbf{x}_A, t)$, as defined in equation (21), is shown in Figure 3a, with $\mathbf{x}_A$ fixed at the center of the array and $\mathbf{x}$ variable along the array. Next we add random variations (uniform between + and -50%) to the 0.25 s time interval between the sources and evaluate again the point-spread function $\Gamma_{\rm sim}(\mathbf{x}, \mathbf{x}_A, t)$, see Figure 3b. Note that the band-limited delta function around $\mathbf{x} = \mathbf{x}_A$ and $t = 0$ remains intact, whereas the crosstalk disperses in space and time. Inverting noisy point-spread functions like the one in Figure 3b may be a more stable process than inverting nearly periodic point-spread functions like the one in Figure 3a.

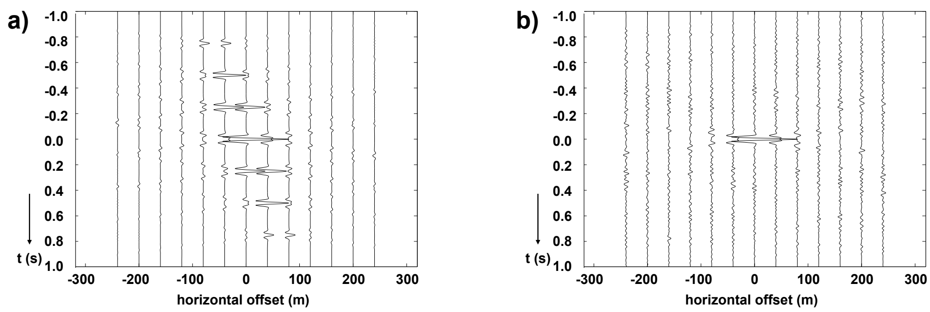

Figure 3: *(a) Point-spread function for the simultaneous-source method, using 64 groups of four sources each, with a regular time interval of 0.25 s. (b) Idem, after adding random variations to the time interval.*

Conclusions

We have shown that deblending can be seen as a form of seismic interferometry by MDD. The discussed representation for the simultaneous-source method has two interesting limiting cases. When each source group $\boldsymbol{\sigma}^{(m)}$ consists of a single source, then we obtain the expressions for interferometry with sequential transient sources. On the other hand, when there is only one source group containing all sources and when the source wavelets are replaced by mutually uncorrelated noise signals, then we obtain the expressions for interferometry with simultaneous noise sources.

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