

TITLE:

“Green’s function representation of virtual reflectors (VR)”

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Introduction

Virtual seismic signals are synthesized by composing and stacking traces recorded by a plurality of sources and receivers. Seismic interferometry typically uses cross-correlation to compose the recorded traces (Wapenaar, 2004), and to get virtual sources at the position of receivers (Calvert, 2004; Bakulin et al 2007). Recently a method was proposed that is based on cross-convolution to simulate virtual reflectors. The Virtual Reflector (VR) method (Poletto, 2008; Poletto and Farina, 2008a) simulates new seismic signals by processing recorded traces from a plurality of sources and receivers. The approach makes it possible to obtain virtual reflected signals as if in the position of receivers (or sources) there was a real reflector even if said reflector is not present there. The novel method exploits its utility in combination with interferometry (Poletto and Farina, 2008b). In this paper we present the formulation of the Virtual Reflector theory for an acoustic inhomogeneous medium by using the Kirchhoff - Helmholtz integral representation theorem (Poletto and Wapenaar, 2009).

Theory

Consider an acoustic inhomogeneous medium in which the seismic wavefields propagate. Let S be the total surface encompassing a volume of interest VO , and ' n ' the outward normal direction to the surface. The Green's integral theorem states that

$$\int_{VO} (G\nabla^2 U - U\nabla^2 G) dV = \int_S (G\nabla U - U\nabla G) \cdot n dS, \quad (1)$$

where the scalar fields $U=U(\omega)$ and $G=G(\omega)$ are Fourier transforms of causal functions, and ω is the angular frequency. In the common use, U and G represent the propagating wavefield and the medium response to an impulse, Green's function, respectively.

In the proposed approach, the virtual reflector signal is the result of the composition of scalar fields from different sources included in the volume VO surrounded by receivers located at the outer surface S_o (Poletto and Farina, 2008a). An equivalent approach is obtained by interchanging sources and receivers for reciprocity.

In the VR application, in general, the sources are assumed to be impulsive and with known delay (waveform), which is not a significant limitation for seismic exploration applications.

Assume that two point sources at internal points r_1 and r inside S_o (Figure 1a) generate the scalar fields U and G , respectively. Under these conditions, equation (1) can be reformulated in the distributional sense, and it can be shown that it can be expressed as (Bleistein, 1984; De Hoop, 1995)

$$U - G = \frac{1}{4\pi} \int_{S_o} \left(G \frac{\partial U}{\partial n} - U \frac{\partial G}{\partial n} \right) dS_o, \quad (2)$$

where we have used $\nabla \cdot n = \partial/\partial n$ for the normal differentiation operator acting at the surface S_o . Equation (2) is similar to the result obtained by Bleistein (1984) for integral representation of the scattered wavefield. The advantage of equation (2) is that we can represent the wavefields in the propagation space with different boundary conditions set for G and U on S_o . Equation (2) gives a cross-convolutive equation expressed in the Fourier frequency domain. If we assume that the scalar field $U(r_o, r_1)$ recorded at r_o is generated by a unit source at inner point r_1 , it represents by definition the medium transfer function, and we can formally substitute $G(r_o, r_1)$ for $U(r_o, r_1)$ in equation (2), to obtain the wavefield representation as

$$V(r, r_1) = \frac{1}{4\pi} \int_{S_o} \left[G(r_o, r) \frac{\partial G(r_o, r_1)}{\partial n} - G(r_o, r_1) \frac{\partial G(r_o, r)}{\partial n} \right] dS_o, \quad (3)$$

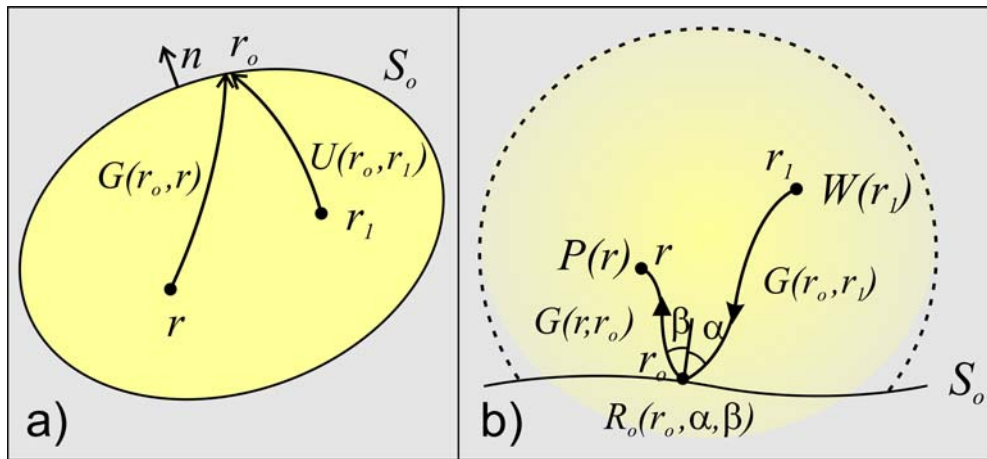


Figure 1: Acoustic model used for VR integral representation: .a) With sources at r and r_1 and receivers at S_o . b) For scattered wavefields (modified after Poletto and Wapenaar, 2009).

where the scalar function

$$V(r, r_1) = G(r, r_1) - G(r_1, r), \quad (4)$$

and where it is intended that the boundary conditions on S_o are different for the scalar functions $G(r_o, r)$ and $G(r_o, r_1)$ at the right hand side of Eq. (3). Integral equation (3) is used to represent the wavefields of a virtual reflector (Poletto and Wapenaar, 2009).

VR wavefield synthesis

The VR acoustic representation theorem is discussed with different boundary conditions at the enclosing surface S_o by Poletto and Wapenaar (2009). They show that the representation with a perfect reflector at the surface S_o is obtained by imposing suitable boundary conditions for the function $G(r_o, r)$ at surface S_o . The *rigid boundary* model is expressed by the *Neumann* boundary condition, for which we have on S_o

$$\frac{\partial G(r_o, r)}{\partial n} = 0. \quad (5)$$

In this model the contrast medium at the interface is a perfectly rigid medium, with reflection coefficient $R = +1$. The *free boundary* model is expressed by the *Dirichlet* boundary condition, for which we have on S_o

$$G(r_o, r) = 0 \quad (6)$$

In this case the reflection coefficient $R = -1$. Equations (5) and (6) are used in equation (3). With a free surface, it is assumed that $G(r_o, r)$ is approximated by its counterpart function without boundary. In general, the Virtual Reflector synthesis can be represented in the form (Poletto and Wapenaar, 2009)

$$V_{RF} = \frac{\mp i\omega}{4\pi c} \int_{S_o} G(r_o, r) G(r_o, r_1) dS_o. \quad (7)$$

where $i = \sqrt{-1}$, c is the medium velocity, and we have used the 'normal ray' approximation $\partial / \partial n \approx -i\omega / c$. In equation (7) we have neglected a factor $\cos \alpha$ where α is the surface

impact-angle, which is dependent on the surface point coordinate and is in general unknown for waves propagating in an arbitrary medium. The result of equation (7) corresponds, apart from a scaling factor, to the integral of the cross-convolutions (expressed in the Fourier frequency domain) over the enclosing surface S_o , where the scalar field functions are recorded, that is the definition of virtual reflector as proposed by Poletto (2008).

The VR representation is extended by a generalization to acoustic scattered wavefields (Poletto and Wapenaar, 2009). The scattered wavefield at r from, without loss of generality, a unit source $W(r_1) = 1$ at r_1 can be expressed, by using reciprocity for $G(r, r_o)$ (Figure 1b) as

$$P(r, r_1) = \int_{S_o} G(r_o, r) R_o(r_o, \alpha, \beta) G(r_o, r_1) dS_o \quad (8)$$

where $R_o(r_o, \alpha, \beta)$ is a variable reflection coefficient function (Taylor, 1975) of the scattering point r_o on S_o and is dependent on scattering ray angles α and β (Fig. 1b). The VR representation is obtained by choosing a suitable reflection coefficient R_o in equation (8).

The reciprocity condition enables us to apply the same reasoning when we interchange sources and receivers.

Examples

VR results are shown with synthetic examples, in inhomogeneous acoustic media, with and without boundary by Poletto and Farina (2008a and 2008b). Figure 2 (left panel) shows a 2D 1400 m \times 1000 m acoustic model used for VR representation. The synthetic seismograms are calculated by a 2D acoustic finite-difference code. The virtual seismic results are obtained with illumination from the shallower source line (red dots). The source wavelet, approximating an impulsive ideal source, has 30 Hz peak frequency. The wavefields propagate in the top uniform medium of velocity $c=1000$ m/s, and are recorded by receivers placed at the bottom boundary profile (represented by green dots). The boundary contrast medium is inhomogeneous, with lateral velocity gradient (c ranging between 800 m/s and 3300 m/s), to simulate variable boundary conditions at the reflecting interface.

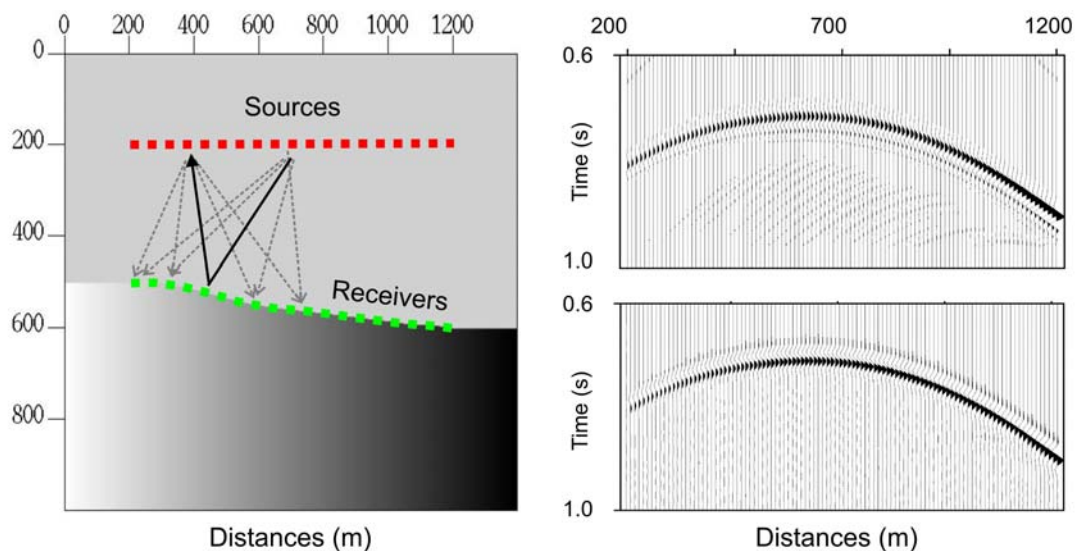


Figure 2: Acoustic model used for VR representation (left panel) and simulated virtual results (bottom right panel) compared to control synthetic reflection seismograms (top right panel). Some differences arise in the waveforms of the source (top) and cross-convolved source (bottom).

The seismic panel at the right-top side of Figure 2 is the reflection gather obtained with a source at central point (position 700 m), and recorded by a control receiver line located at the source line positions. The right-bottom side gather represents the virtual signal – approximating the reflection Green's function – obtained by using the VR equation (reference convolution trace at 700 m) for the model without contrast medium. No knowledge about the propagation medium is needed to obtain the VR signal. The top and bottom panels represent estimates of the reflection Green's function composed with the source and cross-convolved source wavelets, respectively. Some differences in the waveforms of the cross-convolved (virtual) and original source wavelets can be observed. The VR traces are here scaled by variable reflection coefficients, to account for the amplitudes of the model reflections. The agreement between the original synthetic traces and the virtual reflection results can be appreciated.

Conclusions

We derive the Kirchhoff integral representation of the virtual signal produced in an inhomogeneous acoustic medium by a reflecting surface surrounding two points where sources are located. We show that the reflection representation is equivalent to composing the wavefields of the two sources recorded on the encompassing surface. The analysis demonstrates that the Kirchhoff integral represents the cross-convolution term of the virtual reflector signal. We generalize the representation for the scattered wavefield from a virtual reflector with variable reflection coefficients.

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