

Introduction

Recently many researchers have focused on Cross-Correlation-based redatuming methods, like Reverse Time Acoustics (RTA), Interferometric Imaging and the Virtual Source (VS) method (Schuster & Zhou, 2006; Bakulin and Calvert, 2006). Most of these methods have been derived from time-reversal arguments or Seismic Interferometry, requiring a closed boundary of sources to retrieve the Green's function between two receiver locations. In practice these methods are often applied to cases of one-sided illumination, causing an incomplete retrieval of the reflection response and the emergence of spurious events (Wapenaar 2006; Snieder et al., 2006). A different approach is to interpret these Cross-Correlation-based redatuming methods as attempts to solve the following integral equation (Wapenaar & Verschuur, 1996; Amundsen, 1999; Holvik & Amundsen, 2005; Schuster & Zhou, 2006; Wapenaar et al., 2008):

$$\hat{\mathbf{p}}^-(\mathbf{x}_A, \mathbf{x}_S, \omega) = \iint_{\partial D_B} \hat{\mathbf{R}}_0^+(\mathbf{x}_A, \mathbf{x}_B, \omega) \hat{\mathbf{p}}^+(\mathbf{x}_B, \mathbf{x}_S, \omega) d^2 \mathbf{x}_B. \quad (1)$$

Here $\hat{\mathbf{p}}^+(\mathbf{x}_B, \mathbf{x}_S, \omega)$ is the multi-component downgoing wave field emitted by a source at location \mathbf{x}_S , recorded by a receiver at \mathbf{x}_B . The integral is carried out over a surface ∂D_B through the receiver coordinates \mathbf{x}_B that should capture the entire downgoing wave field. Matrix $\hat{\mathbf{R}}_0^+(\mathbf{x}_A, \mathbf{x}_B, \omega)$ represents the multi-component reflection response of the half space below ∂D_B between receiver locations \mathbf{x}_B and \mathbf{x}_A , with the upper half space replaced by a homogeneous medium. The left-hand side $\hat{\mathbf{p}}^-(\mathbf{x}_A, \mathbf{x}_S, \omega)$ is the upgoing wave field emitted by the same source, recorded by a receiver at location \mathbf{x}_A . Various authors have shown how $\hat{\mathbf{R}}_0^+(\mathbf{x}_A, \mathbf{x}_B, \omega)$ can be retrieved by cross-correlation of the upgoing wave field at \mathbf{x}_B with the downgoing wave field at \mathbf{x}_A and summation over the source locations \mathbf{x}_S (Schuster & Zhou, 2006; Mehta et al., 2007). In this paper we solve equation (1) by multi-dimensional deconvolution (Wapenaar et al., 2008), which is closely related to least-squares redatuming (Schuster & Zhou, 2006). We compare the results of multi-dimensional deconvolution with cross-correlation-based interferometry for retrieved PP, SS and PS converted reflection responses for a 1D elastic model with multi-component receivers.

Controlled-source interferometry by multi-dimensional deconvolution

We evaluate equation (1) for various source types and locations and rewrite the resulting equations in matrix form as

$$\hat{\mathbf{P}}^- = \hat{\mathbf{R}}_0^+ \hat{\mathbf{P}}^+. \quad (2)$$

Here $\hat{\mathbf{P}}^\pm$ is a matrix of vectors $\hat{\mathbf{p}}^\pm(\mathbf{x}_{A/B}, \mathbf{x}_S, \omega)$, where the columns have fixed source type and location but variable receiver type and location and the rows have fixed receiver type and location but variable source type and location. $\hat{\mathbf{R}}_0^+$ is a matrix of reflection matrices $\hat{\mathbf{R}}_0^+(\mathbf{x}_A, \mathbf{x}_B, \omega)$, holding the different wave mode reflections as its components. Equation (2) can be solved with multi-dimensional deconvolution (MD):

$$\hat{\mathbf{R}}_0^+ \approx \hat{\mathbf{P}}^- (\hat{\mathbf{P}}^+)^{\dagger} \left[\hat{\mathbf{P}}^+ \{ \hat{\mathbf{P}}^+ \}^{\dagger} + \varepsilon^2 \mathbf{I} \right]^{-1}, \quad (3)$$

where ε is introduced as a stabilization factor, superscript \dagger denotes the complex-conjugate transpose and \mathbf{I} is the identity matrix. If the term between square brackets is approximated by the identity matrix, equation (3) converges to the cross-correlation-based (CC) solution:

$$\hat{\mathbf{R}}_0^+ \approx \hat{\mathbf{P}}^- (\hat{\mathbf{P}}^+)^{\dagger}. \quad (4)$$

If we rewrite equation (4) in integral form we find:

$$\hat{\mathbf{R}}_0^+ (\mathbf{x}_A, \mathbf{x}_B, \omega) = \iint_{\partial D_S} \hat{\mathbf{p}}^- (\mathbf{x}_A, \mathbf{x}_S, \omega) \{ \hat{\mathbf{p}}^+ (\mathbf{x}_B, \mathbf{x}_S, \omega) \}^{\dagger} d^2 \mathbf{x}_S, \quad (5)$$

which can be interpreted as a multi-component version of the Virtual Source (VS) method (Bakulin and Calvert, 2006).

Example: 1D Elastic Model

We test our methodology to a 1D elastic model, inspired by a middle eastern field. In this model, 321 sources are situated at the earth surface with a spacing of 5m. The upper 200 meter of the subsurface consists of finely layered material. A receiver array is situated in a homogeneous layer at 250 m depth. Below the array we find four strong reflectors that we want to image. The data are decomposed into flux-normalized up- and downgoing P- and S-wave fields. Equations (3) and (4) are applied to retrieve the reflection response between fixed source location \mathbf{x}_B and floating receiver location \mathbf{x}_A . The result is then transformed to the time domain. For comparison, we also generated the Ground Truth (GT) response by placing an active vertical force source at location \mathbf{x}_B and receivers at \mathbf{x}_A . The GT response is also decomposed into flux-normalized up- and downgoing P- and S-wave fields, from where the PP, SS and PS converted reflection responses are constructed. In Figure 1A we show the retrieved PP reflection response using CC as prescribed by equation (4). In Figure 1B we show the result of MD, dictated by equation (3). Note that we have achieved better amplitude preservation and a reduction of spurious ringing and artifacts compared to the CC-based results. The improvements can also be visualized in the FK-domain – see Figure 2. While CC-data is subject to the irregular amplitude radiation pattern of the retrieved source, this effect is strongly suppressed by MD. The retrieved SS reflections for both CC and MD are shown in Figure 3. MD yields a better convergence to the kinematics of the GT response but does still not retrieve correct amplitudes at large offsets. This can also be visualized in the FK-domain, where we see improved convergence for low wavenumbers but a remaining lack of illumination at high wavenumbers (Figure 4). Finally, we retrieve PS converted reflections – see Figure 5. Once more MD yields better amplitudes and a reduction of spurious artifacts and ringing compared to the results of CC. The effects can also be studied in the FK-domain (Figure 6), showing broader coverage of MD than the CC-based results.

Conclusion

We have compared retrieved reflection data by controlled-source Seismic Interferometry from multi-component receivers in a 1D elastic model as generated by Cross-Correlation (CC) with their counterpart as generated by multi-dimensional deconvolution (MD) in both space-time and the FK-domain. MD results in better amplitude retrieval and a reduction of spurious artifacts compared to CC for all PP, SS and PS converted reflections. MD can therefore be regarded as a significant step forward in the retrieval of single- or multi-component reflection data by controlled-source Seismic Interferometry with one-sided illumination.

References

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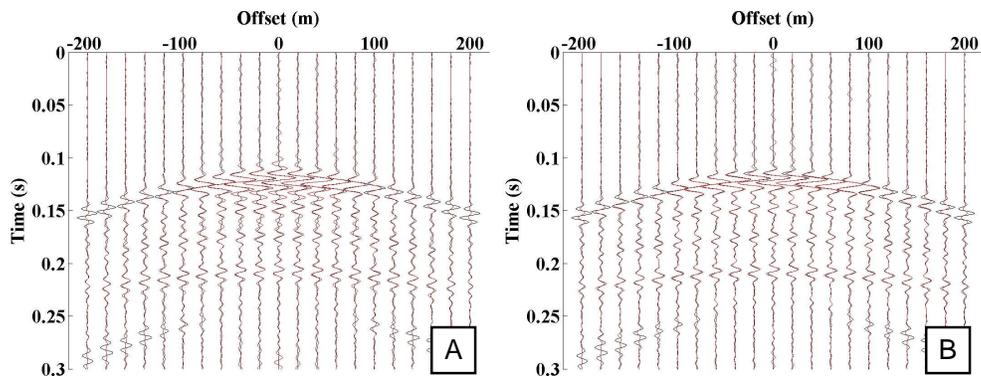


Figure 1A: PP reflection response by CC (red) versus GT (black);
-B: PP reflection response by MD (red) versus GT (black).

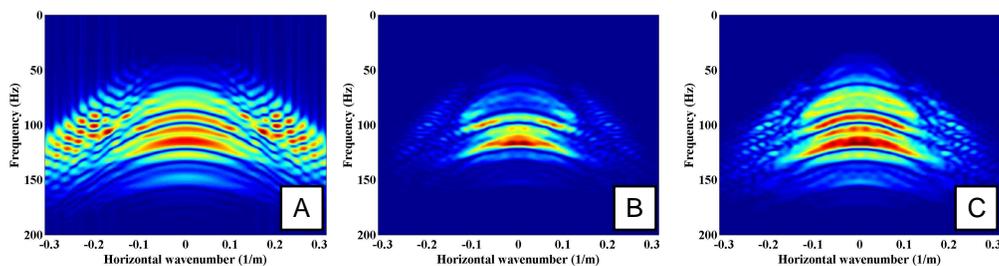


Figure 2: FK-representation of the PP reflection response; GT (panel A), CC (panel B) and MD (panel C).

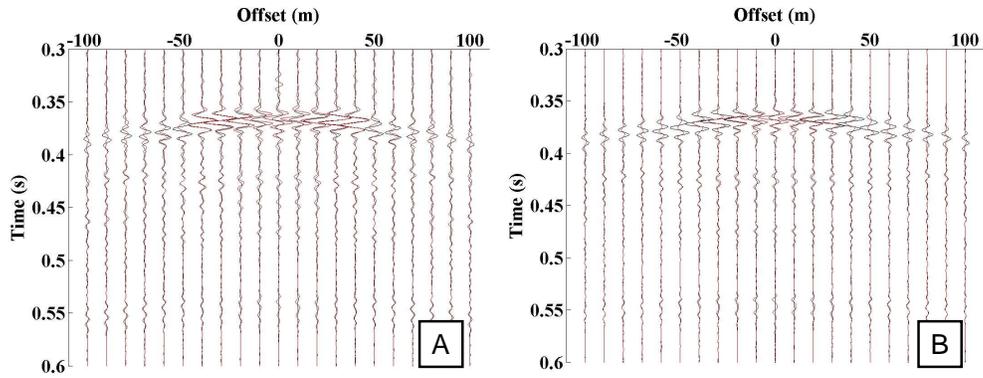


Figure 3A: SS reflection response by CC (red) versus the GT (black);
-B: SS reflection response by MD (red) versus the GT (black).

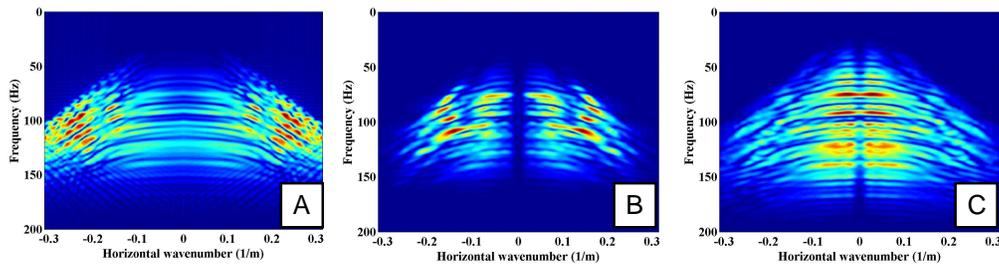


Figure 4: FK-representation of the SS reflection response; GT (panel A), CC (panel B) and MD (panel C).

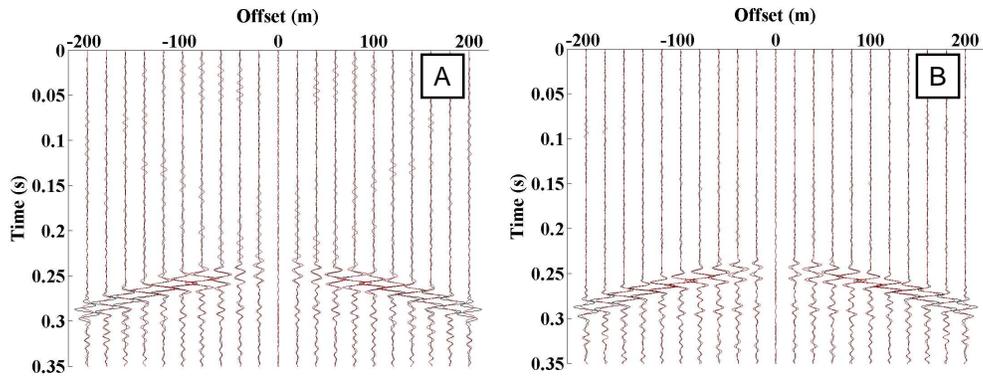


Figure 5A: PS reflection response by CC (red) versus GT (black);
-B: PS reflection response by MD (red) versus GT (black).

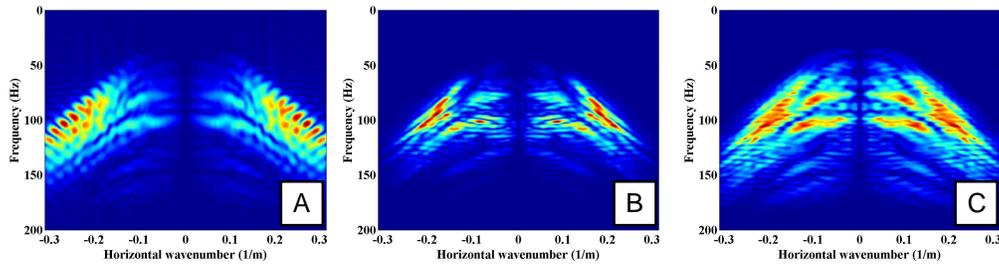


Figure 6: FK-representation of the PS reflection response; GT (panel A), CC (panel B) and MD (panel C).