Seismic interferometry-by-deconvolution for controlled-source and passive data

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Summary:

We discuss seismic interferometry by multi-dimensional deconvolution for controlled-source data as well as for passive data and compare both approaches with the corresponding correlation-based interferometric methods. For the controlled-source situation we derive the virtual source method as an approximation of the multi-dimensional deconvolution method. For the passive data situation we show that the deconvolution method and the cross-correlation method are essentially different, and we discuss the merits and drawbacks of both approaches.

Introduction

Seismic interferometry is a cross-correlation based process to generate new seismic responses from existing ones. It can be applied to passive data as well as to controlled-source data, see the supplement of the 2006 July-August issue of Geophysics. Recently it has been recognized that in specific cases it can be advantageous to replace the correlation process by deconvolution. One of the main advantages is that deconvolution compensates for the properties of the source wavelet; another advantage is that is not necessary to assume that the medium is lossless. Snieder *et al.* (2006) deconvolve passive wave fields observed at different depth levels and show that this changes the boundary conditions of a system. They applied it to earthquake data recorded at different heights in the Millikan library in Pasadena and obtained the impulse response of the building. Mehta *et al.* (2007) used a somewhat similar approach to estimate the near-surface properties of the Earth from passive recordings in a vertical borehole. Both approaches employ a 1-D deconvolution process.

Various authors have shown that multi-dimensional deconvolution processes, applied to controlled-source data with the receivers at a constant depth level (for example at the ocean bottom or in a horizontal borehole), can also be used to change the boundary conditions of a system. Wapenaar and Verschuur (1996), Amundsen (1999), Wapenaar *et al.* (2000) and Holvik and Amundsen (2005) use multi-dimensional deconvolution of down going and up going waves at the ocean bottom to suppress ocean-bottom and surface-related multiples. Berkhout's data processing in the inverse data space (Berkhout, 2006) is another form of multi-dimensional deconvolution. Schuster and Zhou (2006) and Wapenaar *et al.* (2008) discuss multi-dimensional deconvolution in the context of seismic interferometry. Slob *et al.* (2007) apply interferometry-by-deconvolution to modelled CSEM data and demonstrate the insensitivity to dissipation as well as the potential of changing the boundary conditions: the effect of the air wave, a notorious problem in CSEM prospecting, is largely suppressed. In the following we review interferometry by multi-dimensional deconvolution of controlled-source data and we discuss a modification for passive data. We compare both approaches with correlation-based interferometric methods.

Interferometry by multi-dimensional deconvolution of controlled-source data

Consider the situation depicted in Figure 1. State B represents a configuration with sources at $\mathbf{x}_S$ at or below the Earth's surface $\partial\mathbb{D}_0$, and receivers at a constant depth level (the solid line just above $\partial\mathbb{D}_1$). This receiver depth level can be for example the ocean bottom or a horizontal borehole. The wave field at the receiver level is decomposed into down going and up going fields $\hat{p}^+$ and $\hat{p}^-$, respectively (the circumflex denotes the frequency domain; the angular frequency will be denoted by ω). $\hat{R}_0^+(\mathbf{x}, \mathbf{x}_A, \omega)$ in state A represents the reflection response of the medium below $\partial\mathbb{D}_1$ for a source at $\mathbf{x}_A$ and a receiver at $\mathbf{x}$. State A is defined such that the half-space above $\mathbb{D}_1$ is reflection free, hence, $\hat{R}_0^+(\mathbf{x}, \mathbf{x}_A, \omega)$ does not contain any multiple reflections related to the medium above $\partial\mathbb{D}_1$, including the ocean bottom and/or the Earth's free surface. The subscript 0 in $\hat{R}_0^+(\mathbf{x}, \mathbf{x}_A, \omega)$ denotes the absence of these multiples.

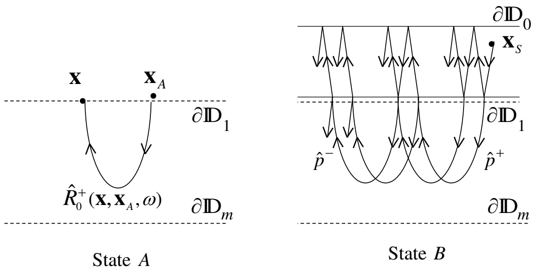

Figure 1: Configuration for controlled-source data.

The fields in states A and B are related according to

$$\hat{p}^-(\mathbf{x}_A, \mathbf{x}_S, \omega) = \int_{\partial\mathbb{D}_1} \hat{R}_0^+(\mathbf{x}_A, \mathbf{x}, \omega)\hat{p}^+(\mathbf{x}, \mathbf{x}_S, \omega)\mathrm{d}^2\mathbf{x}. \tag{1}$$

This is an integral equation of the first kind for $\hat{R}_0^+(\mathbf{x}_A, \mathbf{x}, \omega)$ [due to source-receiver reciprocity we have $\hat{R}_0^+(\mathbf{x}_A, \mathbf{x}, \omega) = \hat{R}_0^+(\mathbf{x}, \mathbf{x}_A, \omega)$]. Note that $\hat{R}_0^+$ is the Fourier transform of an impulse response, whereas $\hat{p}^+$ and $\hat{p}^-$ are proportional to the source spectrum $\hat{s}(\omega)$ of the source at $\mathbf{x}_S$. For laterally invariant media equation (1) can easily be solved via a scalar division in the wavenumber-frequency domain. For 3-D inhomogeneous media it can only be solved when the down going and up going fields $\hat{p}^+(\mathbf{x}, \mathbf{x}_S, \omega)$ and $\hat{p}^-(\mathbf{x}_A, \mathbf{x}_S, \omega)$ are available for a sufficient range of source positions $\mathbf{x}_S$. In matrix notation (Berkhout, 1982), equation (1) can be written as

$$\hat{\mathsf{P}}^- = \hat{\mathsf{R}}_0^+\hat{\mathsf{P}}^+. \tag{2}$$

For example, the columns of matrix $\hat{\mathsf{P}}^+$ contain $\hat{p}^+(\mathbf{x},\mathbf{x}_S,\omega)$ for fixed $\mathbf{x}_S$ and variable $\mathbf{x}$ at $\partial\mathbb{D}_1$, whereas the rows of this matrix contain $\hat{p}^+(\mathbf{x},\mathbf{x}_S,\omega)$ for fixed $\mathbf{x}$ and variable $\mathbf{x}_S$. Inversion of equation (2) involves matrix inversion, according to

$$\hat{\mathsf{R}}_0^+ = \hat{\mathsf{P}}^-(\hat{\mathsf{P}}^+)^{-1} \tag{3}$$

(Wapenaar and Verschuur, 1996). The matrix inversion in equation (3) can be stabilized by least-squares inversion, according to

$$\hat{\mathsf{R}}_0^+ = \hat{\mathsf{P}}^-(\hat{\mathsf{P}}^+)^\dagger[(\hat{\mathsf{P}}^+)(\hat{\mathsf{P}}^+)^\dagger + \epsilon^2\mathsf{I}]^{-1}, \tag{4}$$

where the superscript $\dagger$ denotes transposition and complex conjugation, I is the identity matrix and ϵ is a small constant. Berkhout and Verschuur (2003) used a similar inversion for transforming surface-related multiples into primaries. Equations (3) and (4) describe 3-D interferometry-by-deconvolution. Note that the medium need not be lossless. Moreover, this least-squares inversion compensates for irregularities in the source distribution and for variations in the source spectra. Schuster and Zhou (2006) derived an expression equivalent with equation (4) and called this least-squares redatuming. Wapenaar *et al.* (2008) generalize equations (1) – (4) for general vector fields. Van der Neut *et al.* (2008) discuss numerical examples.

We conclude this section by comparing 3-D interferometry-by-deconvolution with the virtual source method of Bakulin and Calvert (2006). To this end we ignore the inverse matrix in equation (4), according to $\hat{\mathsf{R}}_0^+ \approx \hat{\mathsf{P}}^-(\hat{\mathsf{P}}^+)^\dagger$. If we rewrite this equation again in integral form we obtain

$$\hat{R}_0^+(\mathbf{x}_A,\mathbf{x},\omega) \approx \int_{\partial\mathbb{D}_S} \hat{p}^-(\mathbf{x}_A,\mathbf{x}_S,\omega)\{\hat{p}^+(\mathbf{x},\mathbf{x}_S,\omega)\}^*\mathrm{d}^2\mathbf{x}_S, \tag{5}$$

where the superscript $*$ denotes complex conjugation and $\partial\mathbb{D}_S$ represents the depth level of the sources. Note that $\hat{R}_0^+$ is now proportional to the power spectrum $|\hat{s}(\omega)|^2$ of the sources at $\partial\mathbb{D}_S$. Equation (5) corresponds to the virtual source method of Bakulin and Calvert (2006).

Interferometry by multi-dimensional deconvolution of passive data

Since the underlying assumption of equation (1) is that the medium below $\partial\mathbb{D}_1$ is source-free, it is not a suited starting point for deriving interferometry for passive data with sources in the subsurface. In Wapenaar *et al.* (2004) we derived several relations between reflection and transmission responses. Consider equations (22) and (23) in that paper:

$$\hat{T}_0^+(\mathbf{x}'_A,\mathbf{x}_B,\omega) - \hat{T}^+(\mathbf{x}'_A,\mathbf{x}_B,\omega) = \int_{\partial\mathbb{D}_0} \hat{T}_0^+(\mathbf{x}'_A,\mathbf{x},\omega)\hat{R}^+(\mathbf{x},\mathbf{x}_B,\omega)\mathrm{d}^2\mathbf{x}, \tag{6}$$

$$\hat{T}_0^+(\mathbf{x}'_A,\mathbf{x}_B,\omega) - \hat{T}^+(\mathbf{x}'_A,\mathbf{x}_B,\omega) = \int_{\partial\mathbb{D}_0} \hat{T}^+(\mathbf{x}'_A,\mathbf{x},\omega)\hat{R}_0^+(\mathbf{x},\mathbf{x}_B,\omega)\mathrm{d}^2\mathbf{x}. \tag{7}$$

Here $\hat{T}_0^+(\mathbf{x}'_A,\mathbf{x}_B,\omega)$ is the transmission response of a source at $\mathbf{x}_B$ at the Earth's surface $\partial\mathbb{D}_0$, observed at $\mathbf{x}'_A$ in the subsurface. The subscript 0 denotes again that no free-surface multiples are included. $\hat{T}^+(\mathbf{x}'_A,\mathbf{x}_B,\omega)$ is the transmission response with free-surface multiples. Via source-receiver reciprocity these responses are identical with $\hat{T}_0^-(\mathbf{x}_B,\mathbf{x}'_A,\omega)$ and $\hat{T}^-(\mathbf{x}_B,\mathbf{x}'_A,\omega)$, respectively, each of which represents an up going response at the surface $\partial\mathbb{D}_0$ due to a source in the subsurface. $\hat{R}_0^+(\mathbf{x},\mathbf{x}_B,\omega)$ and $\hat{R}^+(\mathbf{x},\mathbf{x}_B,\omega)$ represent the reflection responses observed at the surface $\partial\mathbb{D}_0$, without and with free surface multiples, respectively. Given the transmission response $\hat{T}^-(\mathbf{x}_B,\mathbf{x}'_A,\omega)$, the aim is to find the reflection response. In matrix notation, equations (6) and (7) become

$$\hat{\mathsf{T}}_0^+ - \hat{\mathsf{T}}^+ = \hat{\mathsf{T}}_0^+\hat{\mathsf{R}}^+ \quad \Rightarrow \quad \hat{\mathsf{R}}^+ = \mathsf{I} - \{\hat{\mathsf{T}}_0^+\}^{-1}\hat{\mathsf{T}}^+, \tag{8}$$

$$\hat{\mathsf{T}}_0^+ - \hat{\mathsf{T}}^+ = \hat{\mathsf{T}}^+\hat{\mathsf{R}}_0^+ \quad \Rightarrow \quad \hat{\mathsf{R}}_0^+ = \{\hat{\mathsf{T}}^+\}^{-1}\hat{\mathsf{T}}_0^+ - \mathsf{I}, \tag{9}$$

with $\hat{\mathsf{T}}_0^+ = \{\hat{\mathsf{T}}_0^-\}^t$ and $\hat{\mathsf{T}}^+ = \{\hat{\mathsf{T}}^-\}^t$, where superscript t denotes matrix transposition. Given $\hat{\mathsf{T}}^-$, the aim is to find $\hat{\mathsf{R}}^+$ or $\hat{\mathsf{R}}_0^+$. Note that we have two equations with three unknowns ($\hat{\mathsf{R}}^+$, $\hat{\mathsf{R}}_0^+$ and $\hat{\mathsf{T}}_0^+$). A third equation is Berkhout's relation between the reflection responses with and without free surface multiples, i.e.,

$$\hat{\mathsf{R}}_0^+ - \hat{\mathsf{R}}^+ = \hat{\mathsf{R}}_0^+\hat{\mathsf{R}}^+ \quad \Rightarrow \quad \hat{\mathsf{R}}_0^+ = \hat{\mathsf{R}}^+(\mathsf{I} - \hat{\mathsf{R}}^+)^{-1}. \tag{10}$$

Unfortunately equations (8) – (10) are not independent, which is most easily seen by substituting the right-hand side versions of equations (8) and (9) into the left-hand side version of equation (10). Hence, the system of equations is underdetermined, so we have to make an additional assumption if we want to resolve the reflection response from $\hat{\mathsf{T}}^-$. In the following we assume that the subsurface consists of an inhomogeneous target below a relatively weak inhomogeneous overburden. The buried sources are located below the target. For this situation the transmission response without surface-related multiples, i.e., $\hat{\mathsf{T}}_0^-$, can be estimated from the transmission response with surface-related multiples, $\hat{\mathsf{T}}^-$, by applying a time window in the time domain. Now, given $\hat{\mathsf{T}}_0^- = \{\hat{\mathsf{T}}_0^+\}^t$ and $\hat{\mathsf{T}}^- = \{\hat{\mathsf{T}}^+\}^t$, the reflection response $\hat{\mathsf{R}}^+$ or $\hat{\mathsf{R}}_0^+$ can be resolved from equation (8) or (9), respectively. Since the inverse of $\hat{\mathsf{T}}^+$ is more difficult to determine than that of $\hat{\mathsf{T}}_0^+$, we propose to resolve $\hat{\mathsf{R}}^+$ from equation (8). Using $\hat{\mathsf{R}}^+ = \{\hat{\mathsf{R}}^+\}^t$ and the symmetries of the transmission matrices, we rewrite equation (8) as $\hat{\mathsf{R}}^+ = \mathsf{I} - \{\hat{\mathsf{T}}^-\hat{\mathsf{S}}\}\{\hat{\mathsf{T}}_0^-\hat{\mathsf{S}}\}^{-1}$, where $\hat{\mathsf{S}}$ is a diagonal matrix containing the spectra of the sources below the target zone. The stabilized version reads

$$\hat{\mathsf{R}}^+ = \mathsf{I} - \hat{\mathsf{P}}^-(\hat{\mathsf{P}}_0^-)^\dagger[(\hat{\mathsf{P}}_0^-)(\hat{\mathsf{P}}_0^-)^\dagger + \epsilon^2\mathsf{I}]^{-1}, \tag{11}$$

where $\hat{\mathsf{P}}^- = \hat{\mathsf{T}}^-\hat{\mathsf{S}}$ and $\hat{\mathsf{P}}_0^- = \hat{\mathsf{T}}_0^-\hat{\mathsf{S}}$ represent the observed up going transmission responses at the surface, with and without surface-related multiples. The process described by equation (11) can be followed by surface-related multiple elimination (equation 10) to obtain $\hat{\mathsf{R}}_0^+$. The generalization for vector fields is similar to that described by Wapenaar *et al.* (2008) for controlled source data. Although equation (11) is primarily meant for multi-dimensional deconvolution, we illustrate it for a 1-D medium. The medium consists of a homogeneous overburden with a propagation velocity of 2000 m/s, and a horizontally layered target between $z = 1600$ m and $z = 2300$ m. Figure 2a shows the transmission response observed at the free surface of a source at $z_S = 2400$ m. Figure 2b is a time-windowed version, without the surface-related multiples (this separation is possible due to the homogeneous overburden). Applying the 1-D version of equation (11) gives the reflection response at the free surface, including the surface-related multiples, see Figure 2c. Figure 2d shows the difference with the directly modelled reflection response.

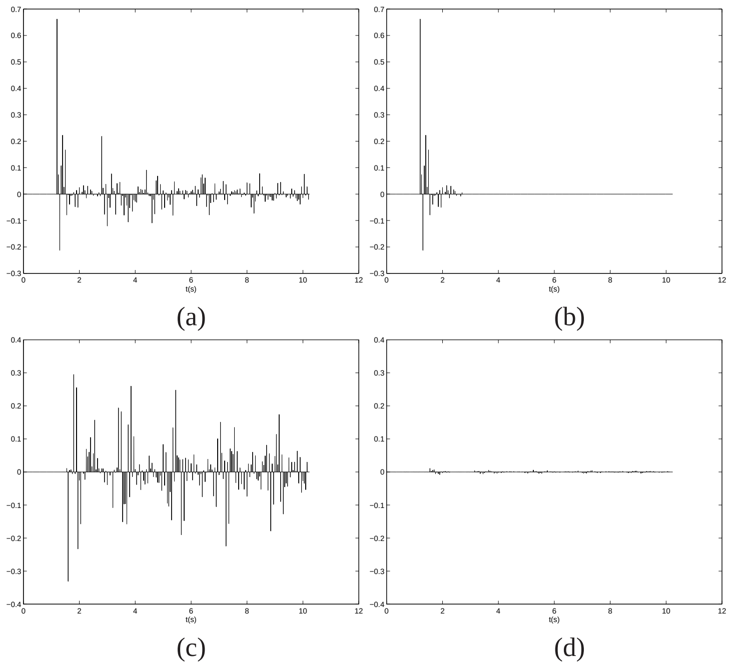

Figure 2: 1-D example of interferometry by deconvolution of passive data (see text for details).

We conclude this section by comparing equation (11) with interferometry by cross-correlation of passive data. To this end we ignore the inverse matrix on the right-hand side of equation (11) and we correct the remaining part for the source spectra, hence $\hat{\mathsf{R}}^+ \approx \mathsf{I} - \hat{\mathsf{T}}^-(\hat{\mathsf{T}}_0^-)^\dagger$. If we rewrite this equation again in integral form we obtain

$$\hat{R}^+(\mathbf{x}_B, \mathbf{x}_A, \omega) \approx \delta(\mathbf{x}_B - \mathbf{x}_A) - \int_{\partial\mathbb{D}_m} \hat{T}^-(\mathbf{x}_B, \mathbf{x}_S, \omega)\{\hat{T}_0^-(\mathbf{x}_A, \mathbf{x}_S, \omega)\}^* \mathrm{d}^2\mathbf{x}_S. \tag{12}$$

Compare this with our expression for interferometry by cross-correlation (Wapenaar *et al.* , 2004),

$$2\Re\big[\hat{R}^+(\mathbf{x}_B, \mathbf{x}_A, \omega)\big] = \delta(\mathbf{x}_B - \mathbf{x}_A) - \int_{\partial\mathbb{D}_m} \hat{T}^-(\mathbf{x}_B, \mathbf{x}_S, \omega)\{\hat{T}^-(\mathbf{x}_A, \mathbf{x}_S, \omega)\}^* \mathrm{d}^2\mathbf{x}_S. \tag{13}$$

Although there is resemblance, there are some important differences. Apart from the difference in the left-hand sides ($\Re$ in equation (13) denotes the real part), in the right-hand sides we have $\hat{T}_0^-(\mathbf{x}_A, \mathbf{x}_S, \omega)$ in equation (12) versus $\hat{T}^-(\mathbf{x}_A, \mathbf{x}_S, \omega)$ in equation (13). Hence, interferometry by cross-correlation of passive data (equation 13) is *not* a special case of interferometry by multi-dimensional deconvolution (equation 11). Both approaches have their own merits and drawbacks. Interferometry by cross-correlation (equation 13) has the advantage that it is applied to the full transmission responses (no need for time-windowing), and that all multiple reflections (internal and surface-related) are properly included in the reconstructed reflection response, no matter the complexity of the overburden. Moreover, for the situation of uncorrelated noise sources, the integral collapses to a single cross-correlation of observations at $\mathbf{x}_A$ and $\mathbf{x}_B$, according to

$$2\Re\big[\hat{R}^+(\mathbf{x}_B, \mathbf{x}_A, \omega)\big] = \delta(\mathbf{x}_B - \mathbf{x}_A) - \hat{T}^-_{\rm obs}(\mathbf{x}_B, \omega)\{\hat{T}^-_{\rm obs}(\mathbf{x}_A, \omega)\}^*. \quad (14)$$

A drawback is that the medium is assumed to be lossless and that the transmission responses in equation (13) are assumed to be regularly sampled impulse responses, although this assumption can be relaxed. Draganov *et al.* (2007) obtained good results by applying equation (14) to real data. The main advantage of equation (11) is that it accounts more accurately for irregular sampling and variations in the source spectrum. Moreover, equation (11) is valid for dissipative media. A drawback is that it requires the transmission response without surface-related multiples, which should be estimated in one way or another from the full transmission response. Another drawback is the more complex processing (large matrix inversion) and the fact that it can not be applied in the situation of simultaneously acting uncorrelated noise sources (the sources should be transients and their responses should be measured sequentially in time, similar as in equation (13), but opposed to equation (14)).

Conclusions

We discussed seismic interferometry by multi-dimensional deconvolution for controlled-source data as well as for passive data and compared both approaches with correlation-based interferometric methods. For the controlled-source situation the virtual source method was derived as an approximation of the deconvolution method. For the passive data situation it was shown that the deconvolution method and the correlation method are essentially different, each of them having their own merits and drawbacks.

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