Introduction

The exact spatial extent of potential reservoirs is a key descriptor, but only indirectly present in the reflected wavefields which we are able to measure at the surface. Therefore the analysis of seismic data is always an inversion problem. This holds especially for very thin reservoirs, whose thickness remains entirely at the sub-wavelength scale. In a standard seismic experiment a source wavelet with a central frequency in the range of 30 to 50 Hz is used. In this case the vertical resolution is in the order of tens of meters, which means that layers with smaller thicknesses are not easily recognizable on a seismic image as a result of wavelet interference. Ideally one would like to go to the sub-seismic resolution, i.e., to meter scale or even sub-meter scale.

Method

In this research we use a Bayesian based inversion approach, which employs a nonlinear least-squares estimation for maximizes the a posteriori probability. As optimization methodology the quasi-Newton method with the Broyden-Fletcher-Goldafarb-Shanno (BFGS) update formula for the Hessian matrix is used. A simple but very useful configuration to demonstrate the sub-seismic resolution ability for thin-bed layers is one where a homogeneous wedge of sandstone is pinched out by surrounding homogeneous shale and limestone layers. Based on such a simple three-layer model, the resolution limits and optimization technique can be easily and successfully tested. The problem is a one-dimensional seismic inverse problem and concerns the estimation of the parameters.

1.1 Simulated field data

The (simulated) geological "field model" consists of three homogeneous layers (fig.1). We assume the presence of a nearby well. The main idea is to start at the well and use the well data (in addition to the seismic data) and then move step-by-step away from the well, in the direction of decreasing thin–bed layer thickness. The rock types and properties for the layers are taken from the well. They are typical for a reservoir. Their acoustic impedance contrasts are large enough to cause sufficient reflections (Table 1). The thickness of the pinched-out sandstone layer is taken smaller than the wavelength used in the seismic experiment. It has been varied from 1/5th of the wavelength (typical seismic resolution) to 1/100th of the wavelength during the experiment.

In order to simulate the "seismic field data" more realistically, some disturbance layers were added. These layers are situated within a wavelength distance from the target sandstone layer, have small acoustic impedance contrasts with the shale and limestone layers, are quite thin in comparison with the pinched-out sandstone layer and the interfaces between these layers have negative reflection coefficients (Fig.1). These reflectors complicate the inversion since they are not visible on the seismic image due interference. For the sake of clarity the parameters were chosen to be close to those of the surrounding rocks. Layer 2 (disturbance) and layer 4 (disturbance) were added between the shale respectively the limestone and the pinched-out sandstone layer. The thicknesses of the disturbing layers were uniformly distributed over the interval ranging from 0m to 1m along the profile.

Table1: Properties of the (simulated) field data

Rock type	P-velocity, *m/s*	Density, kg/m^3	Thickness, *m*
Layer 1. Shale	2900	2400	100
Layer 2 (disturbance). "Shale"	2700	2400	Uniformly distributed over the interval [0,1]
Layer 3. Sandstone (20% porosity, water saturated)	3850	2320	varying
Layer 4 (disturbance). "limestone"	5300	2540	Uniformly distributed over the interval [0,1]
Layer 5. Limestone (10% porosity ,water saturated)	5200	2540	Half-space

The forward model used consists of a 1D convolution method with primaries only. The source wavelet is convolved with the normal-incidence reflectivity time-series to simulate a 1D seismic trace. The convolution is carried out in the frequency domain as a multiplication. Parameters of acquisition: time sampling step dt =1ms, number of samples is 1000.

The Ricker wavelet is a zero-phase wavelet, and corresponds to the second derivative of a Gaussian function. In our experiment the central frequency is 30Hz (Fig.2). In a real seismic experiment the wavelet is generally not known precisely, which complicates the inversion process. Either the amplitude spectrum or the phase spectrum (or both) can be disturbed. In order to simulate such an affect, we introduced phase distortion. The Ricker wavelet s(t) may in the frequency domain be represented as: $S(f)=|S(f)|\cdot\exp(j\cdot(\gamma(f)+\theta(f)))$, $|S(f)|$ and $\gamma(f)+\theta(f)$ are the amplitude and the phase spectrum of $s(t)$ respectively. In our experiment we used $\theta(f)=a\bullet sign(f)$, which is a phase distortion with a expressing the amount of phase distortion. We have applied phase-shifts in a range from 0 to 2π. Testing as well as analysis of the seismic practice showed that a phase-shift with $a=\pi/6$ is quite adequate representing the real seismic situation (Fig.3 (left)).

Again, to be more realistic, noise has been added to the "seismic field data". The noise is uncorrelated and the amplitude is Gaussian distributed with zero mean. Tests for different noise levels were done and the one with the most realistic signal-to-noise ratio was chosen. For the given reflection coefficient, a noise standard deviation of 0.005 was selected.

1.2 The forward model for the "synthetic seismic"

Since the main idea of this paper is to go to sub-seismic resolution and estimate the thin-bed sandstone layer properties, the seven parameters to be estimated were: (1) the thickness, (2) the P-velocity, and (3) the density of the thin bed sandstone layer, (4) the thickness of the top shale layer, (5) the phase shift *a* in a Ricker wavelet and (6 and 7) the thicknesses of the additional reflector-disturbances (layers 2 and 4). This model quite well reflects the situation when the task is to extrapolate thin-bed layer properties starting from a well.

For the synthetic data the wavelet was chosen to be the same as for "seismic field" data, with a central frequency of 30 Hz. The only difference is that this wavelet is initially set to zero-phase (Fig.2). This simulates the real situation when no knowledge of the phase-shift in the seismic data is available. Of course, no noise added to the synthetic data.

1.3 Construction of the a priori

The information about the parameters that is available independently from the data can be used as a priori information. A priori knowledge about parameters often consists of an idea about values (mean) and the uncertainties (standard deviation) in these values. An often used probability density function to describe this type of information is the Gaussian distribution.
The distribution of the prior should include the true value of the parameter. This distribution should not be too wide because then it would have no effect on the minimization. On the other hand it should not be too narrow and exclude the *real* value, because in this case it would direct the solution to the wrong point. For the a priori values corresponding to the layer thicknesses we made the standard deviation dependent on the wavelength, instead of on the layer thickness itself. An exception has been made for the additional reflector-disturbances. The standard deviation of the velocity's and density's priors were set to be a percentage of the true values (see Figure 1 for the designation of the parameters): $\sigma(H_1)=0.1(V_1/fc)=9.6m$; $\sigma(H_2)=0.1m$; $\sigma(H_3)=0.05(V_3/fc)=6.4m$; $\sigma(H_4)=0.1m$; $\sigma(V_3)=0.05(V_3)=192.5m/s$; $\sigma(\rho_3)=0.01(\rho_3)=24kg/m^3$; $\sigma(phase)=\pi/8$, where f_c is the central frequency of the Ricker wavelet.

A priori information makes the solution stronger only when it is close to the real value, otherwise it just shifts the solution to the wrong direction. Therefore, the means of priors should be close to the true parameters values. For the sake of clarity, we set the estimated parameter means α times the standard deviation away from the true value:

$H_j^i = H_j +/- \alpha\sigma(H_j)$; $V_3^i = V_3 +/- \alpha\sigma(V_3)$; $\rho_3^i = \rho_3 +/- \alpha\sigma(\rho_3)$; $phase^i = 0$, where the superscript i denotes the priors, the index j=1-4 denotes the layer and α is a number that is changed, depending on the test.

It is obvious that having a priori information as close as possible to the true parameters, with small standard deviation, would help a lot to find a global minimum that corresponds as close as possible to the true solution. We used the output of the minimization procedure to update the a priori information for the next iteration. We started the inversion process with a trace close to the well where all the parameter values are known. Then we moved, step-by-step (trace-by-trace), towards the pinched-out point. At the starting point, the exact values of the parameters were used as the means of the priors (known from the well). At all successive steps, the estimated values of the parameters obtained in the previous step were used as the means of the priors. The standard deviations of the distributions for all priors remained fixed throughout the complete analysis. Finally, the lateral position of pinched-out point has been found with an uncertainty of 0.8m.

1.4 Test and results

The objective of this research is to obtain sub-seismic resolution and to estimate the parameters of a thin-bed sandstone layer. Therefore the main parameter that was varied in the different tests is the thickness of the thin-bed sandstone layer. The source wavelet with central frequency of 30 Hz normally used in a seismic experiment implies that the vertical resolution is around 20 m, which means that layers with thickness less than 20 m would not be clearly visible in the seismic data due to the interference. The thickness of the pinched-out sandstone layer has been varied from 1/5th of the wavelength to 1/100 of the wavelength during the experiments. For the above mentioned set of the parameters and considerations the results of these experiments are in Table 2 (relative errors have been averaged for the 10 iterations).

Table 2: The relative errors (over 10 iteration) of the estimated parameters for thin beds with a thickness of 1 and 2 meters. Note that all errors are in percentage except the wavelet's phase.

H_3	H_1 (%)	H_2 (%)	H_3 (%)	H_4 (%)	V_3 (%)	ρ_3 (%)	phase *a*
2m	<1	5	15	8	4	<1	1
1m	<1	15	18	9	5	<1	3

Conclusions

The presented thin-bed seismic inversion method, which makes use a priori information obtained from a well, shows encouraging results for very thin simulated reservoirs. The resolution of the thin-bed inversion is far superior to the resolution of the input seismic data, which permits characterization of thin reservoirs. Incorporating a source wavelet phase disturbance in the inversion process makes this method applicable to real seismic data where the wavelet is never precisely known. By adding geological/structural noise to the model with quite low signal-to-noise ratio, the model became more realistic. Updating the priors with the results from the previous iteration yields better convergence and more accurate parameter estimation.

Acknowledgements

The authors would like to acknowledge Delft Research Center "Earth" and Norsk Hydro for sponsoring this research.

References

Aki, K., and P.G. Richards, 1980 Quantative seismology: Theory and methods, W.H.Freeman and Co

Duijndam, A.J.W., 1988a, Bayesian estimation in seismic inversion, Part 1: Principles, Geoph. Prosp., 36, 878-898

Duijndam, A.J.W., 1988b, Bayesian estimation in seismic inversion, Part 2: Uncertainty analysis, Geoph. Prosp., 36, 899-918

Gunning, J and Glinsky, 2004, Delivery: an open-source model-based Bayesian seismic inversion program, Computers and Geosciences, 30, 619-636
Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, 49, 1259-1266
Tarantola, A., 1984, Linearized inversion of seismic reflection data, Geophysical prospecting, 32, 998-1015
Tarantola, A., 2005, Inverse problem theory and methods for model parameter estimation: SIAM
Shanon, C.E., 1948, A mathematical theory of communication, Bell System Technical Journal, 27, 379-423 and 623-656

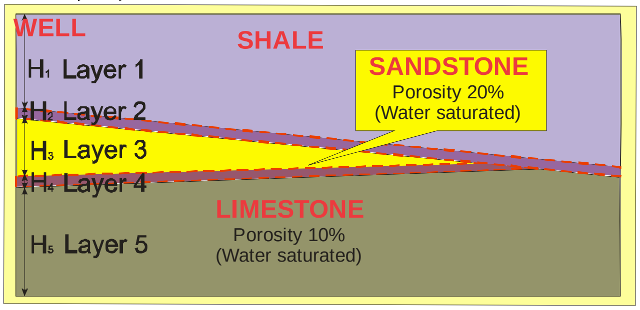

Fig.1 Geological model with geological/structural noise added (red dashed layers)

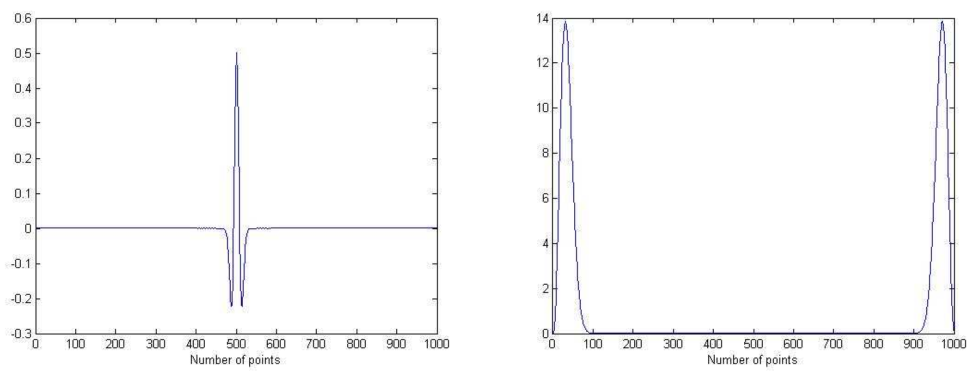

Fig.2 (Left) Ricker wavelet (central frequency 30Hz), (right) Spectrum of Ricker wavelet

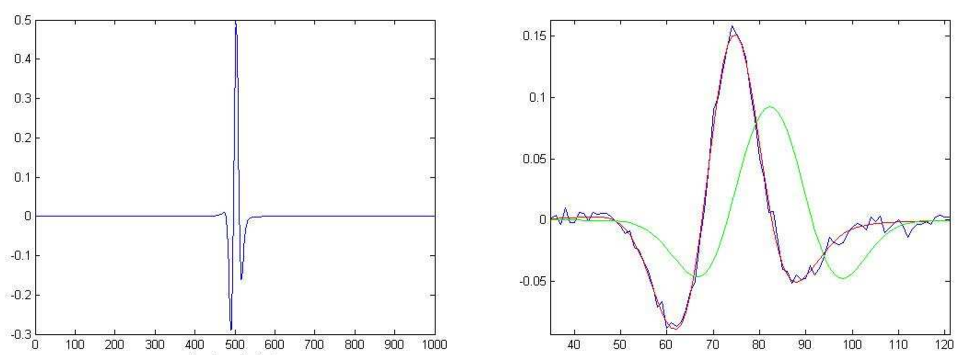

Fig.3 (left) Ricker wavelet with a phase shift, a= π /6, (right) blue: "seismic field data" (noisy, with phase shift a= π /6); green: initial guess of the synthetic subsurface model response (zero phase Ricker wavelet) based on the means of the priors; red: final response of the model after convergence. The thickness of the pinched out sandstone layer H_3 is 1m.