Z-99 DIRECT MIGRATION OF PASSIVE SEISMIC DATA

BRAD ARTMAN[1], DEYAN DRAGANOV[2], KEES WAPENAAR[2] and BIONDO BIONDI[1]
[1]*Geophysics, Stanford Exploration Project, Stanford, CA 94305-2215, USA*
[2]*Delft University of Technology, Department of Applied Earth Sciences, Mijnbouwstraat 120, 2628 RX Delft, The Netherlands*

Introduction

General relations between the reflection and the transmission responses of a 3-D inhomogeneous medium [4] have been developed to forward model the transmission coda, suppress multiples, provide the basis of seismic interferometry, and forward model the reflection response from the transmission response. The final relation, exploited for acoustic daylight imaging, shows that by cross-correlating traces of the transmission response of a medium one can synthesize the reflection response collected in a conventional active source experiment. Having generated shot-gathers in this manner, they can be processed with conventional techniques to enhance signal, remove artifacts, or create a migrated image. Here, we present the theory of direct migration of the measured transmission response wave fields without the need to first generate the reflection response wave fields by cross-correlation. Removing this step allows for significant time savings when dealing with these inherently large data sets and produces an equivalent image.

Passive imaging

The relation between the passively collected transmission response of a medium, T^-, and its equivalent reflection response, R^+ (where superscripts indicate one-way direction), [4] is

$$R^+\left(\boldsymbol{x}_A,\boldsymbol{x}_B,\omega\right)+\left\{R^+\left(\boldsymbol{x}_A,\boldsymbol{x}_B,\omega\right)\right\}^*=\delta\left(\boldsymbol{x}_{H,A}-\boldsymbol{x}_{H,B}\right)-\int_{\partial\mathcal{D}_m}T^-\left(\boldsymbol{x}_A,\boldsymbol{x},\omega\right)\left\{T^-\left(\boldsymbol{x}_B,\boldsymbol{x},\omega\right)\right\}^*d\boldsymbol{x}. \tag{1}$$

Here an arbitrary, lossless (so as to neglect the dynamics of attenuation) earth volume is bounded by the free surface $\partial\mathcal{D}_0$ and a depth level $\partial\mathcal{D}_m$, below which homogeneous half space is assumed. All wave fields include all surface and internal multiples and are functions of source and receiver location, $\boldsymbol{x}$, and frequency, ω. A and B are arbitrary points at the surface, and subscript H indicates only the two horizontal coordinates of the spatial vector. The asterisk denotes complex conjugation.

If the noise sources are discretely distributed in the subsurface and are white and uncorrelated, the surface integral on the RHS of equation (1) can be removed via an observational redefinition of the transmission wave fields to produce

$$R^+\left(\boldsymbol{x}_A,\boldsymbol{x}_B,\omega\right)+\left\{R^+\left(\boldsymbol{x}_A,\boldsymbol{x}_B,\omega\right)\right\}^*=\delta\left(\boldsymbol{x}_{H,A}-\boldsymbol{x}_{H,B}\right)-T^-_{obs}\left(\boldsymbol{x}_A,\omega\right)\left\{T^-_{obs}\left(\boldsymbol{x}_B,\omega\right)\right\}^*\ , \tag{2}$$

where

$$T^-_{obs}\left(\boldsymbol{x}_A,\omega\right)=\sum_i T\left(\boldsymbol{x}_A,\boldsymbol{x}_i,\omega\right)N_i\left(\omega\right) \tag{3}$$

$$T^-_{obs}\left(\boldsymbol{x}_B,\omega\right)=\sum_j T\left(\boldsymbol{x}_B,\boldsymbol{x}_j,\omega\right)N_j\left(\omega\right) \tag{4}$$

are the transmission responses from all possible noise sources, $N_i\left(\omega\right)$, present in the subsurface. After muting the acausal part on the LHS of equation (2), we are left with the reflection response of the earth as it would have been measured at the surface during a conventional reflection survey. Significant energy in the correlations arises at zero-lag (the delta function in equation (2)) and when an energy packet reflected at the free-surface is recorded again after a two-way trip to a subsurface reflector. Thus, the correlation removes the unknown time offset when energy first arrives at the array and removes complicated wavelet characteristics while constructing familiar reflection and diffraction hyperbolas. This processed data, in the form of shot-gathers, can then be manipulated with standard processing techniques to remove multiples, enhance signal-to-noise ratio and image the subsurface.

Direct migration

Although in a passive seismic experiment, only the transmission response at the surface is known, we saw in relation (2) that we can calculate the reflection response at the surface by cross-correlating the transmission response. Similarly, the total reflection wave field in the conventional reflection experiment is the cross-correlation of the source, D^+, and receiver, U^-, wave fields which can be redatumed with a wave-field extrapolation kernel:

$$\tilde{R}^+_{j+1} = \tilde{R}^+_j e^{+i(k^s_z+k^r_z)\delta z} = \tilde{U}^-_j e^{+ik^r_z\delta z}\left\{\tilde{D}^+_j e^{-ik^s_z\delta z}\right\}^* \ , \quad (5)$$

where the tilde represents transformation to the wavenumber-frequency domain. The commutability of extrapolation and cross-correlation steps, that gives rise to the equivalence of shot-profile and source-receiver migration [1], can also be applied to the migration of passive data. Substituting the transmission wave field for source and receiver, we can use a shot-profile migration scheme to propagate the transmission wave field causally and acausally through all model depths, j, with a one-way propagator

$$\begin{array}{ccccccc} \tilde{U}^-_j & \tilde{D}^+_j & \longrightarrow & \tilde{R}^+_j & \longleftarrow & \tilde{T}^+_j & \tilde{T}^+_j \\ \downarrow & \downarrow & & \downarrow & & \downarrow & \downarrow \\ \tilde{U}^-_{j+1} & \tilde{D}^+_{j+1} & \longrightarrow & \tilde{R}^+_{j+1} & \longleftarrow & \tilde{T}^+_{j+1} & \tilde{T}^+_{j+1} \end{array} \ ,$$

where the vertical arrows represent propagation steps, and the horizontal arrows indicate cross-correlation. Within this scheme, the difference between redatuming and migration is simply the extraction of the zero-lag coefficient of the correlation [2] in the evaluation of the imaging condition after propagation.

For rigorous proof, following [3], we can write the redatuming relationship between the reflection response collected at the surface, $R^+(\boldsymbol{x}_A, \boldsymbol{x}_B, \omega)$, and the wave field that would have been collected at some deeper level, $R^+(\boldsymbol{\xi}_A, \boldsymbol{\xi}_B, \omega)$, as

$$R^+(\boldsymbol{\xi}_A, \boldsymbol{\xi}_B, \omega) = \int_{\partial\mathcal{D}_0}\int_{\partial\mathcal{D}_0} \left\{W^+(\boldsymbol{\xi}_A, \boldsymbol{x}_A, \omega)\right\}^* R^+(\boldsymbol{x}_A, \boldsymbol{x}_B, \omega)\left\{W^-(\boldsymbol{x}_B, \boldsymbol{\xi}_B, \omega)\right\}^* d\boldsymbol{x}_A d\boldsymbol{x}_B \ , \quad (6)$$

where $W^+(\boldsymbol{\xi}_A, \boldsymbol{x}_A, \omega)$ is a forward-extrapolation operator from the surface to a subsurface level; $W^-(\boldsymbol{x}_B, \boldsymbol{\xi}_B, \omega)$ is a forward-extrapolation operator from the same subsurface level to the surface. The cascade of the complex-conjugate of these operators (which represent the matched inverse-extrapolation operators) propagate the surface reflection response to a deeper level.

From equation (2) we see that in the case of white and uncorrelated noise sources in the subsurface we can approximate the reflection response with

$$R^+(\boldsymbol{x}_A, \boldsymbol{x}_B, \omega) \approx -T^-_{obs}(\boldsymbol{x}_A, \omega)\left\{T^-_{obs}(\boldsymbol{x}_B, \omega)\right\}^* \quad (7)$$

by omitting the acausal part of the reflection response and the delta function. Combining equations (6) and (7), we can write

$$R^+(\boldsymbol{\xi}_A, \boldsymbol{\xi}_B, \omega) \approx -\int_{\partial\mathcal{D}_0}\int_{\partial\mathcal{D}_0} \left\{W^+(\boldsymbol{\xi}_A, \boldsymbol{x}_A, \omega)\right\}^* T^-_{obs}(\boldsymbol{x}_A, \omega)\left\{T^-_{obs}(\boldsymbol{x}_B, \omega)\right\}^* \left\{W^-(\boldsymbol{x}_B, \boldsymbol{\xi}_B, \omega)\right\}^* d\boldsymbol{x}_A d\boldsymbol{x}_B \ . \quad (8)$$

Further, we can split the double integral into separate integrals over the surface points $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$, and incorporate the reflection coefficient of the free surface, $r = -1$, to find

$$R^{+}(\boldsymbol{\xi}_A, \boldsymbol{\xi}_B, \omega) \approx \int_{\partial\mathcal{D}_0} \left\{W^{+}(\boldsymbol{\xi}_A, \boldsymbol{x}_A, \omega)\right\}^{*} T^{-}_{obs}(\boldsymbol{x}_A, \omega)\, d\boldsymbol{x}_A \left\{\int_{\partial\mathcal{D}_0} W^{+}(\boldsymbol{\xi}_B, \boldsymbol{x}_B, \omega)\, r T^{-}_{obs}(\boldsymbol{x}_B, \omega)\, d\boldsymbol{x}_B\right\}^{*} . \quad (9)$$

Relation (9) shows that by inverse-extrapolating the transmission response at all $\boldsymbol{x}_A$ at the surface to a deeper subsurface level, and forward-extrapolating the transmission response at all $\boldsymbol{x}_B$ to the same subsurface level, followed by cross-correlating the resultant wave fields, we can reconstruct the reflection response at that level. This process is migration without evaluating the imaging condition, or the production of datumed reflection data. In the light of shot-profile migration, the first integral can be seen as the downward-continued receiver one-way wave field and the second as the upward-continued source one-way wave field. The delta function that we omitted in our analysis shows itself at depth 0 in the migrated image (see lower right picture on figure 1) and the omitted acausal part of the reflection response has contributions in the acausal part of the migrated picture, which is non-physical and we do not image it.

Synthetic experiment

A synthetic passive data set was generated with a finite difference code over the earth model shown in the lower central part of figure 1. Images were generated with a split-step Fourier shot-profile migration scheme following two migration flows. During the first migration flow, the raw transmission noise data (left part of figure 1) was directly migrated. During the second flow, the transmission data was first cross-correlated to produce simulated reflection shot-gathers (upper central part of figure 1) and then migrated. Identical images resulted from both procedures (lower right part of figure 1).

Correlation of each trace with every other trace produces N shot-gathers each with N traces from a survey with N receivers. If correlations are performed in the frequency domain, N^2 traces must be inverse Fourier transformed after multiplication to produce the simulated reflection shot-gathers. This requirement is especially onerous in the case of passive seismic data, where the record length is at least minutes long, in contrast to a few seconds in the conventional experiment. Then, another Fourier transform, though of traces only seconds long, must be performed to make a $f-k$ based migration. Finally, the increased I/O associated with migrating N shots in a shot-profile migration are substantial compared with the single shot, even with many more frequencies, for direct migration utilizing an algorithm parallelized over frequency.

Conclusions

Cast in terms of the WRW model of seismic redatuming, we have shown the theoretical justification for direct migration of passive seismic data to image the subsurface. Propagating the transmission wave field forward and backward in time with the wave equation followed by correlation produces the equivalent of reflection seismic data redatumed to some subsurface depth level. In this case, the wave equation is propagating all complicated aspects of the energy captured in the data without the need for the surface correlation. Evaluation of the last step of the imaging condition - the extraction of the zero-lag of the correlation, completes the migration procedure to provide the subsurface image. This result is identical to first creating simulated reflection shot-gathers from the transmission data and migrating them as data from a conventional reflection experiment. This method saves substantial computing costs compared to first creating the shot-gathers.

Acknowledgments

This research is being performed as part of research projects financed by the Stanford Exploration Project SEP, the Dutch Science Foundation STW, the Netherlands Research Centre for Integrated Solid Earth sciences ISES.

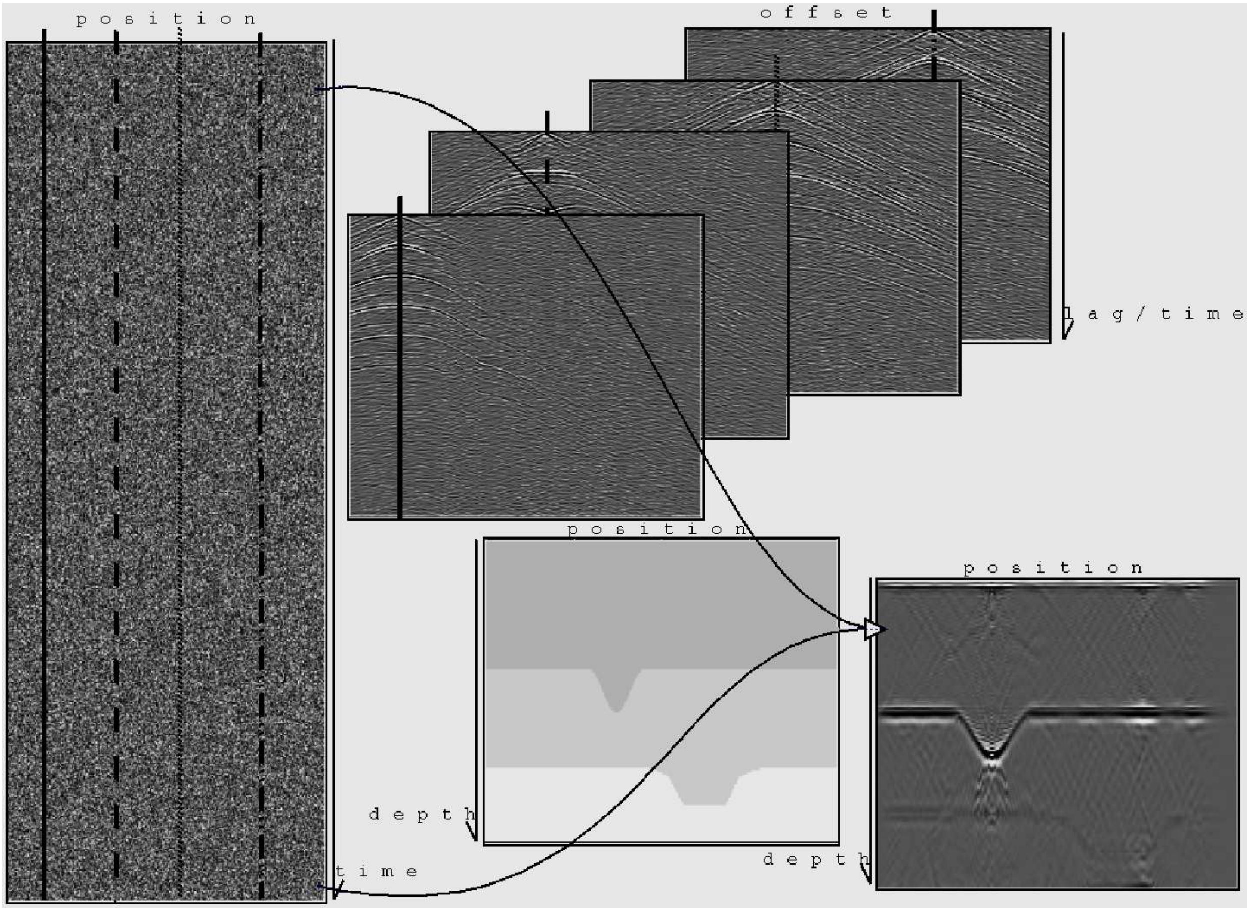

Figure 1: *Raw passive seismic data can be imaged with two processing flows. Cross-correlation of each trace with every other builds shot-gathers, which can then be migrated using a velocity model. Lines on the raw data correspond to the traces used for correlation to produce the four representative simulated shot-gathers (acausal part muted). Alternatively, the raw transmission wave field can be used as up- and down-going wave field in a shot-profile migration scheme to produce identical image.*

References

[1] B. Biondi. Equivalence of source-receiver migration and shot-profile migration. *Geophysics*, 68:1340–1347, 2003.

[2] J. F. Claerbout. Toward a unified theory of reflector mapping. *Geophysics*, 36:467–481, 1971.

[3] C. P. A. Wapenaar and A. J. Berkhout. Elastic wavefield extrapolation. *Elsevier*, 2, 1989.

[4] Kees Wapenaar, Jan Thorbecke and Deyan Draganov. Relations between reflection and transmission responses of 3-D inhomogeneous media. *Geophys. J. Intern.*, in print.