0-000 **SYNTHESIS OF THE REFLECTION RESPONSE FROM THE TRANSMISSION RESPONSE IN THE PRESENCE OF WHITE NOISE SOURCES**

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## Introduction

One of the applications of the general relations between the reflection and the transmission response of a medium is in acoustic daylight imaging. Claerbout [1] derived the relation for a horizontally layered medium. In [2] a relation was derived between the reflection and transmission response for a 3-D inhomogeneous medium in the presence of sources continuously spaced in the subsurface. Here, we present the way to synthesize the reflection response from the transmission response in the presence of white noise sources randomly spaced in the subsurface and show some numerical modeling results.

## Relation between Transmission and Reflection Response

Let us have a lossless, source free 3-D inhomogeneous domain $\mathcal{D}$ (see figure 1 (a) and (b)), embedded between surfaces $\partial\mathcal{D}_0$ and $\partial\mathcal{D}_m$. Just above $\partial\mathcal{D}_0$ we have a free surface and below $\partial\mathcal{D}_m$ the half space is homogeneous. For this configuration, the reflection response can be calculated from the transmission response in the time domain using the relation [2]

$$R\left(\boldsymbol{x}_A, \boldsymbol{x}_B, t\right) + R\left(\boldsymbol{x}_B, \boldsymbol{x}_A, -t\right) = \delta\left(\boldsymbol{x}_{H,B} - \boldsymbol{x}_{H,A}\right)\delta\left(t\right) - \int_{\partial\mathcal{D}_m} T\left(\boldsymbol{x}_A, \boldsymbol{x}, -t\right) * T\left(\boldsymbol{x}_B, \boldsymbol{x}, t\right) d\boldsymbol{x}, \tag{1}$$

where $R\left(\boldsymbol{x}_B, \boldsymbol{x}_A, t\right)$ denotes the reflection response including all free-surface and internal multiples of the domain $\mathcal{D}$ in the presence of a source at $\boldsymbol{x}_A$ and a receiver at $\boldsymbol{x}_B$ (figure 1 (a)); $T\left(\boldsymbol{x}_A, \boldsymbol{x}, t\right)$ denotes the transmission response including all free-surface and internal multiples of the domain $\mathcal{D}$ in the presence of a source at $\boldsymbol{x}$ and a receiver at $\boldsymbol{x}_A$ (1 (b)); $*$ symbolizes convolution; $\boldsymbol{x}_{H,A}$ symbolizes the horizontal coordinates $x_1$ and $x_2$ of point A. The points with position vector $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$ are situated just above the surface $\partial\mathcal{D}_0$. In the derivation of this relation, the evanescent wave modes have been neglected. Due to source-receiver reciprocity, we may replace $R\left(\boldsymbol{x}_B, \boldsymbol{x}_A, -t\right)$ in equation (1) with $R\left(\boldsymbol{x}_A, \boldsymbol{x}_B, -t\right)$. In this form, equation (1) states that the reflection response and its time-reversed version measured at $\boldsymbol{x}_A$ in the presence of a source at $\boldsymbol{x}_B$ equals minus the cross-correlation between the transmission responses measured at $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$ in the presence of all sources on the surface $\partial\mathcal{D}_m$ plus a delta function. Since the reflection response is a causal function, it can be obtained by taking the causal part of the left side of equation (1).

In practice, the sources are not distributed continuously over a certain surface. That is why, we can write the integral in equation (1) as a discrete sum

$$R\left(\boldsymbol{x}_A, \boldsymbol{x}_B, t\right) + R\left(\boldsymbol{x}_A, \boldsymbol{x}_B, -t\right) = \delta\left(\boldsymbol{x}_{H,B} - \boldsymbol{x}_{H,A}\right)\delta\left(t\right) - \sum_{\boldsymbol{x}_i \in \partial\mathcal{D}_m} T\left(\boldsymbol{x}_A, \boldsymbol{x}_i, -t\right) * T\left(\boldsymbol{x}_B, \boldsymbol{x}_i, t\right), \tag{2}$$

where $\boldsymbol{x}_i$ denotes the position vector of the sources in the subsurface. The exchange of the continuous source distribution in relation (1) with the discrete source distribution in relation (2) implies that to compute a good approximation of the reflection response we need to have many sources.

Let us now assume that we have in the subsurface uncorrelated white noise sources with source signatures $N_i\left(t\right)$. The transmission response from a source in the subsurface measured at the surface is $T\left(\boldsymbol{x}_A, \boldsymbol{x}_i, t\right) * N_i\left(t\right)$. Then, the cross-correlation of two transmission responses can be written as

$$T\left(\boldsymbol{x}_A, \boldsymbol{x}_i, -t\right) * N_i\left(-t\right) * T\left(\boldsymbol{x}_B, \boldsymbol{x}_j, t\right) * N_j\left(t\right) = \delta_{ij} T\left(\boldsymbol{x}_A, \boldsymbol{x}_i, -t\right) * T\left(\boldsymbol{x}_B, \boldsymbol{x}_j, t\right). \tag{3}$$

Using relation (3), equation (2) can be rewritten as

$$R(\boldsymbol{x}_A, \boldsymbol{x}_B, t) + R(\boldsymbol{x}_A, \boldsymbol{x}_B, -t) = \delta(\boldsymbol{x}_{H,B} - \boldsymbol{x}_{H,A})\,\delta(t) - \sum_{\boldsymbol{x}_i \in \partial\mathcal{D}_m} \sum_{\boldsymbol{x}_j \in \partial\mathcal{D}_m} T(\boldsymbol{x}_A, \boldsymbol{x}_i, -t) * N_i(-t) * T(\boldsymbol{x}_B, \boldsymbol{x}_j, t) * N_j(t)\,. \quad (4)$$

Writing

$$T_{obs}(\boldsymbol{x}_A, -t) = \sum_{\boldsymbol{x}_i \in \partial\mathcal{D}_m} T(\boldsymbol{x}_A, \boldsymbol{x}_i, -t) * N_i(-t) \quad (5)$$

$$T_{obs}(\boldsymbol{x}_B, t) = \sum_{\boldsymbol{x}_j \in \partial\mathcal{D}_m} T(\boldsymbol{x}_B, \boldsymbol{x}_j, t) * N_j(t) \quad (6)$$

we can finally write equation (4) as

$$R(\boldsymbol{x}_A, \boldsymbol{x}_B, t) + R(\boldsymbol{x}_A, \boldsymbol{x}_B, -t) = \delta(\boldsymbol{x}_{H,B} - \boldsymbol{x}_{H,A})\,\delta(t) - T_{obs}(\boldsymbol{x}_A, -t) * T_{obs}(\boldsymbol{x}_B, t)\,. \quad (7)$$

In relation (7), $T_{obs}(\boldsymbol{x}_A, t)$ can be seen as the transmission response observed at $\boldsymbol{x}_A$ on the surface $\partial\mathcal{D}_0$ due to discretely distributed uncorrelated white noise sources at a number of positions $\boldsymbol{x}_i$ in the subsurface on $\partial\mathcal{D}_m$ (see figure 1 (c)).

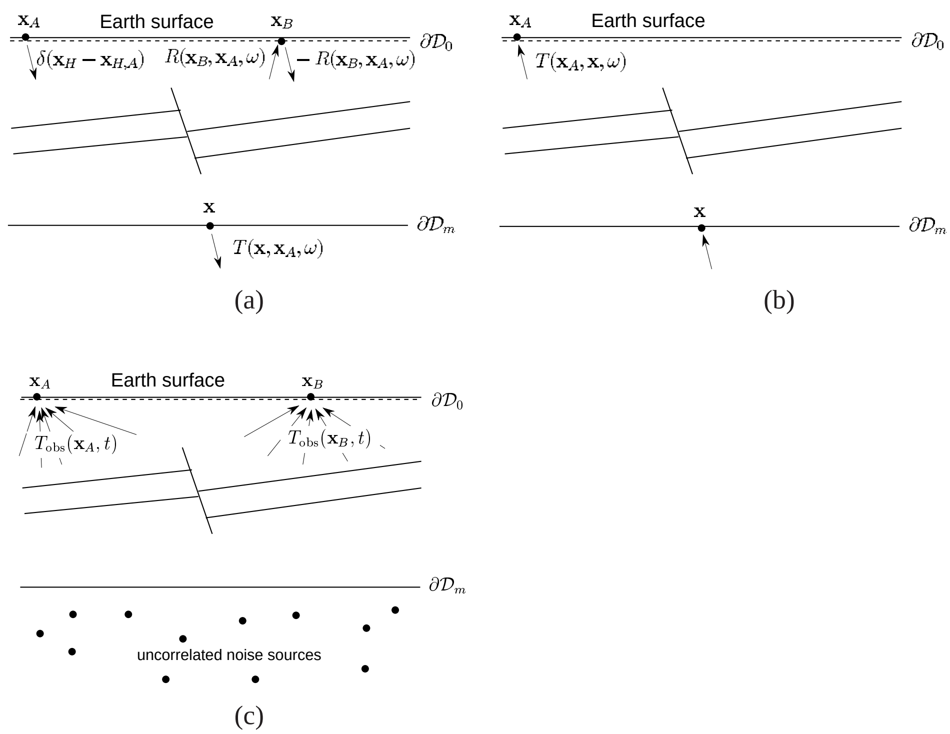
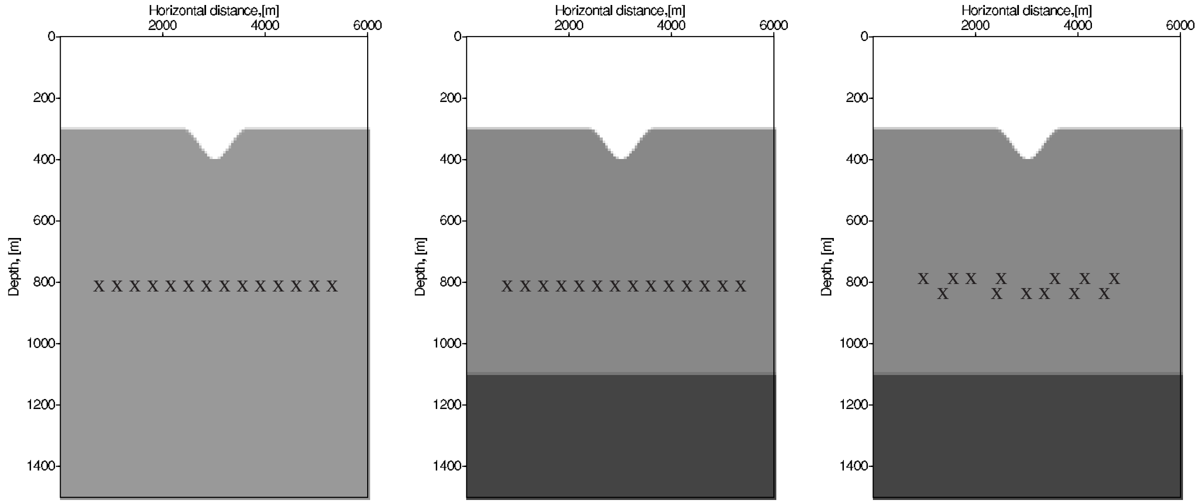



Figure 1: *(a) Domain $\mathcal{D}$ with its reflection response observed at the surface and with its transmission response observed in the subsurface. (b) Domain $\mathcal{D}$ with its transmission response observed at the surface. (c) Transmission response recorded at positions $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$ in the presence of white noise sources in the subsurface. The cross-correlation of these registrations yields the reflection response of the subsurface as it would have been recorded in the presence of an acoustic seismic source at $\boldsymbol{x}_B$ and a receiver at $\boldsymbol{x}_A$, see equation (7).*

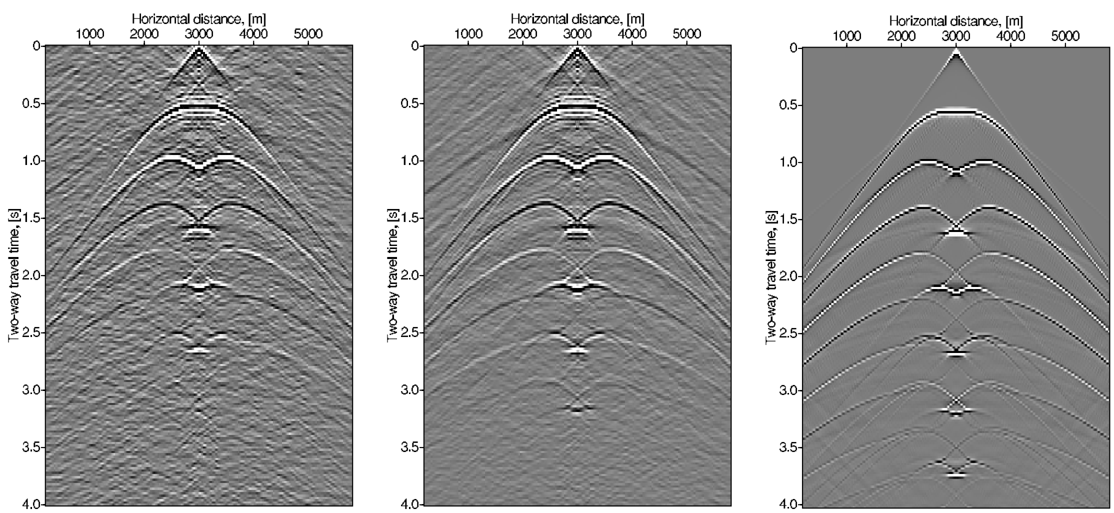
In the following, we are showing several 2-D numerical modeling results calculated using relation (7). Figure 2 (a) shows a syncline model. For this model, short transmission response recordings with a duration of 4 seconds ($T(\boldsymbol{x}, \boldsymbol{x}_i, t)$) were created, using finite difference modeling, for 225 sources with positions at 800 m depth. After that, each of the transmission responses was convolved with a different long white noise record. The created transmission panels in the presence of white noise sources ($T(\boldsymbol{x}, \boldsymbol{x}_i, t) * N_i(t)$) were summed giving $T_{obs}(\boldsymbol{x}, t)$. At the end, the $T_{obs}(\boldsymbol{x}, t)$ panel was correlated with one of its traces $T_{obs}(\boldsymbol{x}_A, t)$. In the examples, the correlation was done with trace at $\boldsymbol{x}_{H,A} = 3000$ m. The quality of the simulated reflection response depends on how good the conditions for relation

(3) are fulfilled. The longer the recording time the better the result, as the white noise sources become less correlated. Figures 3 (a) and (b) show the simulated reflection response for recording times of $T_{obs}(\boldsymbol{x}, t)$ of 10 minutes and 66 minutes, respectively. Figure 3 (c) shows for comparison the directly modeled reflection response.




(a) (b) (c)

Figure 2: *(a) Syncline model with sources at 800 m depth; (b) Syncline model with sources at 800 m depth and a reflector below the sources at 1100 m depth; (c) Syncline model with sources with randomly distributed depths between 725 m and 875 m.*




(a) (b) (c)

Figure 3: *(a) Simulated reflection response for recording times of $T_{obs}(\boldsymbol{x}, t)$ of 10 minutes; (b) Simulated reflection response for recording times of $T_{obs}(\boldsymbol{x}, t)$ of 66 minutes; (c) Directly modeled reflection response.*

In the derivation of the relation (7) it was assumed that below the sources we have a homogeneous half space. In the presence of reflectors below the sources, additional events will appear in the transmission response recordings that will cause ghost events in the simulated reflection response. This can be seen on figure 4 (a), which shows the simulated reflection response for the model in figure 2 (b). The figure shows a syncline model with a reflector at 1100 meters depth, the sources are at 800 meters depth. As a result of this reflector, there are two ghost events that can be seen on figure 4 (a) with apexes at 0.22 seconds and at 0.78 seconds, respectively. On 4 (b) is shown the simulated reflection response for the model in figure 2 (c) with sources with randomly distributed depths between 725 and 875 meters. The ghost events, mentioned above, are weakened, while all the other reflections are correctly mapped. Figure 4 (c) shows for comparison the directly modeled reflection response.

To compute the correct amplitudes of the simulated reflection response the transmission response needs to be flux-normalized, since in the one-way reciprocity theorem of the correlation type, used to derive equation (7), the one-way wave fields were flux-normalized. Further, as we deal with discrete sums, the correlated transmission traces need to be normalized to their variance [3]:

$$T\left(\boldsymbol{x}_A, \boldsymbol{x}_i, -t\right) * T\left(\boldsymbol{x}_B, \boldsymbol{x}_j, t\right) = \frac{cov\left(T\left(\boldsymbol{x}_B, \boldsymbol{x}_j, t\right), T\left(\boldsymbol{x}_A, \boldsymbol{x}_i, -t\right)\right)}{\left\{var\left(T\left(\boldsymbol{x}_B, \boldsymbol{x}_j, t\right)\right) var\left(T\left(\boldsymbol{x}_A, \boldsymbol{x}_i, -t\right)\right)\right\}^{\frac{1}{2}}} \tag{8}$$

## Conclusion

The results from the numerical modeling shown here confirm relation (7) between the reflection and the transmission response of a 3-D inhomogeneous lossless medium in the presence of white noise sources. By cross-correlating noise recordings at two positions at the surface, we obtain the simulated reflection response, as if the source were at one position and the receiver at the other. The longer the recording time of the noise signals the better the simulated reflection response will be. In the presence of reflectors below the sources, ghost events appear in the simulated reflection response. However, white noise sources with randomly distributed depths weaken theses ghost events.

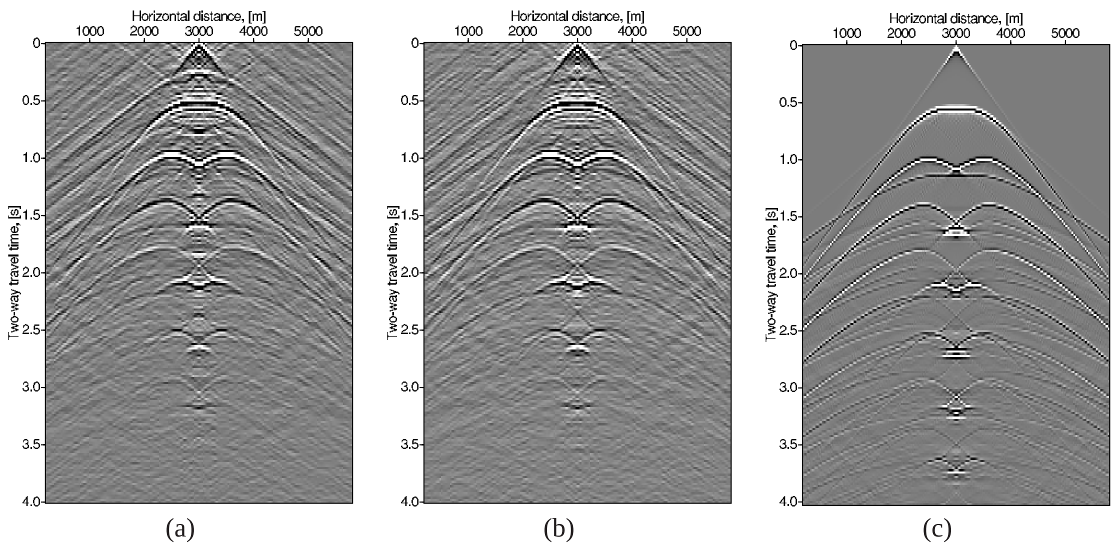



Figure 4: *(a) Simulated reflection response with sources at 800 meters and a reflector at 1100 meters; (b) Simulated reflection response with sources with randomly distributed depths between 725 and 875 meters; (c) Directly modeled reflection response.*

## References

[1] J.F. Claerbout. Synthesis of a layered medium from its acoustic transmission response. *Geophysics*, 33:264–269, 1968.

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[3] M. B. Priestley. Spectral analysis and time series. *Academic press*, 1, 1981.